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ELEMENTS OF MECHANISM.

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PREFACE.

THE main subject-matter of this work was written during 1885 by Peter Schwamb and has been used since then, in the form of printed notes, at the Massachusetts Institute of Technology, as a basis for instruction in mechanism, being followed by a study of the mechanism of machine tools and of cotton machinery. The notes were written because a suitable text-book could not be found which would enable the required instruction to be given in the time available. They have accomplished the desired result, and numerous inquiries have been received for copies from various institutions and individuals desiring to use them as text-books. This outside demand, coupled with a desire to revise the notes, making such changes and additions as experience has proved advisable, is the reason for publishing at this time.

Very little claim is made as to originality of the subject-matter which has been so fully covered by previous writers. Such available matter has been used as appeared best to accomplish the object desired. Claim for consideration rests largely on the manner of presenting the subject, which we have endeavored to make systematic, clear, and practical.

Among the works consulted and to which we are indebted for suggestions and illustrations are the following: "Kinematics of Machinery" and "Der Konstrukteur," by F. Reuleaux, the former for the discussion of linkages, and the latter for various illustrations of mechanisms; "Principles of Mechanism," by S. W. Robinson, for the discussion of non-circular wheels; "Kinematics," by C. W. MacCord, for the discussion of annular wheels and screw-gearing; "Machinery and Millwork," by Rankine; "Elements of Mechanism," by T. M. Goodeve; and "Elements of Machine Design," by W. C. Unwin.

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ELEMENTS OF MECHANISM.

CHAPTER I.

INTRODUCTION.

1. The science of Mechanism treats of the designing and construction of machinery.

2. **A Machine** is a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to produce some effect or work accompanied with certain determinate motions.* In general, it may be properly said that a machine is an assemblage of moving parts interposed between the source of power and the work, for the purpose of adapting the one to the other.

No machine can move itself, nor can it create motive power; this must be derived from external sources, such as the force of gravitation, the uncoiling of a spring, or the expansion of steam. As an example of a machine commonly met with, an engine might be mentioned. It is able to do certain definite work, provided some external force shall act upon it, setting the working parts in motion. We shall find that it consists of a fixed frame, supporting the moving parts, some of which cause the rotation of the engine shaft, others move the valves distributing the steam to the cylinder, and still others operate the governor which controls the engine. These moving parts will be so arranged that they make certain definite motions relative to each other when an external force, as steam, is applied to the piston.

3. The operation of any machine depends upon two things: first, the transmission of certain forces, and second, the production of determinate motions. In designing, due consideration must be given to both of these, so that each part may be adapted to bear the stresses imposed on it, as well as have the proper relative motion in regard to the other parts of the machine. But the nature of the movements does not depend upon the strength or absolute dimensions of the moving parts, as can be shown

* Reuleaux: Kinematics of Machinery.

by models whose dimensions may vary much from those requisite for strength, and yet the motions of the parts will be the same as those of the machine. Therefore, the *force* and the *motion* may be considered separately, thus dividing the science of Mechanism into two parts, viz.:

1° **Pure Mechanism,** which treats of the motions and forms of the parts of a machine, and the manner of supporting and guiding them, independent of their strength.

2° **Constructive Mechanism,** which involves the calculation of the forces acting on different parts of the machine; the selection of materials as to strength and durability in order to withstand these forces, taking into account the convenience for repairs, and facilities for manufacture.

In what follows, we shall, in general, confine ourselves to the first part, *pure mechanism,* or what is sometimes called "the geometry of machinery"; but shall in some cases consider the forces in action.

Then our definition of a machine might be modified to accord with the above, as follows:

A Machine is an assemblage of moving parts so connected that when the first, or recipient, has a certain motion, the parts where the work is done, or effect produced, will have certain other definite motions.

4. **A Mechanism** is a term applied to a portion of a machine where two or more pieces are combined, so that the motion of the first compels the motion of the others, according to a law depending on the nature of the combination. For example, the combination of a crank and connecting-rod with guides and frame, in a steam engine, serving to convert reciprocating into circular motion, would thus be called a *mechanism.*

The term *Elementary Combination* is sometimes used synonymously with *A Mechanism.*

A machine is made up of a series or train of mechanisms.

5. **Motion and Rest** are necessarily relative terms within the limits of our knowledge. We may conceive a body as fixed in space, but we cannot know that there is one so fixed. If two bodies, both moving in space, remain in the same relative position in regard to each other, they are said to be at rest, one relatively to the other; if they do not, either may be said to be in motion relatively to the other.

Motion may thus be either *relative,* or it may be *absolute,* provided we assume some point as fixed. In what follows, the earth will be assumed to be at rest, and all motions referred to it will be considered as *absolute.*

Path.—A point moving in space describes a line called its path, which may be rectilinear or curvilinear. The motion of a body is determined by the paths of three of its points selected at pleasure. If the motion is in a plane, two points suffice, and if rectilinear, one point suffices to determine the motion.

Direction.—In a given path, a point can move in either of two directions only, which may be designated in various ways: as up, +, or down, −; to the right, +, or left, −; with the clock, +, or the reverse, −; direction, as well as motion, being relative.

6. Continuous Motion.—When a point goes on moving indefinitely in a given path in the same direction, its motion is said to be *continuous*. In this case the path must return on itself, as a circle or other closed curve. A wheel turning on its bearings affords an example of this motion.

7. Reciprocating Motion.—When a point traverses the same path and reverses its motion at the ends of such path the motion is said to be *reciprocating.*

Vibration and **Oscillation** are terms applied to *reciprocating circular motion*, as that of a pendulum.

8. Intermittent Motion.—When the motion of a point is interrupted by periods of rest, its motion is said to be *intermittent.*

9. Revolution and Rotation.—A point is said to *revolve* about an axis when it describes a circle of which the centre is in, and the plane is perpendicular to, that axis. When all the points of a body thus move the body is said to revolve about the axis. If the axis passes through the body, as in the case of a wheel, the word *rotation* is used synonymously with *revolution*. It frequently occurs that a body not only rotates about an axis passing through itself, but also moves in an orbit about another axis. In order to make the distinction between the two motions more clear, we shall consider the first as a *rotation*, and the second as a *revolution;* just as we say, the earth rotates on its axis and revolves around the sun.

An Axis of Rotation is a line whose direction is not changed by the rotation; a *fixed axis* is one whose position, as well as its direction, remains unchanged.

A Plane of Rotation is a plane perpendicular to the axis of rotation.

Right-handed Rotation is the same in direction as the motion of the hands of a watch, and is generally considered to be positive. *Left-handed rotation* is in the opposite direction and is consequently considered as negative.

10. Cycle of Motions.—When a mechanism is set in motion and its parts go through a series of movements which are repeated over and over, the relations between and order of the different divisions of the series being the same for each repetition, we have in one of these series what is called a *cycle of motions*. For example, one revolution of the crank of a steam engine causes a series of different positions of the piston-rod, and this series of positions is repeated over and over for each revolution of the crank.

The Period of a motion is the interval of time elapsing between two successive passages of a point through the same position in the same direction.

11. Driver and Follower.—That piece of a mechanism which is supposed to cause motion is called the *driver*, and the one whose motion is effected is called the *follower*.

12. Frame.—The frame of a machine is a structure that supports the moving parts and regulates the path, or kind of motion, of many of them directly. In discussing the motions of the moving parts, it is convenient to refer them to the frame, even though it may have, as in the locomotive, a motion of its own.

13. Velocity.—*Velocity* is the rate of motion of a point in space. When the motion is referred to a point in the path of the body its velocity is expressed in linear measure. When the point is rotating continuously, or for the instant, about some axis, its motion may be referred to the axis when its velocity is expressed in angular measure. In the first case it has *linear velocity* and in the second case *angular velocity*.

Velocity is *uniform* when equal spaces are passed over in equal times, however small the intervals into which the time is divided. The velocity in this case is the space passed over in a unit of time, and if s represent the space passed over in the time t, the velocity v will be

$$v=\frac{s}{t}. \qquad (1)$$

Velocity is *variable* when unequal spaces are passed over in equal intervals of time, increasing spaces giving *accelerated* motion and decreasing spaces giving *retarded* motion. The velocity when variable is the limit of the space passed over in a small interval of time, divided by the time, when these intervals of time become infinitely small. If Δs represent the space passed over in the time Δt, then

$$v=\text{limit of } \frac{\Delta s}{\Delta t} \text{ as } \Delta t \text{ diminishes indefinitely,}$$

or

$$v=\frac{ds}{dt}. \qquad (2)$$

The uniform linear velocity of a point is measured by the number of units of linear distance passed over in a unit of time, as feet per minute, inches per second, etc. When the velocity is variable it is measured by the distance which would be passed over in a unit of time, if the point retained throughout that time the velocity which it had at the instant considered.

14. Angular Velocity.—The *angular velocity* of a point is measured by the number of units of angular space which would be swept over in a unit of time by a line joining the given point with a point outside of its path, about which the angular velocity is desired. The angular space is here measured by circular measure, or the ratio of the arc to the radius. The unit angle, or *radian*, is one subtended by an arc equal to the radius. The angular velocity of a point is therefore expressed in radians per unit of time. (In what follows l.v. will be used to designate linear velocity and a.v. angular velocity, for the sake of brevity.)

Inasmuch as in circular motion the linear velocity of the point is the velocity along the arc, we may write:

$$\text{a.v.} = \frac{\text{l.v.}}{\text{radius}}, \quad \ldots\ldots\ldots\ldots \quad (3)$$

from which

$$\text{l.v.} = \text{a.v.} \times \text{radius}. \quad \ldots\ldots\ldots\ldots \quad (4)$$

Thus when the a.v. remains the same, the l.v. is directly proportional to the radius. For example, given a line of shafting with pulleys of various diameters, the a.v. of all the pulleys is the same, while the l.v's of points in the rims of the pulleys are directly proportional to the respective radii. If N represents the number of revolutions per minute (r.p.m.) and R represents the radius of one of the pulleys in feet, we have the a.v. equal to $2\pi N$ radians per minute, while the l.v. of the rim of the pulley would be $2\pi NR$ feet per minute (f.p.m.).

15. Modes of Transmission.—If we leave out of account the action of natural forces of attraction and repulsion, such as magnetism, one piece cannot move another, unless the two are in contact or are connected to each other by some intervening body that is capable of communicating the motion of the one to the other. In the latter case, the motion of the connector is usually unimportant, as the action of the combination as a whole depends upon the relative motion of the connected pieces. Thus motion can be transmitted from driver to follower:

1° By direct contact.

2° By intermediate connectors.

16. Links and Bands.—An intermediate connector can be rigid or flexible. When rigid it is called a *link*, and it can either push or pull, such as the connecting-rod of a steam engine. Pivots or other joints are necessary to connect the link to the driver and follower.

If the connector is flexible, it is called a *band*, which is supposed to be inextensible, and only capable of transmitting a pull. A fluid confined in a suitable receptacle may also serve as a connector, as in the hydraulic press. Here we might call the fluid a pressure-organ in distinction from the band, which is a tension-organ.

17. Modification of Motion.—In the action of a mechanism the motion of the follower may differ from that of the driver in kind, in velocity, in direction, or in all three. As the paths of motion of the driver and follower depend upon the connections with the frame of the machine, the change of motion in kind is fixed, and it only remains for us to determine the relations of direction and velocity throughout the motion. Now the laws governing the changes in direction and velocity can be determined by comparing the movements of the two pieces at each instant of their action, and the mode of action will fix the laws. Therefore, whatever the nature of the combination, if we can determine throughout the motion of the driver and follower, the *velocity ratio*, and *directional relation*, the analysis will be complete.

Either the velocity ratio or the directional relation may vary, or remain the same throughout the action of the two pieces.

CHAPTER II.

COMPOSITION AND RESOLUTION OF VELOCITIES.—MOTIONS OF RIGIDLY-CONNECTED POINTS.—INSTANTANEOUS AXIS.—CENTROIDS.

18. **Graphic Representation of Motion.**—We can represent the motion of a point in any given piece of mechanism, graphically, by a right line whose length in units indicates the velocity, and whose direction indicates the direction of motion of the point at the instant considered; an arrow-head is used to indicate the direction in which the point is moving. If the path of the moving point be a curve of any kind, the direction of the curve at any point is that of its tangent at that point, which indicates the direction of motion as well.

19. **Resultant.**—If a material point receives a single impulse in any direction, it will move in that direction with a certain velocity. If it receives at the same instant two impulses in different directions, it will obey both, and move in an intermediate direction with a velocity differing from that of either impulse alone. The position of the point at the end of the instant is the same as it would have been had the motions, due to the impulses, occurred in successive instants. This would also be true for more than two motions. The motion which occurs as a consequence of two or more impulses is called the *Resultant*, and the separate motions, which the impulses acting singly would have caused, are called the *Components*.

20. **Parallelogram of Motion.**—Suppose the point a (Fig. 1) to have simultaneously the two component motions represented in magnitude and direction by ab and ac. Then the resultant is ad, the diagonal of the parallelogram of which the component motions ab and ac are the sides. Conversely, the motion ad may be resolved into two components, one along ab, and the other along ac (Fig. 1), by drawing the parallelogram $abdc$, of which it will be the diagonal.

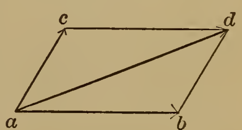


Fig. 1.

Any two component motions can have but one resultant, but a given resultant motion may have an infinite number of pairs of components.

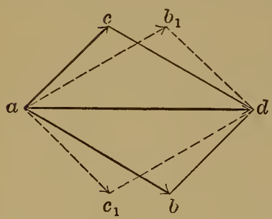


FIG. 2.

In the latter case we have a definite solution provided we know the direction of both components, or the magnitude and direction of one. If we know the magnitude of both components, there are two possible solutions. Thus in Fig. 2, where *ad* is the given resultant, if the two components have the magnitudes represented by *ac* and *ab*, the directions *ac* and *ab* would solve the problem, or the directions ac_1 and ab_1 would equally well fulfil the conditions.

It very often happens that we wish to resolve a motion into two components, one of which is perpendicular, and the other parallel, to a given line, as *ef* (Fig. 3). Here *ad* represents the motion; $ab = ad \cos dab$, the component parallel to *ef*; and $ac = ad \sin dab$, the component perpendicular to *ef*.

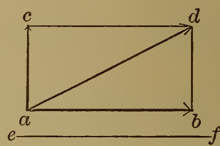


FIG. 3.

21. **Parallelopiped of Motions.**—If the three component motions *ab*, *ac*, and *ad* (Fig. 4) are combined, their resultant *af* will be the diagonal of the parallelopiped of which they are the edges. The motions *ab* and *ac*, being in the same plane, can be combined to form the resultant *ae*; in the same way *ae* and *ad* can be combined, giving the resultant *af*. Conversely the motion *af* may be resolved into the components *ab*, *ac*, and *ad*.

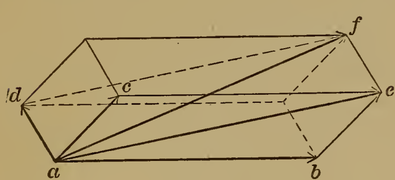


FIG. 4.

If the parallelopiped is rectangular, the case is simpler, and often used; then we have

$$\overline{af}^2 = \overline{ae}^2 + \overline{ad}^2 = \overline{ab}^2 + \overline{ac}^2 + \overline{ad}^2$$

To find the resultant of any number of motions: First, combine any two of them and find their resultant; then, combine this resultant with the third, thus obtaining a new resultant, which can be combined with the fourth; and so on.

22. **Velocities of Rigidly-connected Points.**—If two points are so connected that their distance apart is invariable and if their velocities are resolved into components at right angles to and along the straight line connecting them, the components along this line of connection must be equal, otherwise the distance between the points would change.

In Fig. 5 let *a* and *b* be two rigidly-connected points having the l.v. of *a* represented in magnitude and direction by aa_1 and the l.v. of *b* in direction by bb_1. The components of aa_1 perpendicular to and along ab are ac and ad respectively. The component ad will represent the entire tendency of translation of the line ab in the direction ab due to the l.v. aa_1 at the point a. Since the points a and b are rigidly connected, the l.v. of any point in the line ab must be such that when resolved into components perpendicular to and along ab the component along ab shall be equal to ad. Therefore the l.v. of b must be bb_1, since be must be equal to ad. In the figure the motions are shown in one plane, but the proposition is also true for motions not in one plane.

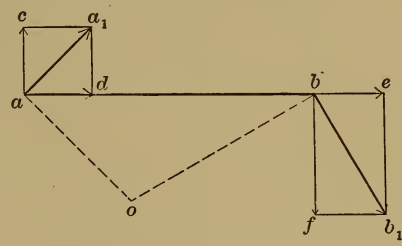


FIG. 5.

For example, in the series of links shown in Fig. 6, c and d are fixed axes and f slides on the line ff_1. If aa_1 represents the l.v. of a, the component of translation along ab will be am, to which the component bn must be equal. Therefore bb_1 will represent the l.v. of b, where bb_1 is

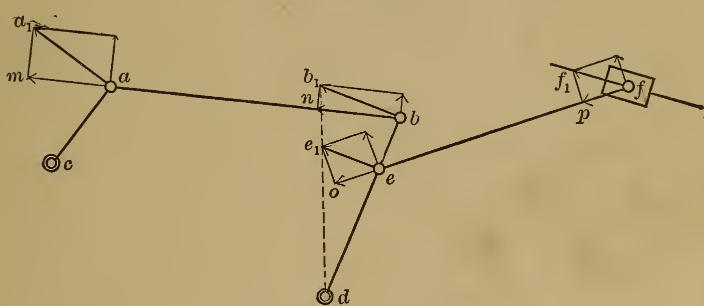


FIG. 6.

tangent to the path of b in the given position. The l.v. of e will be ee_1, where ee_1 is tangent to the path of e in the given position, and where $ee_1 : bb_1 = de : db$, since in any rotating body the l.v's of any points are proportional to their respective distances from the axis.

To find the l.v. of f we have the l.v. of the point e in ef represented by ee_1; therefore the component of translation along ef will be eo. The component fp must be equal to eo, which gives ff_1 as the resulting l.v. of f.

23. Instantaneous Axis.—If a line ao (Fig. 5) is drawn through a perpendicular to the direction of motion aa_1 of the point a, then the motion of a may be the result of a rotation about an axis through any point in the line ao or in ao produced. Similarly, the motion bb_1 may be the result of a rotation about an axis through any point in bo. If a and b are rigidly connected, the piece on which they are situated must have a rotation about one axis, and the a.v. of all points about that axis must be the same. The only point satisfying this condition is o, at the intersection of ao and bo, and the piece ab has a motion at that instant such as it would have if it were rotating about an axis through o. The axis through o, perpendicular to the plane of the motions, is called the *instantaneous axis*, it being the axis about which the body is rotating for the instant in question.

The a.v. about the instantaneous axis being the same for the instant, for the points a and b, the l.v's of a and b will be proportional to their distances from the instantaneous axis;

$$\therefore \; aa_1 : bb_1 = oa : ob.$$

If the motions of the points a and b are not in the same plane, the instantaneous axis would be found as follows: Pass a plane through the point a perpendicular to aa_1; the motion aa_1 might then be the result of a revolution of a about any axis in that plane. In the same manner, the motion of bb_1 might be the result of a revolution of b about any axis in the perpendicular plane through b. The points a and b, being rigidly connected, must rotate about one axis, which in this case will be the intersection of the two perpendicular planes.

Suppose the motions of the two points a and b to be in the same plane and parallel, as in Figs. 7 and 8. Here the perpendiculars through a and b coincide and the above method fails. Let aa_1 and bb_1 be the l.v's of the points a and b respectively. To find the instantaneous axis draw a right line through the points a_1 and b_1 in each case and note the point o where it intersects ab or ab produced. This must be the instantaneous axis, for from the similar triangles aa_1o and bb_1o we have

$$aa_1 : bb_1 = oa : ob,$$

the same equation as was obtained before.

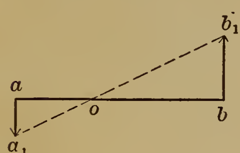


FIG. 7.

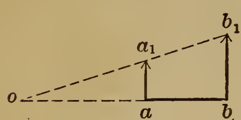


FIG. 8.

In the solid, illustrated by Fig. 9, given the l.v. of a in magnitude and direction aa_1, also the direction of the l.v. of b, bb_1, to find the l.v. of the point c. Knowing the directions of the motions of a and b at the given moment we find the instantaneous centre at o; therefore the direction of the motion of c must be cc_1. To find the magnitude of the l.v. of c we have

$$cc_1 : aa_1 = co : ao.$$

Or we may determine cc_1 by finding the component of aa_1 along ac, which will be ae, and the component of cc_1 along ac must be the same or cf. There is still another method of solution, not using the instantaneous centre. We find that the l.v. of c must have a component cf along ac, its other component being perpendicular to ac; but after determining the l.v. of b, bb_1, we find that we must also be able to resolve the l.v. of c into rectangular components, one of which, ck, shall be along bc, and

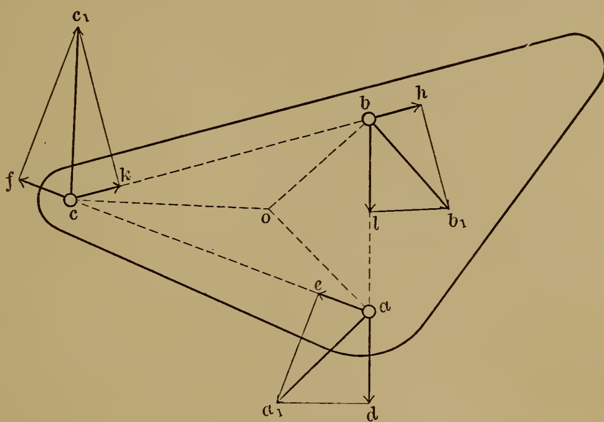


FIG. 9.

equal to bh, the component of bb_1 along bc. Drawing perpendiculars from f and k to ac and bc respectively, their intersection c_1 will give the l.v. of c, cc_1, which will answer the above requirements.

24. **Instantaneous Axis of Rolling Bodies.**—The instantaneous axis of a rigid body which rolls without slipping upon the surface of another rigid body must pass through all the points in which the two bodies touch each other; for the points in the rolling body which touch the fixed body at any given instant must be at rest for the instant, and must, therefore, be in the instantaneous axis. As the instantaneous axis is a straight line, it follows that rolling surfaces which touch each other in more than one point must have all their points of contact in the same straight line in order that no slipping may occur between them. This property is possessed by plane, cylindrical, and conical surfaces only; the terms *cylindrical* and *conical* being used in a general sense, the bases of the cylinders and cones having any figure as well as circles.

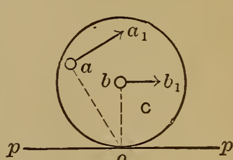


FIG. 10.

Let Fig. 10 represent a section of the rolling surfaces by a plane perpendicular to their straight line of contact, and assume pp as fixed; then o is a point in the instantaneous axis, as it is for the instant at rest, and all points on C, as a and b, are rotating about it for the instant. To find the direction of motion of any point, as a, draw ao, and through a perpendicular to ao draw aa_1, which is the direction of motion of a

for the instant. The linear velocities of a and b are proportional to their distances from o, the instantaneous axis.

25. Motion of Translation.—If, in Fig. 8, the two parallel motions aa_1 and bb_1 become equal to each other, then ob will be infinite and the consecutive positions of ab will be parallel to each other. This is also true if the motions are at any angle with ab, so long as they are equal and parallel, as in Fig. 11.

The motion of a line, or of a body containing that line, at any instant when it is thus revolving about an axis at an infinite distance, is called *translation*. All points in such a body move in the same direction with the same velocity; the paths of the points may be rectilinear or curvilinear. Straight or rectilinear translation is commonly called *sliding*. As an example of straight translation, we have the cross-head of a steam engine; of curvilinear translation, the parallel-rod of a locomotive.

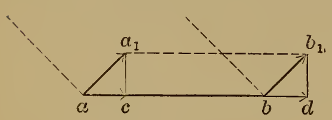


FIG. 11.

26. Periodic Centre of Motion.—It very often happens that we know two positions of a line, as ab and a_1b_1 (Fig. 12), moving in the plane of the paper, and we wish to find an axis about which this line could revolve to occupy the two given positions. Draw aa_1 and bb_1, and find the intersection o of the perpendiculars drawn at their middle points. Thus ab can be brought to the position a_1b_1 by revolving it about an axis through o perpendicular to the plane of the paper, the paths of a and b being arcs of circles drawn from o as a centre, and with radii equal to oa and ob respectively.

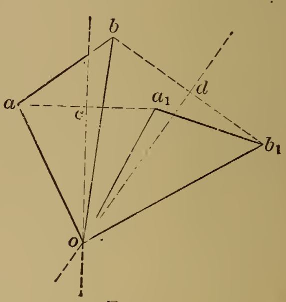


FIG. 12.

When the two positions of ab are taken infinitely near each other, o becomes the instantaneous centre.

27. Centroid.—The curve passing through the successive positions of the instantaneous centre of a body having a combined motion of rotation and translation is called a *centroid*. The surface formed by the successive positions of the instantaneous axis is called an *axoid*.

Suppose we know the relative motions of two links as ab and cd in the mechanism given in Fig. 13, where the motion of ab relative to cd, cd being considered as fixed, is such that a moves in the path a_3aa_2, and b moves in the path b_3bb_2, a_3b_3, a_2b_2, etc., being positions of ab. If in any of these positions, as ab, we draw from a and b normals to their respective paths, their intersection o will be the instantaneous centre of ab for that position. A smooth curve passed through the successive positions of the instantaneous centre, o, o_1, o_2, etc., will be the centroid of ab. In § 24 we saw that the instantaneous axis of one body rolling on another was at their point of contact. From this it would follow that, considering one body as fixed relative to the other, its surface would be the axoid of the moving body. Therefore, in Fig. 13, the axoid of ab, which is represented by the centroid o_1oo_2, may be taken as the surface of a fixed body, containing dc, on which the surface of a moving body, containing ab, shall be able to roll, giving the same motion to ab as the original links would give.

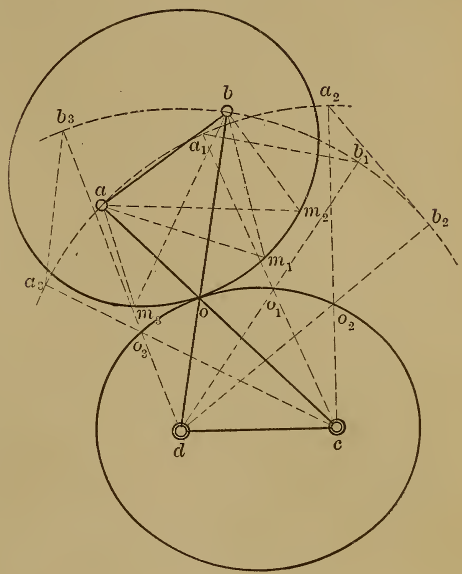


Fig. 13.

To find the trace of the surface of the body containing ab, we have in each of the positions which it may occupy, distances from a and b to its instantaneous centre for that position, which distances are, therefore, distances from a and b to a point in the trace of the surface of the body containing ab. Thus am_1 and bm_1 are equal respectively to a_1o_1 and b_1o_1; similarly am_2 and bm_2 are equal respectively to a_2o_2 and b_2o_2. A smooth curve through these points om_1m_2, etc., would give the trace of the surface of the body containing ab. It will also be found that this curve om_1m_2 is the centroid of cd relative to ab, when ab is assumed fixed.

CHAPTER III.

PAIRS OF ELEMENTS.—BEARINGS AND SCREWS.—WORM AND WHEEL.

28. **Pairs of Elements.**—In order that a moving body, as A (Fig. 14), may remain continually in contact with another body B, and at the same time move in a definite path, B would have a shape which could be found by allowing A to occupy a series of consecutive positions relative to B, and drawing the envelope of all these positions. Thus, if A were a parallelopiped, the figure of B would be that of a curved channel.

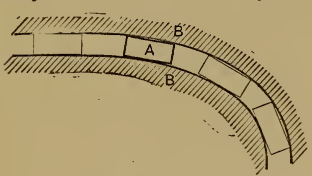


Fig. 14.

Therefore, in order to compel a body to move in a definite path, it must be paired with another, the shape of which is determined by the nature of the relative motion of the two bodies.

A machine consists of elements which are thus connected in pairs, the stationary element preventing every motion of the movable one except the single one desired.

Closed Pair.—If one element not only forms the envelope of the other, but encloses it, the forms of the elements being geometrically identical, the one being solid or full, and the other being hollow or open, we have what may be called a *closed pair*. The pair represented in Fig. 14 is not *closed*, as the elementary bodies A and B do not enclose each other in the above sense.

On the surfaces of two bodies forming a closed pair we may imagine coincident lines to be drawn, one on each surface; and if we suppose these lines to be such in form as will allow them to move along each other, that is, allow a certain motion of the two bodies paired, we shall find that only three forms can exist:

1° A straight line, which allows straight translation.

2° Among plane curves, or curves of two dimensions, a circle, which allows rotation.

3° Among curves of three dimensions, the helix, which allows a combination of rotation and straight translation.

29. Primary and Secondary Pieces.—In order to distinguish between pieces of a machine which are connected directly to the frame and those carried by other moving pieces, the former are called *primary*, and the latter *secondary pieces*.

Thus, if the connection of the primary pieces to the frame be by closed pairs of elements, the following determinate motions can be given to them:

1° Straight translation or sliding;
2° Rotation, motion in a circle, as a wheel on its axis;
3° A helical motion, which might be considered as a combination of 1° and 2°, as a screw.

30. Bearings are the surfaces of contact between the frame and the primary pieces, the name being applied to the surface of each piece; but these surfaces sometimes have distinctive names of their own.

The bearings of primary pieces may be arranged, according to the motions they will allow, in three classes:

1° For straight translation the bearings must have plane or cylindrical surfaces, cylindrical being understood in its most general sense. The surfaces of the moving pieces are called slides; those of the fixed pieces, slides or guides.

2° For rotation, or turning, they must have surfaces of revolution, as circular cylinders, cones, conoids, or flat disks. The surface of the solid or full piece is called a journal, neck, spindle, or pivot; that of the hollow or open piece, a journal, gudgeon, pedestal, plumber- or pillow-block, bush, or step.

3° For translation and rotation combined, or helical motion, they must have a helical or screw shape. Here the full piece is called a screw, and the open piece a nut.

It will be interesting to note the relation that the slide and journal bear to the screw, from which they might be considered as derived. If we suppose the pitch of a screw to be diminished until it becomes zero, or if we suppose the pitch angle to become zero, then the form *A* (Fig. 15) would be changed to that of *B*, which, with a modification of

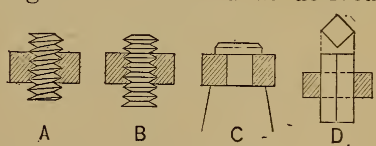


Fig. 15.

the thread outline, would become, like *C*, a common form for a journal. Thus, by making the pitch zero, the motion along the axis of the screw has been suppressed, and only rotation is possible for the nut. If we suppose the pitch angle to increase instead of diminish, the screw will become steeper and steeper. If the angle $=90°$, the screw-threads become parallel to the axis, the screw becomes a prism, and the nut a corresponding hollow prism, as Fig. 15, *D*. Here rotation is suppressed, and only sliding along the axis is possible, giving us the slide. If the angle be made >90°, the screw changes from a right- to a left-handed one, but still remains a screw.

It is very often the case that pulleys or wheels are to turn freely on their cylindrical shafts and at the same time have no motion along them; for this purpose, rings or collars (Fig. 16, *A*) are used, the collars *D* and *E*, held by set screws, prevent the motion of the pulley along the shaft but allow its free rotation. Sometimes pulleys or couplings must be free

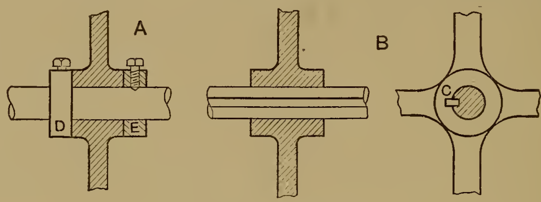


Fig. 16.

to slide along their shafts, but at the same time must turn with them; they must then be changed to a sliding pair. This is often done by fitting to the shaft and pulley or sliding piece a key *C* (Fig. 16, *B*), parallel to the axis of the shaft. The key may be made fast to either piece, the other having a groove in which it can freely slide. The above arrangement is very common, and is called a *feather and groove* or *spline*, or a *key and keyway.*

31. **Screw and Nut.**—A screw might be defined as a solid cylindrical body with a *thread* or projection of uniform section wound around it in successive equidistant coils or helices; a *nut* would be formed by winding the *thread* on the inside of a hollow cylinder. Either the screw or the nut may be the moving piece, the nut being the envelope of the screw in all cases.

The form of the section of a screw-thread varies with the use to which the screw is to be put; Fig. 17 shows some of the common forms.

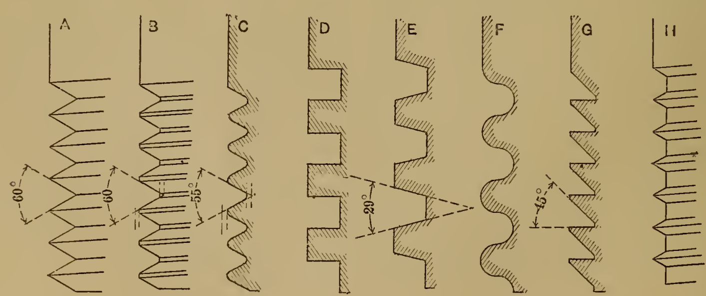


Fig. 17.

The most common form is shown at *A*, and is known as the V thread, its section being an equilateral triangle. As the sharp edges make the thread liable to injury, and less easy to construct, the modified forms *B* and *C* are much used.

Form *B*, known as the Sellers or United States standard, has the angle of the thread 60°, and one-eighth of the depth of the V cut off at the top and at the bottom; this makes a better screw, as more material is left between the bottoms of the threads, the very thin parts removed being of little use as bearing surfaces on account of their weakness. Form *C*, known as the Whitworth, or English standard, has the angle of the thread 55°, and one-sixth of the depth of the V is rounded off at the top and at the bottom of the thread.

As the resistance of pieces in sliding contact is normal to the bearing surfaces, there is a tendency in all V-shaped threads to burst the nut.

D shows the square-threaded screw, most commonly used to produce motion, as it has large wearing surfaces perpendicular to the motion given by the screw; it is, however, not so strong as *A*, as it has only one-half the shearing surface of *A* in a given length of the nut. *E* is a combination of *B* and *D*, used for screw gearing and the lead screws of engine-lathes. This thread with an angle of 29° is now known as the Acme standard. With ordinary pitches this angle will permit a clasp-nut to be used on the screw as in engine-lathes, which is not possible with *D*. A modified form of *D*, used for rough work, the screws being cast, is shown at *F*. In *G*, which is used where the force is always applied in the same direction, as in the breech-screws of large guns, the shearing strength of *A* is combined with the flat bearing surface of *D*. Lag-screws, used in wood, have the form of thread shown at *H*; here the wood is the weaker material and has the larger thread.

Right- and Left-handed Screws.—A screw is said to be right-handed or left-handed according as a right-handed or left-handed rotation is required to make it advance; and this is a permanent distinction. In Fig. 18, *R* shows a right-handed and *L* a left-handed screw; it will be noticed in *R* that when a right-handed screw is held vertically, the threads will rise from the left to the right on the visible side; in the left-handed screw the reverse is the case.

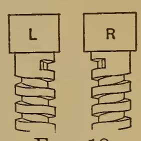


Fig. 18.

32. Screw-pitch.—The *pitch* of a screw is the distance it advances for one complete turn, and, in single-threaded screws, is measured by the distance between two similar points on successive threads measured on a line parallel to the axis of the screw. Such screws are commonly designated by the number of threads to the inch of length; that is, a screw of $\frac{1}{10}''$ pitch, or ten threads to the inch, is called a screw of ten threads per inch.

33. Multiple-threaded Screws.—If, instead of winding one thread around a cylinder, several equidistant threads are wound at the same time, taking care in the winding that the threads are kept the same distance apart, we shall have a multiple-threaded screw. If two threads are used, a double-, and if three threads, a triple-threaded, screw will result, and so on. By the above principle, the pitch can be greatly increased without necessarily increasing the size of the thread. Here the pitch is measured by the axial distance between two similar points on successive coils of the same thread, one point being found from the other by following the thread for one complete turn.

34. Velocity Ratio.—A screw may be used to produce motion in two ways:

1° The nut may be fixed, and the screw be made to turn by applying a force at the end of a lever, or on the circumference of a wheel attached to the screw. While the screw advances through a distance equal to the pitch, the point at which the force is applied describes one coil of a helix of equal pitch. If P represents the pitch of the screw, and R the shortest distance between the point of application of the force and the axis of the screw, called the lever-arm, the velocity ratio is

$$\frac{\sqrt{(\overline{2\pi R}^2+P^2)}}{P}.$$

2° The screw may simply rotate, and the nut may have a motion of translation in a straight line without turning. While the screw makes one turn, the nut will move through a distance equal to the pitch, and the point of application of the force will describe a circle of radius R; the velocity ratio is

$$\frac{2\pi R}{P}.$$

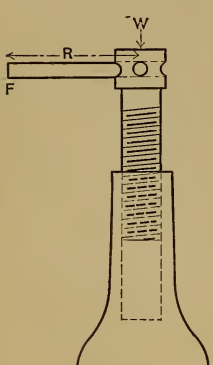


Fig. 19.

The latter form for the velocity ratio is, on account of its simplicity, used as an approximation to the first.

Either of the above combinations may be reversed, that is, the nut may be made to turn and the screw remain stationary in 1°, or have a straight translation in 2°. This does not change the velocity ratio.

For example, in the case of a simple jack-screw as in Fig. 19, if P is the pitch of the screw, and R the length of the lever-arm, we have

$$\frac{\text{l.v. of } F}{\text{l.v. of } W}=\frac{\sqrt{(\overline{2\pi R}^2+P^2)}}{P}=\frac{2\pi R}{P} \text{ nearly. } \quad \ldots \quad (5)$$

35. Compound or Differential Screws.—If A (Fig. 20) is a fixed nut carrying the screw S, and B is a movable nut, also on the screw

S, and free to slide along the guides GG, the pitches of the screw in A and B being P_1 and P_2 respectively, P_2 being smaller than P_1 and both threads being right-handed; we shall have for each turn of the screw in the direction of the arrow an advance of the screw S to the right equal to the pitch P_1. Meanwhile the nut B has moved relatively to the screw a distance P_2 to the left. The absolute motion of B is then to the right and equal to (P_1-P_2), the resultant of its motion relatively to S, and the motion of S. The same result would be obtained by supposing the nuts A and B to act in succession. Thus, suppose B fast to the screw and free to turn, then one turn of the screw in A would advance B a distance $+P_1$ (motion to the right being positive); now suppose the screw fast in A, and turn the nut B back one turn to the position it would have had provided it had not rotated; B will then move a distance $-P_2$. Adding the two motions, we have for the motion of B, (P_1-P_2) as before. This principle of successive movements is very often convenient in determining resultant motions.

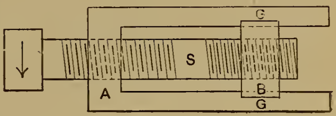


FIG. 20.

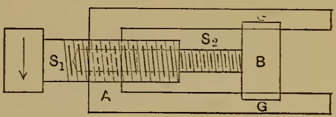


FIG. 21.

When the resultant motion is, as above, the difference of two component motions, the screw is called a *differential screw.*

If one of the threads on S is right-handed and the other left-handed, the motion of B would be (P_1+P_2), its direction depending on the arrangement and rotation of the screw. A right- and left-handed screw are often used in combination to bring together two pieces, not capable of turning, as in the right and left pipe-coupling. Pieces can also be arranged so as to move equal distances in opposite directions in reference to some point located between them.

A more practical form of differential screw than Fig. 20 is shown in Fig. 21, where the screw S_1, working in the fixed nut A, is made hollow, and forms the nut for the smaller screw, S_2, which is fast to the slide B, moving on the guides GG. The action is the same as in the previous case.

In all the previous cases the force has been applied to rotate the screw or nut, and thus cause a straight translation; a force causing translation might be applied to the screw or nut, which would cause the nut or screw to rotate. This is not possible with ordinary pitches, as the frictional resistance is so great; it is well known, however, that nuts and screws subjected to constant jarring, such as those on railway trucks, are very liable to work loose; and double nuts, one serving as a check for the other, are often used. When the pitch is made very long, the screw can be easily turned by moving the nut along it; in this case the screw is formed by a steep spiral groove running along a cylindrical piece. The nut fits this cylindrical piece, and has a projecting feather which fits the groove. This principle is used in a small automatic drill, where the spindle which carries the drill has a multiple-threaded screw of rapid pitch, cut about two-thirds of its length. This screw fits into a tubular handle closed at one end and furnished with a nut which fits the screw: by pushing upon the handle, the screw with the drill is made to rotate; a coiled spring placed between the end of the screw and the closed end of the tube returns the screw to its normal position.

36. Screws are correctly cut in a lathe where the cylindrical blank is made to rotate uniformly on its axis, while a tool, having the same contour as the space between the threads, is made to move uniformly on guides in a path parallel to the axis of the screw, an amount equal to the pitch for each rotation of the blank. The screw is completed by successive cuts, the tool being advanced nearer the axis for each cut until the proper size is obtained. A nut can be cut in the same way by using a tool of the proper shape and moving it away from the axis for successive cuts.

Screws are also cut with solid dies either by hand or power, and with proper dies and care good work will result. Nuts are generally threaded by means of "taps" which are made of cylindrical pieces of steel having a screw-thread cut upon them of the requisite pitch; grooves or flutes are then made parallel to the axis to furnish cutting edges, the tap is then tapered off at the end to allow it to enter the nut, and the threads are "backed off" to supply the necessary clearance.

Screws cut by open dies that are gradually closed in as the screw is being cut are not accurate, as the screw is begun on the outside of the cylinder by the part of the die which must eventually cut the bottom of the thread on a considerably smaller cylinder. Thus, as the angle of the helix is greater the smaller the cylinder, the pitch remaining the same, the die at first traces a groove having a pitch due to the greater angle of the helix at the bottom of the thread. As the die-plates are made to approach each other, they tend to bring back this helical groove to the standard pitch; this strains the material of the threads, and finally produces a screw of a different pitch than that of the die-plates.

37. **Worm and Wheel.**—A worm and wheel (Fig. 22) is a combination of a screw and a wheel furnished with teeth so shaped as to be capable of engaging with the screw placed tangential to the wheel. The continuous rotation of the screw or worm will then impart continuous rotation to the wheel, and it will advance through one, two, or three teeth upon each turn of the screw, according as the thread on the screw is single, double, or triple. On account of the great

reduction of velocity obtainable by this combination, it is extremely valuable as a means of obtaining mechanical advantage, and is much used in hoisting machinery. It is also useful in making fine angular adjustments, as in gear-cutting machines; when thus used for making adjustments, it is sometimes called a "tangent-screw."

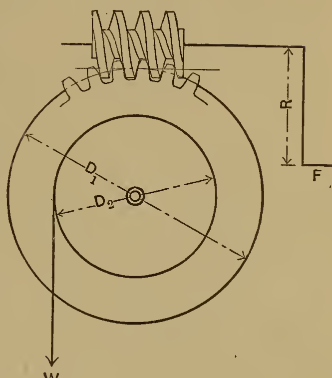


FIG. 22.

Velocity Ratio in a Worm and Wheel.—Two cases may be considered:

1° Let P be the pitch of the worm, D_1 the pitch diameter of the worm-wheel, D_2 the pitch diameter of the drum upon which the resistance W is exerted. For one turn of the worm the point where the force F is applied will move a distance $2\pi R$ and the surface of the drum where W is exerted will move $P\times\frac{D_2}{D_1}$.

$$\therefore \frac{\text{l.v. of } F}{\text{l.v. of } W}=\frac{\text{motion of } F}{\text{motion of } W}=\frac{2\pi R}{P\frac{D_2}{D_1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

2° Let the worm be double-threaded and let N be the number of teeth on the worm-wheel, then we shall have

$$\frac{\text{l.v. of } F}{\text{l.v. of } W}=\frac{2\pi R}{\frac{2}{N}\pi D_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The denominator $\frac{2}{N}\pi D_2$ would be the motion of W for one turn of the worm. If the worm were triple-threaded, the motion of W for one turn of the worm would be $\frac{3}{N}\pi D_2$.

38. Power of a Screw. Relation between Forces and Linear Velocities in Mechanisms.—Since, if we neglect the loss of work by friction or concussion, any mechanical combination must deliver as much work as it receives, we must have the force at the point of application multiplied by the velocity of that point equal to the force at the point of delivery multiplied by its velocity, or the forces are to each other inversely as the velocities of the points at which they act. For example, in the previous paragraph we have

$$\frac{\text{l.v.}F}{\text{l.v.}W}=\frac{W}{F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Combining this fact with the previous statement in regard to the ratio of the linear velocities of the points where the forces act, we can find the forces which may be transmitted in such cases, when losses due to friction are neglected. Thus in case 1° in the preceding paragraph we should have

$$\frac{W}{F}=\frac{\text{l.v.}\,F}{\text{l.v.}\,W}=\frac{2\pi R}{P\frac{D_2}{D_1}} \quad \cdots \cdots \cdots \quad (9)$$

39. Inversion of Closed Pairs.—If, in a closed pair, we exchange the fixed element for the movable one, there is no alteration in the resulting absolute motion; the exchange of the fixedness of an element with its partner is called the inversion of the pair. This has already been noticed in connection with the discussion of the screw, where it made no difference in the resulting motion whether the screw or the nut was considered as fixed. In the ordinary bolt we turn the nut, while in the "tap-bolt" the nut is stationary and the bolt is turned. In the common wagon-wheel the axle is fixed to the body of the wagon, while the wheel turns on it; in the railway truck the bearing is attached to the truck frame, and the axle turns in it with the wheel, which is made fast to the axle.

40. Incomplete Pairs of Elements; Force closure.—Hitherto it has been assumed that the reciprocal restraint of two elements forming a pair was complete, i.e., that each of the two bodies, by the rigidity of its material and the form given to it, restrained the other. In certain cases it is only necessary to prevent forces having a certain definite direction from affecting the pair, and in such cases it is no longer absolutely necessary to make the pair self-closed; one element can then be cut away where it is not needed to resist the forces. When a pair of elements is thus incomplete, and the closure is effected by means of a force or forces, we have what is called a *force-closed* pair of elements.

The bearings for railway axles, the steps for water-wheel shafts, the ways of an iron planer, railway wheels kept in contact with the rails by the force of gravity, are all examples of force-closed pairs.

CHAPTER IV.

ROLLING CYLINDERS AND CONES CONNECTED BY FORCE-CLOSURE.—ROLLING OF NON-CYLINDRICAL SURFACES.—LOBED WHEELS.

41. One of the most common and most useful problems of mechanism is to connect two axes so that they shall have a definite angular velocity ratio. We may have three cases: 1° parallel axes; 2° axes which intersect; 3° axes which are neither parallel nor intersecting. In any case the velocity ratio may be constant or it may vary. The first two cases, with a constant velocity ratio and directional relation, are the most common, and resemble each other in many ways; for that reason they will be considered first; the third case will come up later.

Pure Rolling Contact consists of such a relative motion of two lines or surfaces that the consecutive points or elements of one come successively into contact with those of the other in their order.

42. **Rolling Cylinders.**—Let A and B (Fig. 23) be the sections of two rolling cylinders, and let o_1 and o_2, the centres of the circles A and B, be the traces of their axes. The point c is called the *point of contact*. The common element through c is called the *line* or *element of contact*. The plane passing through the axes, whose trace is the common normal through the point c, is called the *plane of centres;* the common normal o_1o_2 is called the *line of centres*.

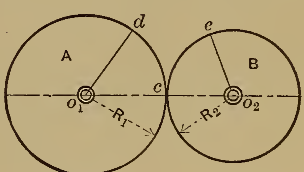


FIG. 23.

According to the definition of rolling contact, the linear velocity ratio of the two rolling surfaces must be unity, or slipping will result between them.

43. **Angular Velocity Ratio.**—In Fig. 23 let R_1 and R_2 be the radii of two rolling cylinders, V_1 and V_2 their angular velocities, and N_1 and N_2 their revolutions per minute. Suppose the two circles to roll in such a way that the point c travels to d in A, and to e in B; then, as no slipping can occur,

$$\text{arc } cd = \text{arc } ce, \quad \text{or} \quad R_1V_1 = R_2V_2.$$

$$\therefore \frac{V_1}{V_2} = \frac{R_2}{R_1}. \qquad (10)$$

That is, when two circles roll together, their uniform angular velocities are inversely as the radii of the circles.

Now if, as is more commonly the case, we take the number of revolutions per minute, N_1 and N_2, as given, the l.v's per minute of the rolling surfaces would be $2\pi R_1N_1$ and $2\pi R_2N_2$, which must be equal.

$$2\pi R_1N_1=2\pi R_2N_2, \quad \text{or} \quad R_1N_1=R_2N_2.$$

$$\therefore \frac{N_1}{N_2}=\frac{R_2}{R_1}. \qquad (11)$$

That is, when two circles roll together, their revolutions in a given time are inversely proportional to the radii of the circles.

44. Given the velocity ratio and the distance between centres of a pair of rolling cylinders, to find their radii.

External Contact (Fig. 23).—If D is the distance between the axes, and A makes N_1 revolutions and B makes N_2 revolutions per minute, we have from equation (11)

$$\left.\begin{array}{l} \dfrac{R_2}{R_1}=\dfrac{N_1}{N_2} \\ R_1+R_2=D. \end{array}\right\}$$

and, from the figure,

Solving the two equations for R_1 and R_2, we have

$$R_1=\frac{N_2D}{N_2+N_1} \quad \text{and} \quad R_2=\frac{N_1D}{N_2+N_1}.$$

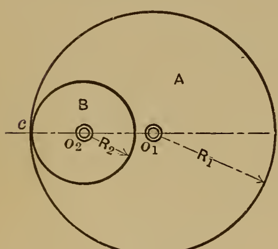


Fig. 24.

Internal Contact (Fig. 24).—Using the same notation as above, we have

$$\left.\begin{array}{l} \dfrac{R_2}{R_1}=\dfrac{N_1}{N_2} \\ R_1-R_2=D. \end{array}\right\}$$

and

$$\therefore R_1=\frac{N_2D}{N_2-N_1} \quad \text{and} \quad R_2=\frac{N_1D}{N_2-N_1}.$$

Graphical Solution.—*External Contact.*—Let o_1o_2 be the line of centres (Fig. 25). Through o_1 draw the line o_1n, and lay off upon it, to some convenient scale, the distances o_1m and mn equal to the number of units in N_2 and N_1 respectively. Draw no_2 through the centre o_2, and then draw mc parallel to no_2. c will be the point of contact and R_1 and R_2 the radii.

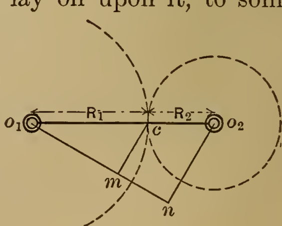


Fig. 25.

Internal Contact (Fig. 26).—Draw the line o_1n as before, and lay off o_1m equal to the units in N_2; then lay off mn toward o_1 from m equal to the units in N_1;

draw no_2, and mc parallel to no_2; R_1 and R_2 will be the radii, and c the point of contact.

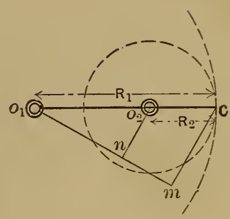


Fig. 26.

It will be noticed that two circles in external contact rotate in opposite directions, while those in internal contact rotate in the same direction.

The various forms of roller bearings, where a series of rolling cylinders occupy the annular space between two coaxial bearing cylinders and roll externally on one and internally on the other are good examples of rolling cylinders.

We shall find in the study of gearing that it is possible to use teeth developed from rolling cylinders, which teeth may be of such shape that the conditions of pure rolling contact of the cylinders shall be practically obtained.

45. **Rack and Pinion.**—If we suppose the circle B (Fig. 23) to be indefinitely increased, its radius will eventually become infinite, and its circumference will become sensibly a straight line. This combination is often used in gearing, and the velocity ratio is properly considered here. As the straight line or rack has straight translation only, its linear motion needs only to be considered. If the radius R, and the a.v. or number of revolutions N, of the rolling circle or pinion are given, the l.v. of the rack L is found by the equation

$$L=2\pi RN.$$

For, to satisfy the conditions of rolling contact, the l.v's must be equal. Here the velocity ratio is constant, but the directional relation must be reversed when the end of the rack reaches the pinion.

46. **Rolling Cones.**—Let A and B (Fig. 27) be a pair of rolling cones; ce, the element of contact; o_1co_2e, the plane of centres. Let o_1cs and o_2ct be planes passing through the point of contact c and respectively perpendicular to o_1e and o_2e; the circles cut from the cones by these planes are called *base circles*. Similar circles could be cut by planes parallel to o_1cs and o_2ct through any other point on the element of contact, as c'. Now in order to have no slipping, the linear velocity ratio of all pairs of points on the common element of contact ce must be unity, that is, the consecutive elements of one cone must roll successively into contact with those of the other. From this it

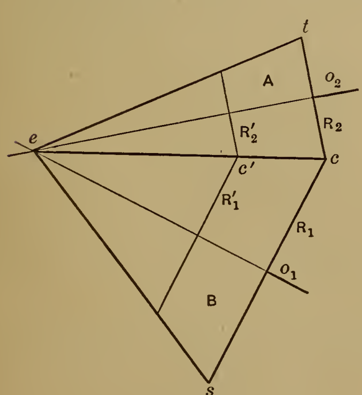


Fig. 27.

also follows that two or more cones in pure rolling contact must have a common apex.

47. **Angular Velocity Ratio.**—Let R_1 and R_2 be the radii of the base circles, N_1 and N_2 be their revolutions per minute, and V_1 and V_2 their angular velocities. Since the l.v's of the two base circles in contact at c are the same, we have, as for rolling circles,

$$\frac{R_1}{R_2}=\frac{V_2}{V_1}=\frac{N_2}{N_1},$$

and for any other point, as c', we must also have

$$\frac{R_1'}{R_2'}=\frac{V_2}{V_1}=\frac{N_2}{N_1},$$

as R_1 and R_1', being in one piece, must revolve with the same a.v. about o_1e. It can also easily be seen by similar triangles that

$$\frac{R_1}{R_2}=\frac{R_1'}{R_2'}.$$

Thus the angular velocity ratio of any two circles in contact on the common element is the same as that of the base circles and that of the two axes.

In practice it is customary to use thin frusta of the rolling cones for rolling conical wheels, and we shall find in this case also that teeth can be developed from the cones and give the same result as pure rolling contact of the original cones. Such toothed wheels are called *bevel gears.*

Given the a.v. ratio, the position of the base of one of the cones, and the angle between the axes: to find the position of the base of the other cone, and the radii of both bases.

Algebraical Solution.—Let o_1e and o_2e (Fig. 28) be the axes of a pair of rolling cones, θ the angle between the axes, R_1 and R_2 the radii of bases, N_1 and N_2 the revolutions per minute, and α_1 and α_2 the semi-vertical angles of the respective cones.

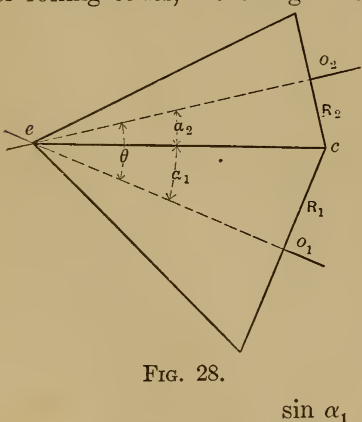


Fig. 28.

$$R_1=ec\sin\alpha_1;$$

$$R_2=ec\sin\alpha_2;$$

$$\therefore\ \frac{\sin\alpha_1}{\sin\alpha_2}=\frac{R_1}{R_2}=\frac{N_2}{N_1}. \quad . \quad (12)$$

substituting for α_2 the value $(\theta-\alpha_1)$ and solving, we have

$$\frac{N_2}{N_1}=\frac{\sin\alpha_1}{\sin(\theta-\alpha_1)}=\frac{\sin\alpha_1}{\sin\theta\cos\alpha_1-\cos\theta\sin\alpha_1}$$

$$=\frac{\dfrac{\sin\alpha_1}{\cos\alpha_1}}{\sin\theta-\cos\theta\dfrac{\sin\alpha_1}{\cos\alpha_1}}=\frac{\tan\alpha_1}{\sin\theta-\cos\theta\tan\alpha_1}.$$

$$\therefore \tan\alpha_1 = \frac{N_2 \sin\theta}{N_1 + N_2\cos\theta} = \frac{\sin\theta}{\frac{N_1}{N_2} + \cos\theta}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In a similar manner we could find for α_2 the equation

$$\tan\alpha_2 = \frac{\sin\theta}{\frac{N_2}{N_1} + \cos\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Now the two angles α_1 and α_2 being known, and the distance o_1e of one base circle from the intersection of the axes being given, the distance o_2e of the other base circle from the intersection of the axes may be readily found and also the radii R_1 and R_2 of the base circles.

The angle θ between the axes is often 90°, in which case we should have

$$\tan\alpha_1 = \frac{N_2}{N_1}; \quad \tan\alpha_2 = \frac{N_1}{N_2};$$

$$R_1 = o_1e \tan\alpha_1 = o_1e\frac{N_2}{N_1}; \quad R_2 = o_1e.$$

Graphical Solution.—Given the angle between the axes of two rolling cones and their a.v. ratio: to find the element of contact of the two cones. Let N_1 and N_2 be the revolutions per minute of the axes o_1e and o_2e respectively (Fig. 29). Then, since the revolutions are

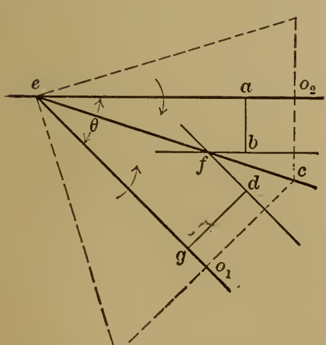


Fig. 29.

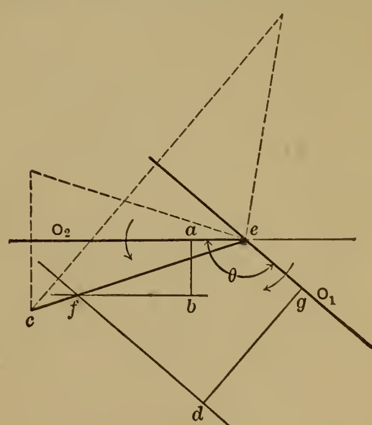


Fig. 30.

inversely as the radii at any point of contact, the element of contact ec may be found by erecting, to any convenient scale, perpendiculars to o_1e and o_2e inversely as their respective revolutions. Thus ab represents N_1 and dg represents N_2. The intersection f of the lines drawn through the ends of these perpendiculars parallel to the respective axes will give one point on the element of contact, which may then be drawn through the point thus found, and through e.

Fig. 30 shows a case in which the data are such that the cones are found to be in internal contact.

48. Rolling Cylinder and Sphere.—Fig. 31 shows an example of a rolling cylinder and sphere as used in the Coradi planimeter. The segment of the sphere A turns on an axis ac passing through a, the centre of the sphere. The cylinder B, whose axis is located in a plane also

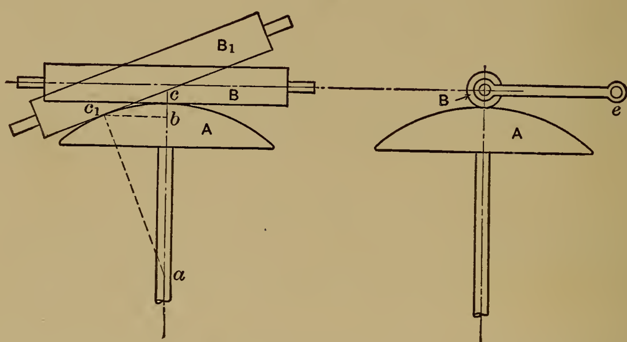


FIG. 31.

passing through the centre of the sphere, is supported by a frame pivoted at e and is held to the cylinder by a spring, not shown. The frame pivots e are movable about an axis at right angles to ac and passing through a, the centre of the sphere. When the roller is in the position B with its axis at right angles to ac, the turning of the sphere produces no motion of B; when, however, the roller is swung so that its axis makes an angle bac_1 with its former position, as shown at B_1 by fine lines, the point of contact is transferred to c_1 in the perpendicular from a to the roller axis. If now we assume the radius of the roller $=R$, the relative motion of roller and sphere, in contact at c_1, is the same as that of two circles of radii R and bc_1 respectively. Transferring the point of contact to the opposite side of ab will result in changing the directional relation of the motion. The action of this device is purely rolling and but very little force can be transmitted. It is used only in very delicate mechanisms.

Disc and Roller.—If in Fig. 31 we assume the radius of the sphere ac to become infinite and the roller B to be replaced by a sphere of the same diameter turning on its axis, we shall have a *disc and roller* as shown in Fig. 32, where AA represents the disc and B the roller, made up of the central portion of the sphere.

If we suppose the rotation of the disc to be uniform, the velocity ratio between B and A will constantly decrease as the roller B is shifted nearer the axis of A, and conversely. If the roller is carried to the other side of the axis, it will rotate in the opposite direction to the first.

This combination is sometimes used in feed mechanisms for machine tools, where it enables the feed to be adjusted and also reversed by simply adjusting the roller on the shaft CC.

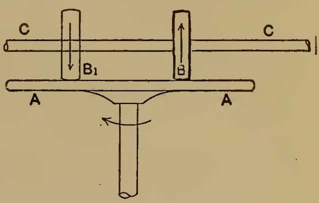


Fig. 32.

49. Friction-gearing. Rolling cylinders and cones are frequently used to transmit force, and constitute what is known as *friction-gearing.* In such cases the axes are arranged so that they can be pressed together with considerable force, and, in order to prevent slipping, the surfaces of contact are made of slightly yielding materials, such as wood, leather, rubber, or paper, which, by their yielding, transform the line of contact into a surface of contact, and also compensate for any slight irregularities in the rolling surfaces. Frequently only one surface is made yielding, the other being usually made of iron. As slipping is likely to take place in these combinations, the velocity ratio cannot be depended upon as absolute.

When rolling cylinders or cones are used to change sliding to rolling friction, that is, to reduce friction, their surfaces should be made as hard and smooth as possible. This is the case in roller bearings previously described and in the various forms of ball bearings where spheres are arranged to roll in suitably constructed races, all bearing surfaces being made of hardened steel and ground.

Friction-gearing is utilized in several forms of speed-controlling devices, among which the following are good examples:

Fig. 33 shows the mechanism of the Evans friction-cones, consisting of two equal cones A and B turning on parallel axes with an endless movable leather belt, C, in the form of a ring running between them, the axis of B being urged toward A by means of springs or otherwise. By adjusting the belt along the cones, as shown by the arrows, their a.v. ratio may be varied at will.

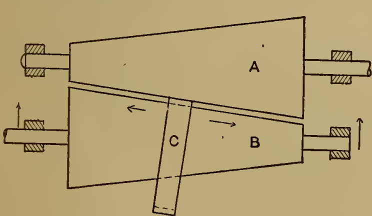


Fig. 33.

It should be observed that there must be some slipping since the a.v. ratio varies from edge to edge of the belt, the resulting ratio approaching that of the mean line of the belt. A leather-faced roller might be substituted for the belt and a similar series of speeds obtained, the cones then turning in the same instead of in opposite directions.

Fig. 34 shows, in principle, another form made by the Power and

Speed Controller Co. Here two equal rollers, C and D, faced with a yielding material, are arranged to run between two equal hollow discs A and B. The rollers with their supporting yokes (only one of which is shown in the elevation) are arranged as indicated in the figure and are made by a geared connection, not shown, to turn opposite each other on the vertical yoke axes, s. The contour of the hollow in the discs must thus be an arc of a circle of radius equal that of the roller drawn from s as a centre. If now the disc B is made fast to the shaft, and A, running loose, is urged against B by a spring or otherwise, a uniform motion of A may be made to give varying speeds to B by turning the rollers as shown. To increase the power two sets of discs are often used.

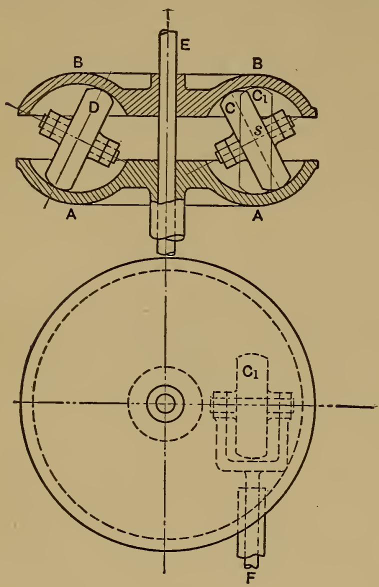


FIG. 34.

Fig. 35 shows the Sellers *feed-discs* used to give a varying a.v. ratio between two parallel shafts, one of them controlling the feed on a machine.

The two outer wheels are thickened on their peripheries and run between two convex discs BB which are constantly urged together by hidden coil springs bearing against the spherical washers clearly shown. The discs BB are supported by the pivoted forked arm D. If now the disc A be given a uniform a.v., the disc C may be made to have a greater or less a.v. as the axis of the discs BB is made to approach or recede from A.

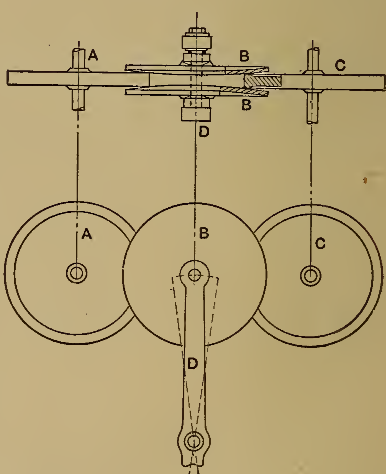


FIG. 35.

In Fig. 36 a modified form of the Sellers discs used by Jones and Lamson is shown. The shaft A is driven by the pulley P and is carried by a forked arm supplied with two bearings CC and swinging about a point near the centre of the pulley driving P by means of a belt.

The externally rubbing discs B are free to slide axially on the shaft A, but turn with it and are constantly urged apart by springs clearly shown. The internally rubbing convex discs are made fast to the driven shaft by set-screws. To vary the speed of D, that of A being constant, it is only necessary to vary the distance between the shafts. In the position shown D has its highest speed, the discs rubbing at a. When the shaft A is urged in the direction of the arrows the rubbing radius on B is diminished and that on E increased, the discs BB approaching each other. The discs BB may be made solid and one of the discs E be urged toward the other by a spring on its hub, which would simplify the construction.

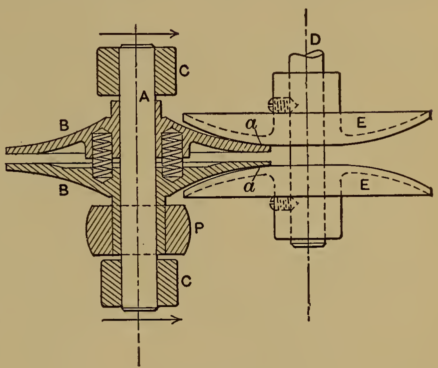


FIG. 36.

Grooved Friction-gearing.—Another form of friction-gearing is shown in Fig. 37. Here increased friction is obtained between the rolling bodies by supplying their surfaces of contact with a series of interlocking angular grooves; the sharper the angle of the grooves, the greater the friction for a given pressure perpendicular to the axes; both wheels are usually made of cast iron. Here the action is no longer that of rolling bodies; but considerable sliding takes place, which varies with the shape and depth of the groove. This form of gearing is very generally used in hoisting machinery for mines, and also for driving rotary pumps; in both cases a slight slipping would be an advantage, as shocks are quite frequent in starting suddenly, and their effect is less disastrous when slipping can occur.

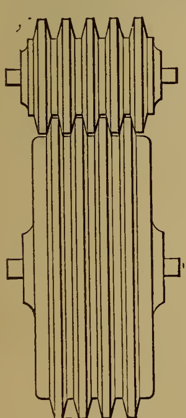


FIG. 37.

The velocity ratio is not absolute, but is substantially the same as that of two cylinders in rolling contact on a line drawn midway between the tops of the projections on each wheel, they being supposed to be in working contact.

50. Rolling of Non-cylindrical Surfaces.—If the angular velocity ratio of two rolling bodies is not a constant, the pitch lines will not be circular. Whatever forms of curves the pitch lines take, the conditions of pure rolling contact should be fulfilled, namely, the point of contact must be on the line of centres, and the rolling arcs must be of equal length. For example, in the rolling bodies represented by Fig. 38, with o_1 and o_2 the axes of rotation, we must find the sum of the radiants in contact, $o_1c + o_2c$, equal to the sum of any other pair, as $o_1d + o_2e$, $o_1f + o_2g$; and also the lengths of the rolling arcs must be equal, $cd = ce$, $df = eg$. This will cause the successive points on the curves to meet on the line of centres, and the rolling arcs, being of equal length, will roll without slipping.

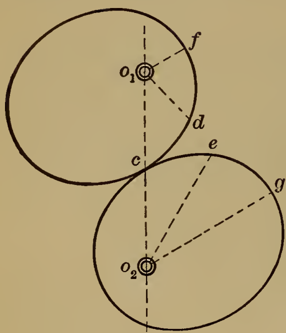


FIG. 38.

There are four simple cases of curves which may be arranged to fulfil these conditions:

A pair of logarithmic spirals of the same obliquity.
A pair of equal ellipses.
A pair of equal hyperbolas.
A pair of equal parabolas.

We shall also find that any of the above curves may be transformed in one way or another and still fulfil the conditions of perfect rolling contact, while allowing a wide range of variation in the angular velocity ratio.

51. The Rolling of two Logarithmic Spirals of Equal Obliquity.—Fig. 39 shows the development of a pair of such spirals, where, if they roll on the common tangent de, the axes o_1 and o_2 will move along the lines fg and hk respectively. The arcs a_1c, cb_1, etc., being equal to a_2c, cb_2, etc., and also equal to the distances ac, cb, etc., on the common tangent, it will be clear that if the axes o_1 and o_2 are fixed, the spirals may turn, fulfilling the conditions of perfect rolling contact; for the arc $cb_1 =$ arc cb_2, and also the radiant $o_1b_1 +$ radiant $o_2b_2 = o_1'b + o_2'b = o_1c + o_2c$; and similarly for successive arcs and radiants.

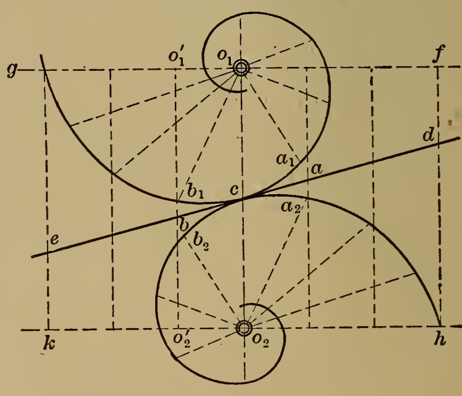


FIG. 39.

52. *To construct two spirals, as in Fig. 39, with a given obliquity.*—The equation for such a logarithmic spiral is

$$r = ae^{b\theta},$$

where a is the value of r when θ is zero; and $b = \frac{1}{\tan \phi}$, ϕ being the constant angle between the tangent to the curve and the radiant to the point of tangency; and where e is the base of the Naperian logarithms.

In Fig. 40 let $oc = a$, and $ocd = \phi$. Taking successive values of θ, starting from oc, we may calculate the values of r and thus plot the curve. If, however, it is desired to pass a spiral through two points on radiants a given angle apart, it is to be noticed from the equation of the curve that if the successive values of θ are taken with a uniform increase, the lengths of the corresponding radiants will be in geometrical progression. To draw a spiral through the points b and e, Fig. 40, bisect the angle boe, and make of a mean proportional between ob and oe; f will be a point on the spiral. Then by the same method bisect foe, and find oh; also bisect bof and find ok, and so on; a smooth curve through the points thus found will be the desired spiral.

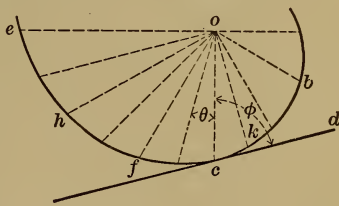


Fig. 40.

53. Since these curves are not closed, one pair cannot be used for continuous motion; but a pair of such curves may be well adapted to sectional wheels requiring a varying angular velocity. For example, in Fig. 41, given the axes o_1 and o_2, the angle co_1e through which A is to turn, and the limits of the a.v. ratio. Make $\frac{o_1c}{o_2c}$ equal to the minimum a.v. ratio and $\frac{o_1d}{o_2d}$ equal to the maximum a.v. ratio. Then o_1e must equal o_1d. Now construct a spiral through the points c and e. The spiral for B is that part of the spiral A constructed about o_1 which would be included between radiants o_1b and o_1a, equal respectively to o_2c and o_2g $(= o_2d)$, which may be found by continuing the spiral about o_1 beyond c or e if necessary. Since these curves (Fig. 41) are parts of the same spiral, and since by construction $o_1c + o_2c = o_1e + o_2g$, A could drive B, the points e and g ultimately rolling together at d on the line of centres. The conditions of rolling contact are evidently fulfilled, as will be seen by referring to § 51.

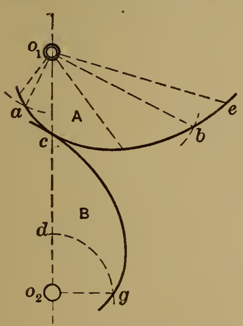


Fig. 41.

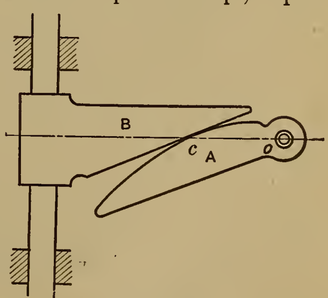


Fig. 42.

54. In Fig. 42 we have a logarithmic spiral sector, A, driving a slide, B. Here the driven surface of the

slide coincides with the tangent to the spiral, the line of centres being from o through c to infinity and perpendicular to the direction of motion of the slide. In this combination the l.v. of the slide will equal the a.v. of A multiplied by the length of the radiant in contact, oc.

55. **Wheels using Logarithmic Spirals arranged to allow Complete Rotations.** — By combining two sectors from the same or from different spirals, unilobed wheels may be found which may be paired in such a way as to fulfil the laws of perfect rolling contact. Taking two equal sectors from the same spiral, we should have a symmetrical unilobed wheel, as A (Fig. 43), and this will run perfectly with a wheel B exactly like A, as shown. If A is the driver, the minimum a.v. of B will occur when the points d and e are in contact, and we shall have

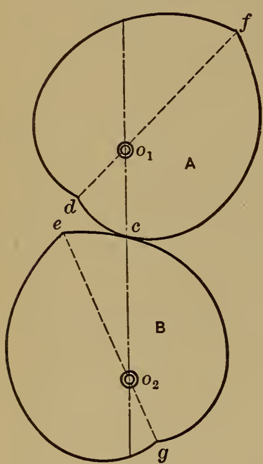


Fig. 43.

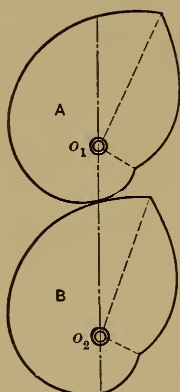


Fig. 44.

$$\frac{\text{a.v. } B}{\text{a.v. } A}=\frac{o_1d}{o_2e}.$$

The maximum a.v. of B will occur when the points f and g are in contact. Such wheels are readily formed, if the maximum and minimum a.v. ratios are known, by the method in § 53, only it is to be noticed that the minimum ratio must be the reciprocal of the maximum ratio, and that the angle which each sector subtends must be 180°. Unilobed wheels need not be formed from equal sectors, in which case the sectors used will not have the same obliquity nor will the subtended angles be equal, but the wheels must be so paired that sectors of the same obliquity shall be in contact. Fig. 44 shows a pair of such wheels in which maximum and minimum a.v. ratios occur at unequal intervals; it will, however, be noticed that the minimum a.v. ratio must here also be the reciprocal of the maximum ratio.

56. By a similar method wheels may be formed which shall give more than one position of maximum and of minimum a.v. ratio; that is, we may have either symmetrical or unsymmetrical bilobed wheels, trilobed wheels, etc. Fig. 45 shows a pair of symmetrical bilobed wheels. Here all the sectors are from the same spiral, all the same length, each subtending an angle of 90°. It will be seen that the

conditions of rolling contact are perfectly fulfilled, and that if A turns uniformly B will have two positions of maximum and two of minimum speed. Similarly a pair of symmetrical trilobed wheels could be formed where each of the sectors subtends an angle of 60°.

Following the method used in obtaining the unsymmetrical unilobed wheels of Fig. 44, a pair of unsymmetrical bilobed wheels could be arranged, providing only that sectors of the same obliquity come into contact and that such sectors subtend equal angles. Fig. 46 shows a pair of trilobed wheels of this form.

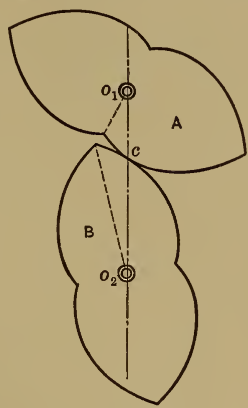


Fig. 45.

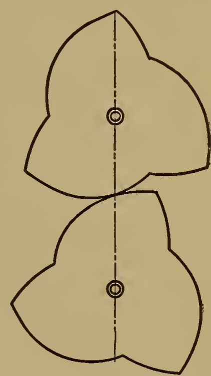


Fig. 46.

57. Such wheels as those just described cannot be interchangeable, but since any two spiral arcs having the same obliquity will roll correctly, a unilobe may be made to roll correctly with a bilobe where the sectors of the unilobe are from a given spiral and each subtending 180°,

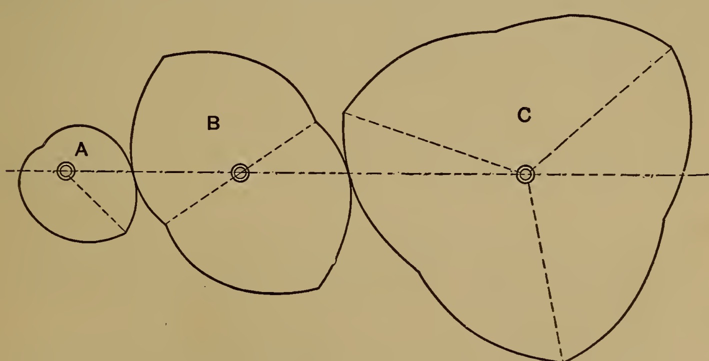


Fig. 47.

and where each of the sectors of the bilobe is of the same length as one of those of the unilobe, and from a spiral of the same obliquity, but where each subtends an angle of 90°. In a similar manner a trilobed wheel may be found which could be driven by the same unilobed wheel as above, hence also by the bilobed wheel found from that unilobe. These wheels would therefore be interchangeable. Fig. 47 shows a set of such wheels which would be symmetrical wheels. A set of unsymmetrical wheels could be found in a similar manner.

58. The Rolling of Equal Ellipses.—If two equal ellipses, each turning about one of its foci, are placed in contact in such a way that the distance between the axes o_1o_2, Fig. 48, is equal to the major axis of the ellipses, we shall find that they will be in contact on the line of centres and that the rolling arcs are of equal length. If the point c is on the line of centres o_1o_2, we should have $o_1c + co_2 = o_1o_2 = o_1c + cd$, and therefore $cd = co_2$. Since the tangent to an ellipse at any point, as c, makes equal angles with the radii from the two foci, $o_1cm = dcn$ and $ecm = o_2cn$; but since $cd = co_2$, the point c is similarly situated in the two ellipses, and therefore the angle o_1cm would equal the angle o_2cn, which would give a common tangent to the two curves at c. Hence if o_1o_2 is equal to the major axis, the ellipses could be in rolling contact on the line o_1o_2. Since the distances cd and co_2, from the foci d and o_2 respectively, are equal, it also follows that the arc cf is equal to the arc cg which completes the requirements for perfect rolling contact. It will also be noticed that the line dce will be straight and that a link could connect d and e, as will be seen when discussing linkwork.

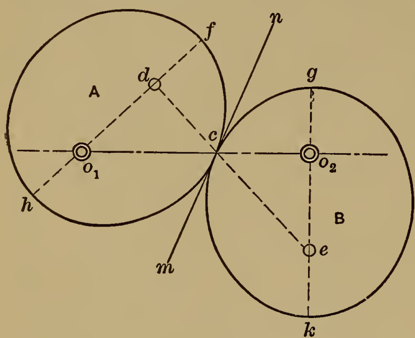


FIG. 48.

If A (Fig. 48) is the driver, the a.v. ratio will vary from a minimum when h and k are in contact, and then equal to $\frac{o_1h}{o_2k}$, to a maximum when f and g are in contact, when it will equal $\frac{o_1f}{o_2g}$. The a.v. ratio will be unity when the major axes are parallel, the point of contact being then midway between o_1 and o_2.

Such rolling ellipses supplied with teeth, thus forming elliptic gears, are sometimes used to secure a quick-return motion in a slotting-machine.

59. Multilobed Wheels Formed from Rolling Ellipses.—Non-interchangeable wheels may be formed from a pair of ellipses by contracting the angles the same amount in each ellipse. Thus, if the angles were contracted to one-half their size, a pair of bilobed wheels could be formed; and if to one-third their size, a pair of trilobed wheels. Such wheels would give perfect rolling contact, but could only be used in pairs as stated.

By a different method of contraction a pair of wheels may be formed, one of which may be, for example, a bilobe and the other a trilobe. By this method only parts of the original ellipses are used; parts which would roll correctly, but which subtend unequal angles in some desired ratio. If the arcs subtend angles in proportion as 2 is to 3, the angles may be contracted or expanded to be 60° and 90°, which are in the same ratio, when we shall have arcs suitable for a trilobe and a bilobe respectively, which will roll correctly. For example, assume the foci o_1 and d (Fig. 49); lay off angles fo_1d and fde as 2 to 3. Then the point f will lie on an ellipse from which a bilobe and a trilobe may be formed by contracting the angle fo_1d to 60° and the angle $go_2d = fde$ to 90°, as shown in the figure.

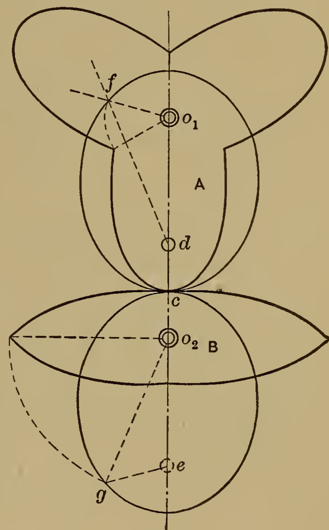


FIG. 49.

60. The Rolling of Equal Parabolas.—Two parabolas may be considered as two ellipses with one focus of each removed to infinity. In

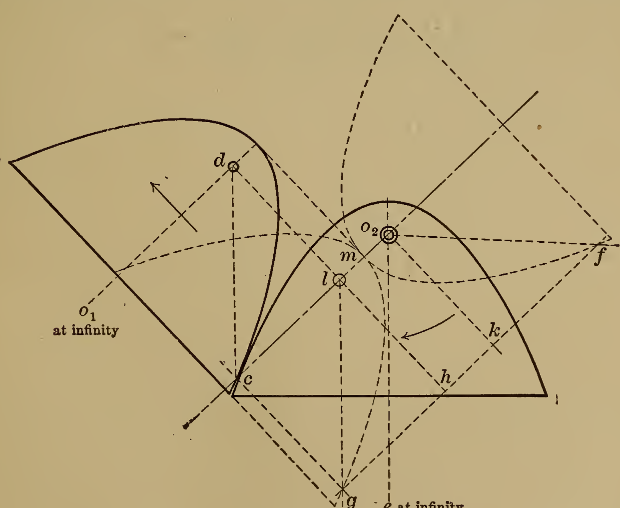


FIG. 50.

the ellipses of Fig. 48 suppose the foci o_1 and e to be so removed; we shall have the parabolas of Fig. 50 in contact at the point c and in perfect rolling contact, one turning about its focus o_2 as an axis, and the other having a motion of translation perpendicular to o_1d.

To prove the rolling action perfect, assume the parabolas with their vertexes in contact at m. Let f be the point on the turning parabola which will move to c, so that $o_2f=o_2c$. Draw fg parallel to o_2c, and since the parabolas are equal we shall have $lg=o_2f$, therefore $lg=o_2c$; but since o_2k is the directrix of the parabola whose focus is now at l, $lg=gk$; therefore $gk=o_2c$, and as this parabola slides perpendicular to o_1d, the point g would also move to c. The rolling arcs mf and mg are equal. Thus the parabola turning about o_2 would cause the other parabola to have translation perpendicular to o_1d, the two moving in perfect rolling contact.

61. The Rolling of Equal Hyperbolas.—If two equal hyperbolas are placed as in Fig. 51, so that the distances between their foci o_1 and o_2, and d and e, are each equal to $fg=hk$, the distance between the vertexes of the hyperbolas, we shall find them in contact at some point c. If

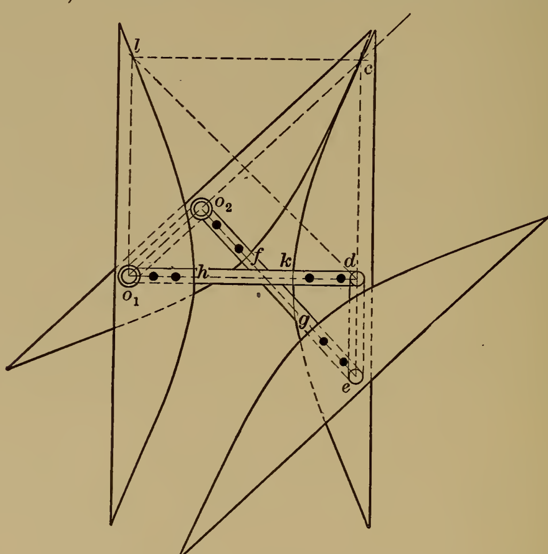


Fig. 51.

the foci o_1 and o_2 are then taken as axes of rotation, the hyperbolas will turn in perfect rolling contact. To prove this take the point l on the hyperbola whose foci are at o_1 and d so that $o_1l=dc$ and $o_1c=dl$. Then since a tangent at any point on a hyperbola makes equal angles with the radii from the two foci, the tangent at l will bisect the angle o_1ld and the tangent at c will bisect the equal angle o_1cd. If now the branch o_1hl is placed tangent to the branch dkc with the points l and c in contact, the radius lo_1 must fall on o_1c and dl on dc. Since the difference between the radii from the two foci to any point on a hyperbola is a constant and equal to the distance between the vertexes, $o_1c - dc = hk$; but o_1l was taken equal to dc, hence $o_1c - o_1l = hk$. Then, since o_1o_2 was originally assumed equal to hk, we shall have $o_1c - o_1l = o_1o_2$, and therefore the line o_1o_2c will be a straight line, and the point of contact c will lie on the line of centres. The arc lh which is equal to ck will also be equal to cf. Therefore the hyperbolas will be in perfect rolling contact. The same reasoning will apply for any position of the point of contact. It is interesting to note that since $o_1o_2 = de =$ a constant, and $o_1d = o_2e =$ a constant, the linkage o_1o_2ed with the axes o_1 and o_2 fixed would cause the same a.v. ratio about o_1 and o_2 as the rolling hyperbolas would give.

If the hyperbola turning about the axis o_2 is the driver, the a.v. ratio will be a minimum when the vertexes f and k are in contact and will be $\frac{o_2f}{o_1k}$; this ratio will increase until the point of contact is at infinity, when the ratio would be unity, and would correspond to the position of the linkage when o_1o_2 and de are parallel. Further rotation would bring the opposite branches of the hyperbolas into contact, the maximum a.v. ratio occurring as the points g and h come together, when its value becomes $\frac{o_2g}{o_1h}$. The construction shown in the figure will allow only a limited motion.

CHAPTER V.

CONNECTION BY BANDS OR WRAPPING CONNECTORS—BELTS, CORDS AND CHAINS.

62. A flexible wrapping connector may be paired by force-closure with a pulley, and two such pairs may be combined to connect two axes, whether parallel, intersecting, or neither parallel nor intersecting. Flexible connectors may be divided into three general classes:

1° **Belts** made of leather, rubber, or woven fabrics are flat and thin, and require pulleys nearly cylindrical with smooth surfaces. Flat ropes may be classed as belts.

2° **Cords** made of catgut, leather, hemp, cotton, or wire are nearly circular in section, and require either grooved pulleys or drums with flanges. Rope gearing, either by cotton or wire ropes, may be placed under this head.

3° **Chains** are composed of links or bars, usually metallic, jointed together, and require wheels or drums either grooved, notched, or toothed, so as to fit the links of the chain. For convenience the word *band* may be used as a general term to denote all kinds of flexible connectors.

Bands for communicating continuous motion are endless.

Bands for communicating reciprocating motion are usually made fast at their ends to the pulleys or drums which they connect.

63. **The Line of Connection** of a pair of pulleys connected by a band is the neutral line or axis of that part of the band which transmits the motion. The neutral line of the band is a line which is neither compressed nor stretched in passing around the pulley.

The Pitch Surface of a pulley over which a band passes is the surface to which the *line of connection* is always tangent; that is, an imaginary surface whose distance from all parts of the acting surface of the pulley is equal to the distance from the acting surface of the band to its neutral line. Belts are commonly used to transmit a nearly constant and continuous velocity ratio, and in this case the acting surfaces are cylindrical.

The Effective Radius of a pulley is the radius of its pitch surface.

The Pitch Line of a pulley is the line on its pitch surface in which the centre line of that part of the band which touches the pulley lies. The line of connection is tangent to the *pitch line.*

64. **Velocity Ratio.**—If we assume the band to be inextensible, and that there is no slipping between it and the pulley, the l.v. of the pitch surfaces of the connected pulleys must be equal. Let D_1 and D_2 be the diameters of the connected pulleys and N_1 and N_2 their revolutions per minute respectively, the distance from the surface of the pulley to the neutral axis of the belt being ρ; then

$$\pi N_1(D_1+2\rho)=\pi N_2(D_2+2\rho).$$

$$\therefore \quad \frac{D_1+2\rho}{D_2+2\rho}=\frac{N_2}{N_1}. \quad \ldots \ldots \ldots \quad (15)$$

That is, the angular velocity ratio, as in rolling cylinders, is inversely proportional to the effective diameters, and is constant for circular pulleys. As the thickness of thin flat bands or belts is generally small compared with the diameters of the pulleys connected, it may be neglected, and the acting surfaces of the pulleys may then be considered to have the same linear velocity. The approximate angular velocity ratio will then be given by the equation

$$\frac{D_1}{D_2}=\frac{N_2}{N_1}. \quad \ldots \ldots \ldots \quad (16)$$

Belts and cords are not suited to transmit a precise velocity ratio, because they are liable to stretch or to slip on the pulleys. This freedom to stretch and slip is an advantage in powerful and quick-running machinery, as it prevents shocks which are liable to occur when a machine is thrown suddenly into gear, or when there is a sudden fluctuation in the power transmitted.

Belts or Thin Flat Bands.

65. **Crowning of Pulleys.**—If we suppose a belt to run upon a revolving conical pulley, it will tend to lie flat upon the conical surface, and, on account of its lateral stiffness, will assume the position shown in Fig. 52. If the belt travels in the direction of the arrow, the point a will, on account of the pull on the belt, tend to adhere to the cone and will be carried to b, a point nearer the base of the cone than that previously occupied by the edge of the belt: the belt would then occupy the position shown by the dotted lines. Now if a pulley is made up of two equal cones placed base to base, the belt will tend to climb both, and would thus run with its centre line on the ridge formed by the union of the

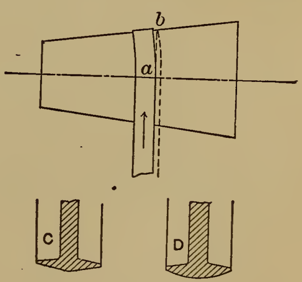


Fig. 52.

two cones. In practice pulley rims are made crowning, except in cases where the belt must occupy different parts of the same pulley. In Fig. 52 two common forms of rim sections are shown at C and D; that shown at C is most commonly met with, as it is the easier to construct. When pulleys are improperly located, the belt will generally work toward the position where it is tightest, or will run toward the high side of the pulley; this is due to the lateral stiffness of the belt, and could be explained in the same way as the climbing on a conical pulley.

66. Tight and Loose Pulleys are used for throwing machinery into and out of gear. They consist of two pulleys placed side by side upon the driven shaft CD (Fig. 53); A, the tight pulley, is keyed to the shaft; while B, the loose pulley, turns loose upon the shaft, and is kept in place by the hub of the tight pulley and a collar. The driving-shaft carries a pulley G, whose width is the same as that of A and B put together, or twice that of A. The belt, when in motion, can be moved by means of a *shipper* that guides its advancing side, either on to the tight or the loose pulley. The tight pulley is sometimes made larger than the loose pulley, so that the belt may be slack when it is on the loose pulley. In such a case the loose pulley has a flange upon the edge next the tight pulley, of a diameter equal to that of the tight pulley; this flange aids in the transfer of the belt from one pulley to the other.

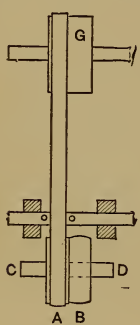


Fig. 53.

The acting surface of a pulley is called its *face;* a pulley with a *six-inch face* is one having a face six inches wide. The faces of pulleys are always made a little wider than the belts which they carry. The pulley G (Fig. 53) has a flat face, because the belt must occupy different positions upon it; while A and B have crowning faces, which will allow the shifting of the belt, and will retain it in position when shifted upon them.

67. Length of Belts connecting Parallel Axes.—Pulleys for belts connecting parallel axes usually have their pitch circles in one plane, which is perpendicular to the axes. The belts may be crossed as in Fig. 54, where the pulleys rotate in opposite directions, or open as in Fig. 55, where the pulleys rotate in the same direction.

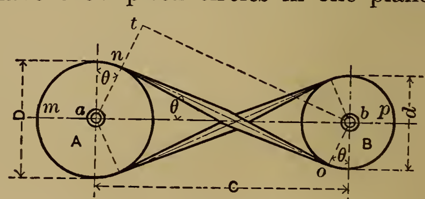


Fig. 54.

Crossed Belts.—Let D and d (Fig. 54) be the diameters of the connected pulleys; C the distance between their axes; L the length of the belt. Also let

$$(D+d)=\Sigma \quad \text{and} \quad (D-d)=\Delta.$$

Then
$$L=2(mn+no+op)$$
$$=\left(\frac{\pi}{2}+\theta\right)D+2C\cos\theta+\left(\frac{\pi}{2}+\theta\right)d$$
$$=\left(\frac{\pi}{2}+\theta\right)\Sigma+2C\cos\theta, \quad \ldots \ldots \ldots \quad (17)$$

where
$$\sin\theta=\frac{at}{ab}=\frac{an+bo}{ab}=\frac{D+d}{2C}=\frac{\Sigma}{2C}.$$

Open Belts.—Using the same notation as for crossed belts, we have (Fig. 55)

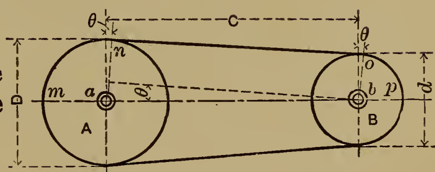


Fig. 55.

$$L=2(mn+no+op)$$
$$=\left(\frac{\pi}{2}+\theta\right)D+2C\cos\theta+\left(\frac{\pi}{2}-\theta\right)d$$
$$=\frac{\pi}{2}(D+d)+\theta(D-d)+2C\cos\theta$$
$$=\frac{\pi}{2}\Sigma+\theta\Delta+2C\cos\theta, \quad \ldots \ldots \ldots \quad (18)$$

where
$$\sin\theta=\frac{an-bo}{C}=\frac{D-d}{2C}=\frac{\Delta}{2C},$$

and
$$\cos\theta=\sqrt{1-\frac{\Delta^2}{4C^2}}.$$

For an open belt, θ is generally small, so that $\theta=\sin\theta$, very nearly; then

$$L=\frac{\pi}{2}\Sigma+\frac{\Delta^2}{2C}+2C\sqrt{1-\frac{\Delta^2}{4C^2}}, \text{ nearly,}$$
$$=\frac{\pi}{2}\Sigma+2C\left\{\frac{\Delta^2}{4C^2}+\sqrt{1-\frac{\Delta^2}{4C^2}}\right\}, \text{ nearly.}$$

If we expand the quantity under the radical sign, and neglect all terms having a higher power of C than the square in the denominator, as C is always large compared with Δ, we have

$$L=\frac{\pi}{2}\Sigma+2C\left\{\frac{\Delta^2}{4C^2}+1-\frac{\Delta^2}{8C^2}\ldots\right\},$$

or
$$L=\frac{\pi}{2}\Sigma+2C+\frac{\Delta^2}{4C}, \text{ very nearly.} \quad \ldots \ldots \ldots \quad (19)$$

68. Speed-cones are contrivances for varying and adjusting the velocity ratio of a pair of rotating parallel axes by means of a shifting belt: they may be continuous cones or conoids, as in Figs. 56 and 58, where the velocity ratio can be varied gradually while they are in motion by shifting the belt, or sets of stepped pulleys, as in Figs. 57 and 59.

In order that the belt may be equally tight in all positions, the length L, previously found, must not vary.

Crossed Belts (Figs. 56 and 57).—Here (equation 17) L depends only on the distance between centres C, the sum of the diameters Σ, and the angle θ, which is itself dependent on C and Σ. As C is a constant, the axes being parallel, therefore the sum of the diameters, $(D+d)=\Sigma$, in

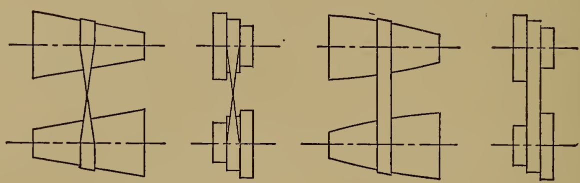


FIG. 56. FIG. 57. FIG. 58. FIG. 59.

all positions of the belt must be constant in order that the length of the belt L may not vary. This condition is fulfilled by two cones with equal angles at their apexes, as in Fig. 56, or a pair of stepped pulleys having the sum of the diameters for each pair of steps a constant.

Open Belts (Figs. 58 and 59).—Here the length of the belt is as found in equation (19),

$$L=\frac{\pi}{2}\Sigma+2C+\frac{\Delta^2}{4C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Hence, if an open belt is to run on a pair of pulleys, the sum and difference of whose diameters are Σ_a and Δ_a, and the same belt is also to run on another pair of pulleys, the sum and difference of whose diameters are Σ_x and Δ_x, we have, since the length of the belt must be the same in both cases,

$$\frac{\pi}{2}\Sigma_a+2C+\frac{\Delta_a^2}{4C}=\frac{\pi}{2}\Sigma_x+2C+\frac{\Delta_x^2}{4C},$$

from which

$$\Sigma_x=\Sigma_a+\frac{\Delta_a^2-\Delta_x^2}{2\pi C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

From this equation and the equation $\frac{D_x}{d_x}=\frac{n_x}{N}$, where N represents the r.p.m. of the driving shaft and n_x the r.p.m. of driven shaft when the belt is on the diameters D_x, d_x, the diameters D_x and d_x could be found. It is, however, generally accurate enough to solve equation (21) for Σ_x by substituting for Δ_x, which should be $D_x - d_x$ for the open belt, its value found by assuming the belt crossed; that is, solving for $D_x - d_x$, assuming $\Sigma_x = \Sigma_a$; thus

$$\frac{D_x}{d_x} = \frac{n_x}{N}.$$

$$\therefore \frac{D_x - d_x}{D_x + d_x} = \frac{n_x - N}{n_x + N},$$

and Δ_x (for a crossed belt) $= \dfrac{n_x - N}{n_x + N}\Sigma_a$. . . . . (22)

Substituting this in equation (21) we have

$$\Sigma_x = \Sigma_a + \frac{\Delta_a^2 - \left\{\dfrac{n_x - N}{n_x + N}\Sigma_a\right\}^2}{2\pi C}. \quad . \ . \ . \ . \ . \ . \ (23)$$

From this value of Σ_x and the fundamental relation $\dfrac{D_x}{d_x} = \dfrac{n_x}{N}$ we may calculate the values of the diameters D_x and d_x.

Hence the process of designing a set of speed pulleys is thus: Having given N, the r.p.m. of the driving shaft, and C, the distance between axes, decide on the speeds n_1, n_2, n_3, etc., of the driven shaft; choose either diameter of any pair of steps, and calculate the other diameter by equation (16), thus finding values for Σ_a and Δ_a. From these the diameters of any other pair of pulleys can be found. In the case of a *crossed* belt, the other diameters are found by using the two equations

$$D_a + d_a = D_x + d_x = \text{a constant}$$

and

$$\frac{D_x}{d_x} = \frac{n_x}{N}.$$

In the case of an open belt find values for Δ_1, Δ_2, Δ_3, etc., assuming the belt crossed as in equation (22). Substituting these values in equation (21) will give sufficiently accurate values for Σ_1, Σ_2, Σ_3, etc. From these values of Σ the successive pairs of diameters may be found.

In the case of conoids (Fig. 58), for an open belt, a series of equidistant diameters could be calculated for each, and curves drawn through the ends of these diameters would give the shapes for the conoids.

In designing a pair of speed-cones or pulleys, it is very often desirable to have them both alike so that they can be made from one pattern, the velocity ratios being thus fixed. The diameters of both pulleys, as in Figs. 56 and 58, would be the same for the central position of the belt; in Figs. 57 and 59, when the number of steps is odd, the middle diameters will also be the same, or $D_2 = d_2$; then $\Delta = D_2 - d_2 = 0$, and $\Sigma = D_2 + d_2 = 2D_2$.

69. Suppose we wish to construct two *equal* speed-cones (Fig. 60) to give to the driven shaft a range of speed between n_1 and n_n r.p.m., n_n being greater than n_1. Let the smallest diameters $D_1=d_n$ be given; it is necessary first to determine the largest diameters $D_n=d_1$, and N, the r.p.m. of the driving shaft. Assuming the belt on D_1, d_1, we may write $\frac{D_1}{d_1}=\frac{n_1}{N}$; and assuming the belt on D_n, d_n would give $\frac{D_n}{d_n}=\frac{n_n}{N}$; but since $D_1=d_n$ and $D_n=d_1$ we have

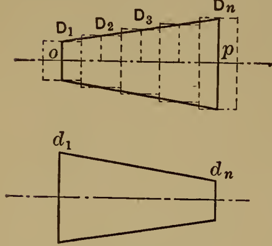


FIG. 60.

$$\frac{n_1}{N}=\frac{N}{n_n}, \quad \text{or} \quad N=\sqrt{n_1 n_n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Knowing the value of N the large diameters $D_n=d_1$ are readily found. If we assume the smallest diameters $D_1=d_n=4''$, $n_n=600$ r.p.m., and $n_1=60$ r.p.m., N will be found to be 189.7 r.p.m. and the largest diameters, $D_n=d_1$, will be $12''.65$.

If the belt were crossed, the cones shown in Fig. 60 would answer. In this case the middle diameters would be equal to $\frac{D_1+d_1}{2}=8''.32$; or if a stepped pulley having n steps were desired, divide op into $(n-1)$ parts, and erect perpendiculars D_2, D_3, etc., which would give the diameters of the successive steps; then draw in the pulley as shown by the dotted lines.

If the belt is open, the largest and smallest diameters are found as above. To find the middle diameters D_m, d_m, which are equal since the pulleys are to be alike, equation (21) is used, Σ_a being D_1+d_1 or D_n+d_n; Δ_a being D_1-d_1 or D_n-d_n; and Δ_x being the difference between the middle diameters D_m-d_m, which difference is zero since $D_m=d_m$. Making these substitutions in equation (21) we have

$$\Sigma_m=\Sigma_1+\frac{\Delta_1^2}{2\pi C}.$$

Using the same data as taken above with the crossed belt, and letting the distance between the axes $C=20''$, we find

$$\Sigma_m=16.65+\frac{74.82}{125.66}=17''.25,$$

and

$$D_m=d_m=8''.65.$$

Having thus determined the middle diameter, the curve of the conoid may be an arc of a circle passing through the extremities of the three diameters, as in Fig. 58. This middle diameter would also be correct for the middle step of any stepped pulley having an odd number of steps. Other steps might be found graphically by use of the principle of Fig. 60, the curve being used in place of the straight line; the results can be checked by calculation.

As a flat belt tends to climb a conical pulley, it is necessary, when speed-cones or conoids are used as in Figs. 56 and 58, to provide a shipper which will guide the advancing side of the belt for each pulley and retain it in its proper place; the guiding forks are moved simultaneously, and are placed as near each pulley as possible.

If the distance between the axes of the pulleys to be connected by an open belt is considerable, as is often the case in belting from an overhead shaft to a machine, or if, is as sometimes the case, one of the axes is adjustable (the proper tension on the belt being obtained by the weight of the pulley on the adjustable axis), the diameters can be calculated as though the belt were crossed. However, the distance between the axes may be large, and yet the range of speed may be so great, that with a fixed distance between the axes, an open belt would not be sufficiently tight toward the middle diameters if it were properly tight at the ends. In this case the diameters would need to be found as in the solution already given for the open belt.

70. Effective Pull.—By the *effective pull* on a belt we mean the pull that is doing the work, that is, the difference in the tension of the two sides of the belt. If in Fig. 55 we assume A the driver and B the follower, T_1 the tension in the tight side and T_2 the tension in the loose side of the belt, the effective pull will be $(T_1 - T_2) = P$. Suppose A to be 30″ in diameter, to make 200 revolutions per minute, and to carry a belt transmitting 4 horse-power or $4 \times 33{,}000$ ft.-lbs. per minute; what is the effective pull on the belt?

Here the work done by the belt, that is, the effective pull on the belt, in lbs., multiplied by its speed in feet per minute, must equal the horse-power transmitted multiplied by 33,000, or

$$\frac{30}{12} \times 200 \times 3.1416 \times P = 4 \times 33{,}000.$$

$$\therefore\ P = 84.03 + \text{lbs.}$$

From the above example it can readily be seen that the quicker a belt travels the smaller the pull for a given horse-power transmitted.

Belts and Pulleys to Connect Non-parallel Axes.

71. The *plane of a pulley* is a central plane through the pulley perpendicular to its axis, or is the plane of the pitch line of the pulley. The *point of delivery* of a pulley is the point in the pitch line at which the belt leaves the pulley.

The general principle governing the arrangement of belt-pulleys upon non-parallel axes may be stated as follows: *The belt must always be delivered in the plane of the pulley toward which it is running.* That is, a belt leaving a pulley may be drawn out of the plane of the pulley; but when approaching a pulley, its centre line must lie in the plane of the pulley.

Guide-pulleys are used to change the directions of belts, and are placed according to the above rule. It is possible, by the use of two guide-pulleys, to connect any pair of pulleys by an endless belt, and the guide-pulleys may be so placed that the belt will travel in either direction, which is sometimes a great advantage.

72. General Case.—Let AA and BB (Fig. 61) be the vertical and horizontal projections of a pair of pulleys to be connected by a flat belt, their planes intersecting on SS. Choose a and c convenient points on the trace SS; from a draw a line ab, and from c draw cd, both in the plane of A and tangent to the pitch line of A; similarly draw ae and cf in the plane of B. The two lines ab and ae determine the plane of the guide-pulley C, it being so placed that its pitch line is tangent to both ab and ae; in this position it can take a belt delivered either at e or b, as both e and b are in its plane; it can also deliver a belt to A or B, as it has its points of delivery in the planes of A and B. In the same manner, the two lines cd and cf in the planes A and B respectively fix the position of D. Having placed C and D, the belt may be put on as shown; the same side of the belt comes in contact with all of the pulleys, and it is immaterial which way it runs.

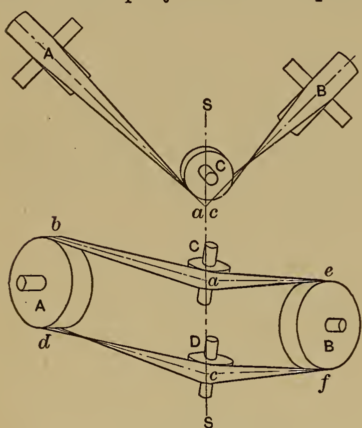


Fig. 61.

As double bending, i.e., bending back and forth, tends to injure a belt, it is desirable, when possible, to have the same side of the belt come in contact with each of the pulleys.

73. Quarter-turn Belt.—When a belt always travels in the same direction, it is only necessary to provide for its running properly in that direction, and in such a case the belt must always be delivered in the plane of the pulley toward which it is running.

Fig. 62 shows a *quarter-turn belt* connecting two pulleys A and B whose axes are in parallel planes and at right angles with each other. SS is the trace of the intersecting planes of A and B, and if we suppose the pulley A to revolve in the direction indicated by the arrow, it delivers the belt at a in the plane of B, while B delivers the belt in the plane of the pulley A; thus the rotation indicated by the arrows is allowable. If, however, we attempt to turn A in the opposite direction, the belt will immediately leave the pulleys, for d, the point of delivery of B, is not in the plane of A, nor is e in the plane of B. If the pulley B with its plane is swung about SS as an axis into any position such as B_1, the belt will still run in the direction indicated by the arrow, as the same conditions exist as before. In fact, if we draw on a piece of paper the pitch lines of two pulleys as A and C, and draw a line SS tangent to them, the paper may then be folded on SS, and it can easily be seen that the point of delivery a of the pulley A is in the plane of C, and that the point of delivery of C is in the plane of A, no matter how the plane of C is turned about SS.

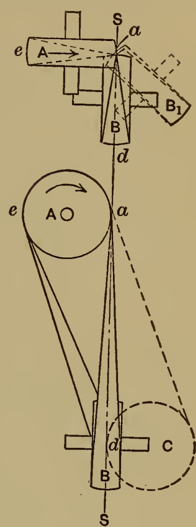


FIG. 62.

It is not well to use quarter-turn belts where there is any liability of the motion being reversed, as when this happens they will immediately leave the pulleys. The angle at which the belt is drawn off from the pulleys should not be great, since when this is the case the belt is much strained, and does not hug the pulley well; power is also lost in side-slipping. To make this angle as small as possible, the pulleys should be placed a sufficient distance apart, and they should be as small in diameter and carry as narrow a belt as practicable.

Fig. 63 shows a quarter-turn belt arranged with a double pulley so as to run in either direction; A is the driving pulley and B is the

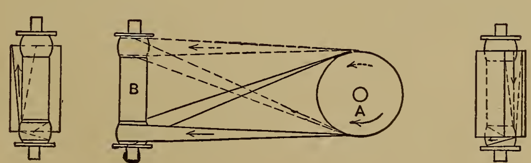


FIG. 63.

driven pulley, which is designed for two positions of the belt. For the rotation in the direction of the full arrow the belt is drawn in full lines; for the opposite rotation the belt is drawn dotted. This arrangement was made use of on a small moulding-machine where the spindle attached to B could be made to turn in opposite directions, its cutters being made to work when rotating in either direction.

Two shafts at right angles to each other are often connected by means of a belt and one guide-pulley, as shown in Fig. 64. Here *ab* is the trace of the two intersecting pulley planes, and the plane of the guide-pulley *C* is found by taking a point in *ab*, as *c*, and from it drawing lines *ce* and *cd* tangent to the pitch lines of *A* and *B* respectively, which fix the plane of *C*. This arrangement is often used to drive millstones, which are carried by the upright shafts of the pulleys like *A*; the faces of the upright shaft pulleys are made straight and considerably wider than the belt, so as to allow a slight motion up or down to adjust the stones, or a movement of the guide-pulley *C* to tighten the belt, which then merely runs on a different part of the pulley *A*.

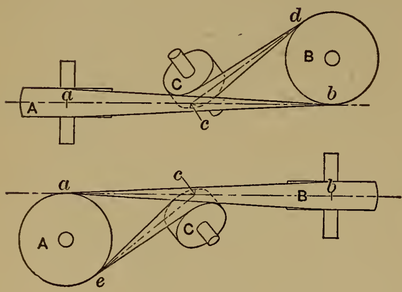


Fig. 64.

The shaft *A* might be arranged to carry a tight-and-loose pulley, and the belt might be shifted by moving the guide-pulley *C*, the motion of which should be such that the belt has the proper tension in each position. Whenever a belt is used in the above manner, the working part of the belt should pass from *a* to *b*, and not over the guide-pulley *C*; this should be true for all cases where one guide-pulley is used, as the extra friction, due to the working pull, is saved on the guide-pulley bearings.

Figs. 65 and 66 show another method of connecting two shafts at right angles, two guide-pulleys being used in each case. The belt can travel in either direction, and the same side of the belt comes in contact with each pulley, except in the case of *D* (Fig. 66), where it is not possible on account of the crossed belt. It will be noticed that the directional relation between *A* and *B* is the same in Figs. 65 and 66.

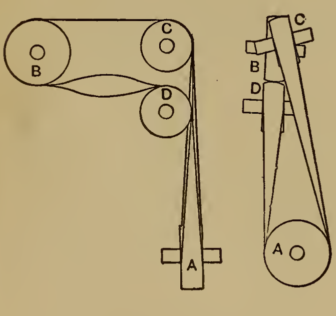


Fig. 65.

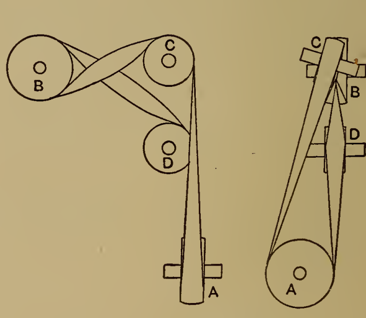


Fig. 66.

When a crossed belt is used to connect two pulleys, as in Fig. 54, it is necessary to give the two straight parts of the belt a *half-twist* to have them pass each other, and to bring the same side of the belt in contact with each pulley.

74. Let A and B (Fig. 67) be two pulleys whose axes are parallel, but whose planes do not coincide. Place the guide-pulleys C and D, whose diameters are equal to the distance between the planes of the pulleys, and whose planes are parallel to the axes of, and tangent to, the pulleys, as shown in the figure. The belt can then be applied as indicated, and can run in either direction. This arrangement could also be used to connect two axes quite near each other, and thus obtain a long belt, which works much better than a short one.

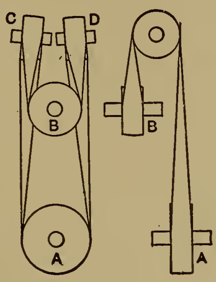


FIG. 67.

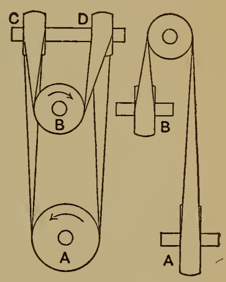


FIG. 68.

It is generally more convenient to place the guide-pulleys C and D on the same axis: such an arrangement is shown in Fig. 68, where the guide-pulley C is in a plane perpendicular to the common axis of the guide-pulleys and tangent to B, while D is in a plane parallel to that of C and tangent to the pulley A. In this case, the belt can run only in the direction indicated by the arrows.

We may now imagine the pulley B to be swung around, its plane still remaining tangent to C and D, and its axis in a plane parallel to that of the axis of A: the method of arranging the guide-pulleys remains the same. This arrangement is often used to connect axes in the same horizontal plane, or in two horizontal planes one a little above the other; the guide-pulleys are then placed on a common vertical shaft on which they turn loosely, being held in position by collars properly placed. The name *mule-pulleys* is often applied to guide-pulleys arranged, as indicated above, upon a vertical shaft.

Fig. 69 shows, in elevation, two shafts at right angles to each other connected by means of a belt and *mule-pulleys*. In order that a belt may run properly on pulleys having other than horizontal axes, they are made more crowning, and, when on perpendicular axes, have flanges on their lower edges; or, better, stationary flanges similarly placed, and carried by the shaft on which the pulleys revolve. The proper radial section for the guiding flange is shown at A, where a projecting lip serves to guide the belt when approaching the mule-pulleys, and the recess

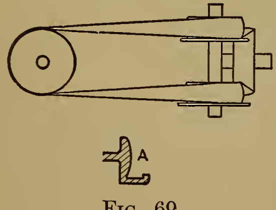


FIG. 69.

behind the lip permits the belt to lie flat on the pulley. A straight flange is liable to cause the belt to climb and strain its edge.

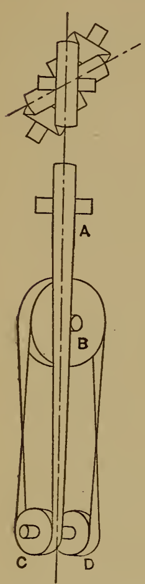


FIG. 70.

Fig. 70 shows a method of connecting two horizontal axes making an angle with each other. The perpendicular distance between the axes must be a little greater than the sum of the radii of the pulleys *A* and *B*. In this case, *A* and *B* are of the same size, and both guide-pulleys can be placed on the same shaft and will revolve together; the belt may also travel in either direction. *A* might drive *D*, and *B* and *C* then act as guide-pulleys.

75. **Binder-pulleys.**—Guide-pulleys are often used, as in Fig. 64, to increase the arc of contact of the belt, and also, as there, to tighten the belt; when so used, they are called *binder-* or *tightening-pulleys*, and are always applied to the loose side of the belt.

Pulleys for belts could be combined in many other ways, but the same principles govern the arrangement in each case. When convenient it is best to arrange the belt to travel in either direction, as engines are sometimes moved backward a part of a turn, thus rendering any belt not admitting of motion in either direction liable to be thrown off.

Cords and Ropes.

76. Pulleys for round ropes or cords must be provided with V grooves to keep the ropes in place; any flat bands not having sufficient lateral stiffness would also require flanges to keep them on the pulley.

Cords of small diameter are used to transmit small amounts of power, as for driving the spindles in spinning-machinery.

Hemp and cotton ropes are now very generally used to transmit quite large amounts of power; when so used they are often run in sets, each pair of pulleys carrying several ropes. The grooves are made V-shaped, and the ropes are drawn into them, thereby increasing the hold upon the pulleys. Rope pulleys are usually made of cast iron, and the grooves are turned. Fig. 71 *A* shows a section of the rim of a pulley for carrying a number of ropes.

Large amounts of power are now successfully transmitted over long distances by rapidly moving wire ropes carried by large, grooved wheels. Wire ropes will not support without injury the lateral crushing due to the V-shaped grooves; hence it is necessary to construct the pulleys with grooves so wide that the rope rests on the rounded bottom of the groove, as shown in Fig. 71 *B*, which shows a section of the rim of a wire-rope pulley. The friction is greatly increased, and the wear of the rope diminished, by lining the bottom of the groove with some elastic material, as gutta-percha, wood, or leather jammed in tight.

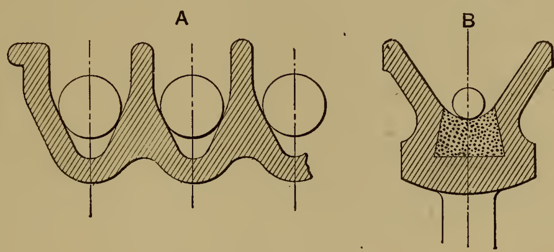


Fig. 71.

Cords and ropes, especially wire ropes, are in general only used to connect parallel axes, but when otherwise used the same rules as were given for flat bands govern the arrangement. As ropes have no lateral stiffness, they are not used to connect vertical axes without supplying guide-pulleys to insure the proper guiding of the ropes into the grooves.

77. **Drum, or Barrel.**—When a cord does not merely pass over a pulley, but is made fast to it at one end, and wound upon it, the pulley usually becomes what is called a *drum*, or *barrel*. A drum for a round rope is cylindrical, and the rope is wound upon it in helical coils. Each layer of coils increases the effective radius of the drum by an amount equal to the diameter of the rope. A drum for a flat rope has a breadth equal to that of the rope, which is wound upon itself in single coils, each of which increases the effective radius by an amount equal to the thickness of the rope.

78. Small cords are often used to connect non-parallel axes, and very often the directional relation of these axes must vary. The most common example is found in spinning frames and mules, where the spindles are driven by cords from a long, cylindrical drum, whose axis is at right angles to the axes of the spindles. In such cases, the common perpendicular to the two axes must contain the planes of the connected pulleys; both pulleys may be grooved, or one may be cylindrical, as in the example given above. Fig. 72 shows two grooved pulleys, whose axes are at right angles to each other, connected by a cord which can run in either direction, provided the groove is deep

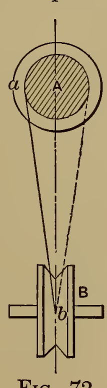


Fig. 72.

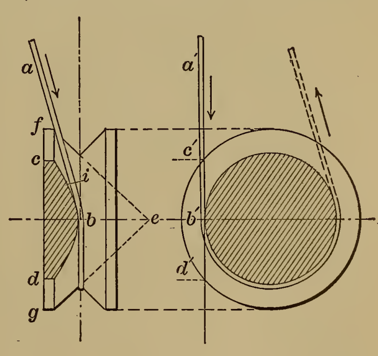


Fig. 73.

enough. To determine whether a groove has sufficient depth in any case, the following construction (Fig. 73) may be used. Let *ab* and *a'b'* be the projections of the approaching side of the cord; pass a plane through *ab* parallel to the axis of the pulley; it will cut the hyperbola *cbd* from the cone *feg*, which forms one side of the groove. The cord will lie upon the pulley from *b* to *i*, where it will leave the hyperbola on a tangent. If the tangent at *i* falls well within the edge of the pulley *c*, the groove is deep enough. It will usually be sufficient to draw a straight line, as *ab* (Fig. 72), and see that it falls well inside of the point corresponding to *c* in Fig. 73.

GEARING-CHAINS.

79. In cases where a considerable amount of power has to be transmitted between two shafts running at a slow speed, metal chains, called *gearing-chains* or *pitch-chains*, are used with toothed wheels or *sprocket-wheels*. When a chain is to drive or be driven by a sprocket-wheel, the acting surface of the wheel must be adapted to the figure of the chain, so as to insure sufficient hold between them. Guide-sheaves for chains are made circular, and grooved to suit the form of the chain. Chain *drums* or *barrels* have one end of the chain made fast to them, and the chain is guided by a suitably formed helical groove. Such drums are used in cranes.

Gearing-chains are usually made of flat links, riveted or pinned together at their ends, so as to allow a free turning of the links at the joints. The most simple form consists of double and single links arranged alternately, the single link being made thick enough to have the same strength as the double links.

A wheel for such a chain (Fig. 74) has a polygonal prism *abc* for its pitch surface, and when the links are of the same length, the pitch line is a regular polygon; each side of the polygon is equal to the effective length of a link, or the distance from centre to centre of the rivets. The teeth of the wheel are arranged to correspond to the chain, and are alternately single and double: the single teeth pass between the two thin flat parts of the double links, and the double teeth pass on both sides of the thicker single links. On larger wheels, the double teeth are sometimes left off, and their places are supplied by short projecting lugs, which serve to keep the chain

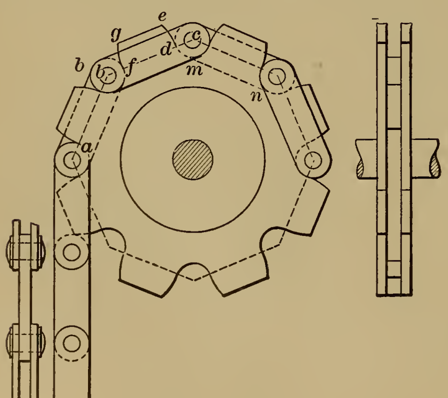


FIG. 74.

in position sidewise. The acting parts of the teeth outside of the pitch line are made up of arcs of circles, whose centres are the adjacent angles of the pitch polygon; thus the arc *de* is drawn with *b* as a centre, and *fg* with *c* as a centre. The space between the teeth is shaped to receive the chain, as *dmn*. The length of the teeth should be such that they come up to, or project a little beyond, the outer edge of the chain when it is in position on the chain-wheel.

The sprocket-wheel may be constructed without the double teeth *mn* when the links are short and the single and double links may have different lengths, the latter being longer to give sufficient strength to the tooth or sprocket. In such case the wheel is made a little thinner than the single links, its teeth decreasing in thickness toward their points to better enter the chain. The pitch line is a polygon having alternate long and short sides, and the pitch is usually taken as the combined length of a long and short link. This form of a chain (known as a block chain) and wheel, of one inch pitch is now very accurately and economically made of hardened steel and is extensively used on bicycles.

A gearing-chain is sometimes made up of two systems of links, separated by an enlargement of the pins connecting the successive links, as shown in Fig. 75. The sprocket-wheel is placed in the space between the two systems of links, and the teeth gear with the enlarged portions of the pins connecting the links; their acting surfaces are arcs of circles, as in the preceding case.

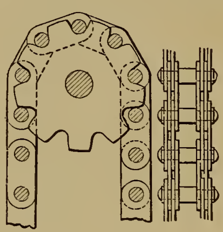


FIG. 75.

Light chains of this form are now extensively used on automobiles and are supplied with rollers to reduce friction and wear. Fig. 76 shows a roller-chain as made by the Whitney Mfg. Co. The rollers *R* turn on the hollow sleeves *S* supplied with flats at their ends to keep them from turning in the inner links. The rivets *P* are fast in the outer links *L* and turn on the inside of the sleeves *S*.

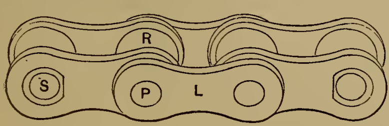


FIG. 76.

Fig. 77 shows a wheel for a chain having oval links; here the pitch line is a polygon, with alternate long and short sides; the pitch of the link lying flatwise is equal to its longer diameter plus the diameter of the link iron, while that of the link standing edgewise is equal to its longer diameter minus the diameter of the link iron. In this case, the short sides of the polygon only are provided with teeth, which act on the ends of the flat-lying links, and receive those standing edgewise between them.

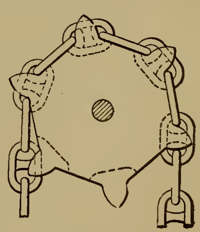


FIG. 77.

Chain-wheels for oval chains, which are the most common, are often made with a groove to receive the links standing edgewise, and a series of pockets into which the links lying flatwise fit. This form of wheel is often used to exert a pull on a chain passing partially around it, as shown in Fig. 78. Here the pull is a downward one on the chain X, which passes under the pocketed chain-wheel A, and over a guide-sheave E, from which it passes downward, and is deposited in a box. A *chain-guide*, C, is placed under the wheel A, to insure the

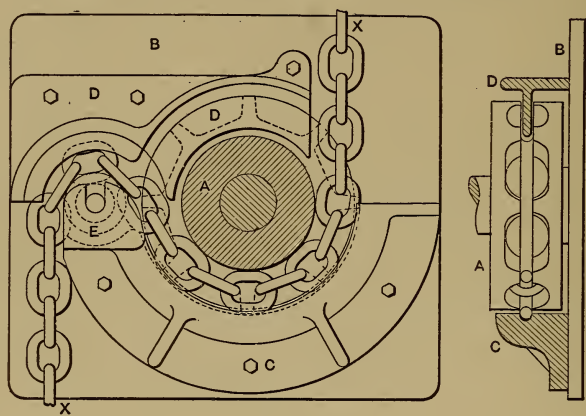


Fig. 78.

proper pocketing of the chain and guidance to the sheave E; this *guide* is provided with a groove in which the links standing edgewise move, and it is placed far enough away from the chain-wheel to allow the links to move freely when properly pocketed. A *chain-stripper*, D, passing into the groove of the wheel A, removes the chain from the wheel at the proper place: its action can be clearly seen from the figure. A prolongation of the *stripper D* covers the guide-sheave E, and insures the proper passing downward of the chain.

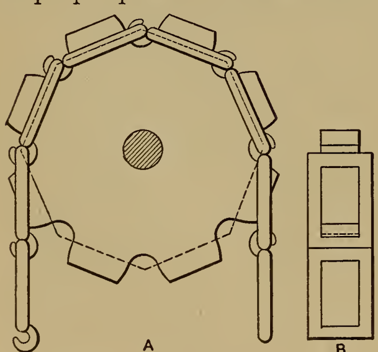


Fig. 79.

The chain could be arranged to pass over a chain-wheel: it would then exert an upward pull. In such cases, the guide-sheave is dispensed with, and a *guide* is placed at the point where the chain comes in contact with the wheel to insure the proper pocketing of the links. This arrangement is often used in small hoisting-machines, the weight being directly lifted by the chain which passes over the chain-wheel.

A geared chain formed of square open links made of malleable iron is now very extensively used in agricultural and other machinery requiring light geared chains. Fig. 79 *A* shows a side elevation of the chain and sprocket, and at *B* the form of the link and the method of fastening are shown. The sides and one end of each rectangular link are round in section, and the remaining end carries a hook-shaped projection which is as wide as the rectangular opening, and hooks over the rounded end of the adjacent link; the rectangular spaces between the hooks and opposite ends of the links receive the teeth of the wheel, which act against the rounded inner parts of the hooks as on round pins. The links are so made that they can be slid together sidewise when their planes are about at right angles.

Very large chain-wheels may have cylindrical faces provided with suitably formed projections or hollows.

The chief objections to chain gearing are the liability of stretching of the links forming the chain, and the wearing of the pins, or links, both of which tend to alter the pitch, and thus cause bad fitting between the chain and the wheel. The necessity for slow motion is also sometimes a disadvantage.

80. High-speed Chains.—None of the above-mentioned chains, even if very accurately made, can be run at high speeds without noise which rapidly increases with the wear. A form of high-speed chain has been developed by Hans Reynold known as the *Reynold silent chain.* It consists of links *C* of a peculiar form with straight bearing edges *a*, *b*, Fig. 80, which run over cut sprocket-wheels with straight-sided teeth

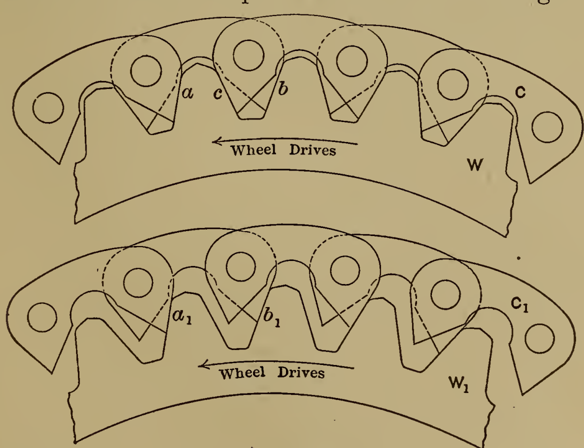


Fig. 80.

whose angles vary with the diameter of the wheel. The chain may be made any convenient width, the pins binding the whole together. One sprocket of each pair is supplied with flanges to retain the chain in place.

The upper figure shows a new chain in position on its sprocket, the bearing points of the links a, b, c being on the straight edges of the links only, not on the tops or roots of the teeth. The chain thus adjusts itself to the sprocket at a diameter corresponding with its pitch, and as any tooth comes into or out of gear there is neither slipping nor noise. The lower figure shows the position taken by a worn chain of increased pitch on the same wheel, a_1 and b_1 being the new bearing surfaces on the wheel W_1.

Morse Rocker-joint Chain.—This chain is an improved form of the silent-running type and is now extensively used in place of belting and gearing at speeds up to 2000 ft. per minute. In it the links are rounded at one end and made to fit the sprocket-wheel at the other, as shown in Fig. 81. The teeth of the sprocket-wheel are unsymmetrical, their

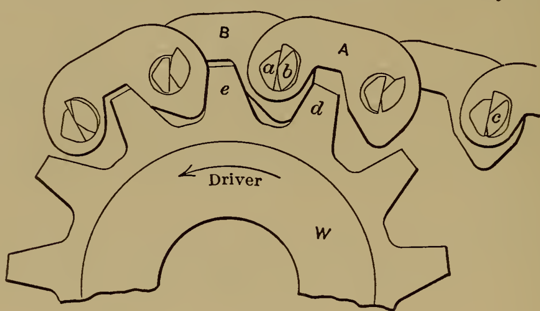


Fig. 81.

working faces d being more nearly radial, thus reducing any tendency to slipping in the chain. The ordinary pin bearings are here replaced by rocker-joints consisting of two pieces of hardened tool-steel, a and b, so shaped and arranged that in passing on and off the sprocket one piece rocks upon the other. Each link A has fast in one end the seat-piece a and at the other a rocker-piece like b, and is so shaped as to be free to move through the required angle on parts similarly held in the adjoining links, such as b in link B. Each outside link is bent laterally so that the large end comes under the small end of the adjoining link to permit of the proper engagement with both seat and rocker piece. The shouldered ends of the seat-pieces a are softened to allow their being riveted either to the outside links or to washers.

To prevent undue vibration under high speeds and consequent wear, the rocker-pieces are so shaped that the contact surfaces are greatly increased when the chain is drawn straight, as shown at c, thus giving a broad bearing surface which, while stiffening the chain, also reduces the pressure on the parts designed for rolling except when the chain is passing over its sprockets. This chain, having so very little friction, has an exceedingly high efficiency.

To keep the chain in place on its sprockets special guiding links are supplied which project below the chain into grooves turned in the sprockets.

CHAPTER VI.

LEVERS.—CAMS.

81. **Levers.**—We often have occasion to transfer some small motion from one line to another; we will consider three cases depending on the relative positions of the lines of motion:

1° Parallel lines.
2° Intersecting lines.
3° Lines neither parallel nor intersecting.

In 1° we have the common lever (Fig. 82), where, when the lever has a small angular motion, the points a and b receive motions proportional to their distances from the centre c, and approximately in the parallel lines aa_1 and bb_1. If we suppose the lever to move, we have

$$\frac{\text{l.v. } a}{\text{l.v. } b}=\frac{ac}{bc}.$$

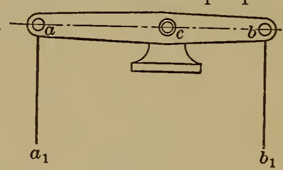


Fig. 82.

If P and W denote the forces applied at a and b respectively, we have, if they are in equilibrium,

$$P\times ac=W\times bc, \quad \text{or} \quad \frac{P}{W}=\frac{bc}{ac},$$

or the forces are inversely proportional to the lengths of their lever-arms.

82. **Bell-crank Lever.**—In 2° we have the *bell-crank lever acb* (Fig. 83), where a and b move approximately in the lines ad and bd, intersecting at d. Here, as in 1°, the velocities are proportional to the lengths of the lever-arms, or are proportional to the sines of the angles adc and cdb made by the line cd with the directions of the motions.

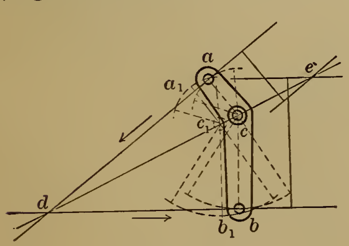


Fig. 83.

Suppose the angle adb to be given and a bell-crank lever is required that will give a motion along ad equal to one-half that along bd.

Draw the line dc, dividing the angle adb into two parts whose sines are directly proportional to the velocities of a and b. This may be done by erecting perpendiculars on ad and bd in the ratio of the required motions along those lines, and drawing through their extremities lines parallel to ad and bd respectively; the intersection of these lines at e determines the line de. Choose any point c in de, and drop the perpendiculars ca and cb on ad and bd respectively; then acb is the bell-crank lever required.

Here, as in the previous case, if we suppose the lever to move, we have

$$\frac{\text{l.v. } a}{\text{l.v. } b}=\frac{ac}{bc}=\frac{\sin cda}{\sin cdb}.$$

It is evident that, for a small angular motion, the movements in ad and bd are very nearly rectilinear, and they will become more and more so the farther we remove c from the point d.

Any slight motion that may occur perpendicular to the lines ad and bd may be provided for by the connectors used. It is to be noticed, however, that for a given motion on the lines ad and bd these perpendicular movements, or deviations, are less when the lever-arms vibrate equal angles each side of their positions when perpendicular to the lines of motion, and they should always be arranged to so vibrate. By moving the point c nearer to d, at the same time keeping the lever-arms the same, this perpendicular deviation could be disposed equally on each side of the lines of motion, which is advisable especially in cases where the deviation is allowed for by the spring of the connecting piece. This is shown in position $a_1c_1b_1$, Fig. 83. In Fig. 83 it will be seen that a and b move in opposite directions, while Fig. 84 shows the result if a and b are to move in the same direction.

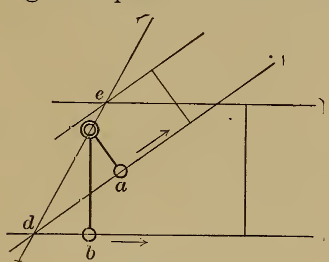


FIG. 84.

In 3° we have the lines of motion ad and bc (Fig. 85) neither parallel nor intersecting. Draw ee_1, the common perpendicular to the lines ad and bc; through e_1 draw e_1d_1 parallel to ed; construct a bell-crank lever a_1f_1b, for the desired movements considered as transferred to the lines bc and e_1d_1 in one plane; draw ff_1 parallel to ee_1 and equal to it, and further make af parallel and equal to ff_1. The piece aff_1b will be the lever required.

What has been done is this: a bell-crank lever has been constructed in a plane, passing through the first line of motion and parallel to the second line, the transferred

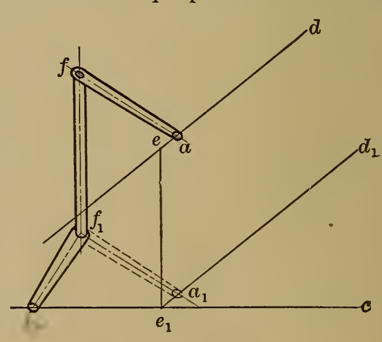


FIG. 85.

second line of motion in this plane being a projection of the second line of motion; or the lever a_1f_1b has been arranged to transfer the motion from bc to a_1d_1; then the motion along a_1d_1 has been transferred to the second line ad parallel to a_1d_1 by moving the lever a_1f_1 parallel to itself an amount equal to the perpendicular distance between the two lines of motion, and connecting f_1 and f by means of a shaft, so that af and bf_1 turn together about ff_1. The *rocker-arm* on a locomotive, which transfers the motion from the link, inside of the engine frame, to the valve-rod, outside of the engine frame, is an example of this form of lever.

83. **Cams and Wipers.**—A cam is a curved plate or groove which communicates motion to another piece by the action of its curved edge. This motion may be transmitted by sliding contact; but where there is much force to be transmitted, it is often by rolling contact.

If the action of the piece is intermittent, it is sometimes called a *wiper;* that is, a cam, in most places, is continuous in its action, while a wiper is always intermittent: but a wiper is often called a cam notwithstanding.

In most cases which occur in practice the condition to be fulfilled in designing a cam does not directly involve the velocity ratio, but assigns a certain series of definite positions which the follower is to assume while the driver occupies a corresponding series of definite positions.

The relations between the successive positions of the driver and follower in a cam motion may always be represented by means of a diagram

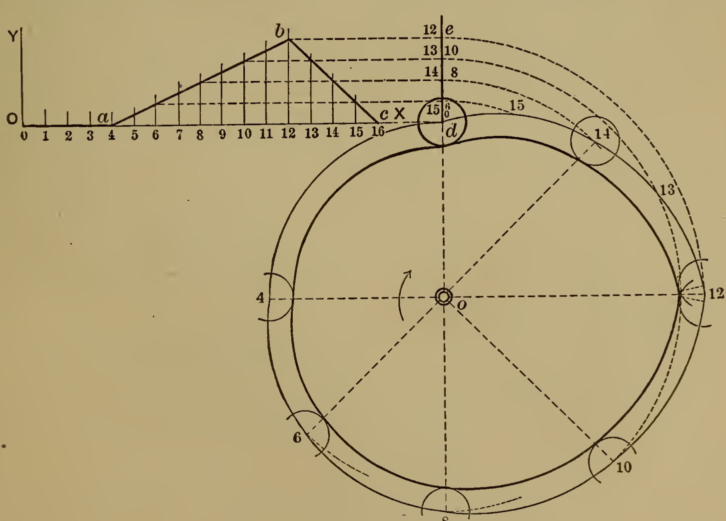


Fig. 86.

such as is given in Fig. 86, where the line $Oabc$ represents the motion given by the cam. The perpendicular distance of any point in the line from the axis OY represents the angular motion of the driver, while the perpendicular distance of the point from OX represents the corresponding movement of the follower, from some point considered as a starting-point. Thus the line of motion $Oabc$ indicates that from the position 0 to 4 of the driver, the follower had no motion; from the position 4 to 12 of the driver, the follower had a uniform upward motion $b12$; and from position 12 to 16 of the driver, the follower had a uniform downward motion $b12$, thus bringing it again to its starting-point.

84. Where the cam acts upon the point for which the motion is given, as d in Fig. 86, and where the motion of the point is in a straight line, passing through the axis of the cam, it is only necessary in constructing the pitch line of the cam to make the radii of such length as to bring the follower to the desired positions after the required intervals of rotation of the cam. Thus in Fig. 86 the point d, the axis of the roller, is to be still for one quarter of a turn; then it is to move up the distance de uniformly in one half a turn; and then down the same distance uniformly in one quarter of a turn. Let the cam turn right-handed. Draw radial lines from the axis of the cam, at intervals corresponding with those taken on OX in the diagram, in this case sixteen lines, at equal intervals. The pitch line from 0 to 4 is an arc of a circle subtending 90° with a radius equal to od. In the next 180° there are eight equal intervals and the radii $o6$, $o8$, etc., are made equal to the distances $o6$, $o8$, etc., on the line of motion ode. The remaining 90° is divided into four equal intervals, and the radii $o13$, $o14$, etc., are made equal to the distances $o13$, $o14$, etc., on ode. The curve through the points 4, 6, 8, etc., will give the pitch line of the cam, which line would cause the axis of the roller to move with the desired motion.

If a roller is used, the cam outline lies inside the dotted line, as shown in Fig. 86, a distance equal to the radius of the roller, and may be found by striking arcs from the pitch line with a radius equal to that of the roller, as shown, and drawing a smooth curve tangent to these arcs. The heavy curve shows the cam outline, which, acting upon the roller, will give its axis the desired motion.

It will be noticed that at the point 12, using the roller, the axis is not moved quite so far as e. The use of a roller will often be found to prevent the exact equivalent of the motion as given by the pitch line, the two normal curves overlapping at their juncture, thus cutting off a part of each.

It will be noticed (Fig. 86) that the cam can only drive the roller in one direction, viz., away from the centre o; in order to provide for the return, a spring or the weight would have to be relied upon, tending to force d toward o. Now if we suppose the cam-plate to be extended beyond the roller, and cut a groove in it which would be the envelope of the successive positions of the roller, by the principle of Fig. 14 it would act upon the roller equally well in either direction. The groove in this case would have parallel sides, and should be a little wider than the roller to prevent binding, the play being allowed by taking material from the non-acting surface of the cam. Cams are usually supplied with rollers, as they greatly reduce the friction and wear.

85. Diagrams for Cams giving Rapid Movements.—It is very often the case that a cam is required to give a definite motion in a short interval of time, the nature of the motion not being fixed. We will now discuss the form of the diagram for such a motion.

In the diagram shown in connection with Fig. 86 the follower had two uniform motions, and if the cam be made to revolve quickly, quite a shock will occur at each of the points where the motion changes, as *a*, *b*, and *c*; to obviate this the form of the diagram can be changed, provided it is allowable to change the nature of the motion.

Suppose we wish a cam to rapidly raise a body from *e* to *f* (Fig. 87), the nature of the motion to be such that the shock shall be as light as possible.

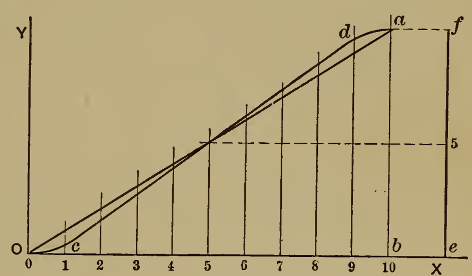


Fig. 87.

If we draw the straight line *Oa*, we have the case of a uniform motion (as in Fig. 86), the body being raised from *e* to *f* in an interval proportional to *ob*; here the motion changes suddenly at *O* and *a* accompanied with a perceptible shock. The line *Ocda* would be an improvement, the follower not requiring so great an impulse at the start or near the end of the motion, each being much more gradual than before.

The body may be made to move with a *simple harmonic motion*, the diagram for which would be drawn as follows (Fig. 88).

Draw the semicircle *e5f* on *ef* as a diameter; divide the time line *Oh* into a convenient number of equal parts (in this case ten), and then divide the semicircle into the same number of equal parts; through the divisions of the semicircle draw horizontal lines intersecting the vertical lines drawn through the corresponding points of division of the time line *Oh*, thus obtaining points, as *a*, *b*, *c*, etc. A smooth curve drawn through these points gives the full curve *Oabcd . . . n*. Here the body or follower receives a velocity increasing from zero at the start to a maximum at the middle of its path, when it is again gradually diminished to zero at *f*, the end of its path.

This form of diagram gives very good results, and is satisfactory in many of its practical applications.

A body dropped from the hand has no initial velocity at the start, but has a uniformly increasing velocity, under the action of gravity, until it

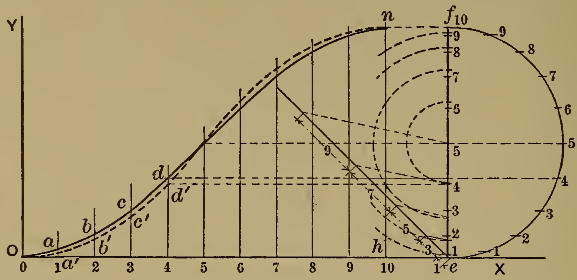


FIG. 88.

reaches the ground; similarly, if the body is thrown upward with the velocity it had on striking the ground, it will come to rest at a height equal to that from which it was dropped, and its upward motion is the reverse of the downward one, that is, a uniformly retarded motion.

In designing a cam for rapid movement the motion of the follower should obey the same law of gravity, and have a uniformly accelerated motion until the middle of its path is reached, then a uniformly retarded motion to the end of its path.

A body free to fall descends through spaces, during successive units of time, proportional to the odd numbers 1, 3, 5, 7, 9, etc., and the total space passed over equals the sum of these spaces.

To develop a line of action according to this law upon the same time line *Oh*, and with the same motion *ef*, as before, proceed as follows:

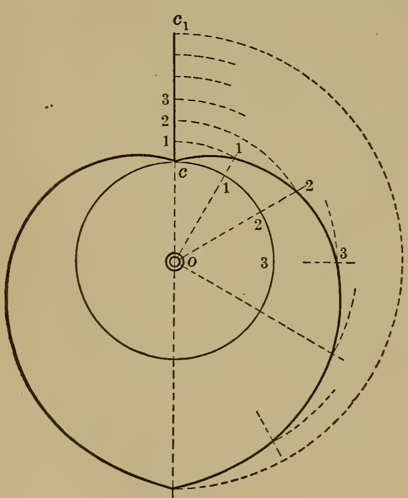


FIG. 89.

Divide the time line *Oh* into any *even* number of equal parts, as ten; then divide the line of motion *ef* into successive spaces proportional to the numbers 1, 3, 5, 7, 9, 9, 7, 5, 3, 1, and draw horizontal lines through the ends of these spaces, obtaining the intersections *a′*, *b′*, *c′*, etc., with the vertical lines through the corresponding time divisions 1, 2, 3, etc.; a smooth curve, shown dotted in the figure, drawn through these points, will give the cam diagram.

86. Heart Cam.—If the desired motion is to be on a line passing through the axis of the cam, uniformly up in one half a turn of the cam and uniformly down in the remaining half-turn, a heart-shaped cam will be the result, as shown in Fig. 89. The curve for the pitch line of such a cam will be found to be the *spiral of Archimedes*, as it fulfils the polar equation for that curve,

$$r = d + m\theta,$$

where r represents the radiant, d the distance of the starting-point c on the *initial line* cc_1 from the *origin of co-ordinates* o, θ the angular motion of the radius vector, and $m\theta$ the increase in the radius vector for the angular motion θ, or the motion of the cam. If the spiral starts at the origin o, $d=0$, and the equation then becomes

$$r = m\theta.$$

87. If the line of motion of the follower-point does not pass through the axis of the cam, the construction shown in Fig. 90 is used.

Here ab is the line of motion of the follower, and the point a is to be uniformly raised through the distance ab while the cam turns uniformly left-handed five eighths of a turn; it is then to be suddenly dropped to its first position a, and remain there for the remaining part of the cam rotation. Draw the small circle cc_1c_6 tangent to the line cab with o as a centre; all the positions of the line ab will be tangent to the small circle if we suppose the line ab to revolve about o right-handed; draw a circle through any point on the line of motion ab, as d, with o as a centre, and make the angle $dod_6 = coc_6$. Divide the arc dd_6 into any convenient number of equal parts, and divide the distance ab into the same number of equal parts. From the divisions on the arc dd_6 draw lines d_1c_1, d_2c_2, etc., tangent to the circle cc_1c_6. These lines, on rotating the cam, will coincide successively with the line of motion ab. To find a point on the cam outline as e_1, draw an arc through a_1 with o as a centre, and note e_1 where it intersects d_1c_1; other points can be found in a similar way. The part of the cam producing no motion is made circular.

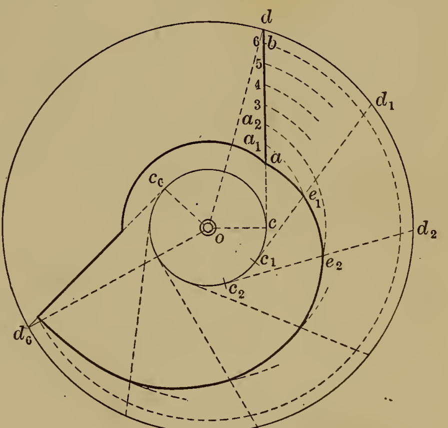


FIG. 90.

88. Involute Cam.—If the distance ab (Fig. 90) is equal to the arc cc_1c_6 of the small circle, the point a would have a motion such as would be derived by unwrapping a band from the small circle, and the curve of the cam would then be the involute of the small circle, thus giving an *involute cam.*

If the distance through which the point is to be raised and the corresponding angular motion θ of the cam are known, we have for the unknown radius r of the involute circle the equation

$$r\theta = \text{Distance}; \text{ or, } r = \frac{\text{Distance}}{\theta},$$

θ being expressed in circular measure.

After determining the radius r of the small circle, the line of motion can be located, and the cam constructed, as in Fig. 90.

This form of cam is often used to raise the stamps of an ore-crusher, or for giving a uniform upward movement to a rod passing by the shaft of the cam.

89. **General Case.**—Suppose we have given (Fig. 91) the position of the axis o of a plate cam; that the cam turns uniformly right-handed and gives motion to the slide D, in a straight line ef by means of the lever dcb centred at c, and the rod ab, the cam to act on the point d, on which as an axis a roller could be placed; that the slide D shall remain stationary for the first quarter of a turn, move uniformly up on ef an amount aa_6 in the next half-turn, and then move with simple harmonic motion down an amount a_6a in the remaining quarter of a turn, to find the pitch line of the cam.

First, draw the motion diagram, where Oh is taken to represent 360° of motion of the cam. For the first quarter of a turn there is no motion of D, thus giving the line $O0 = \frac{1}{4}\,Oh$, coincident with OX; for the next half-turn, from 0 to 6 equal to $\frac{1}{2}\,Oh$, there is a uniform upward motion $g6 = aa_6$; this would be indicated by the straight line $0g$, and any intermediate position of D between a and a_6 could be found by drawing an ordinate at the cam position; similarly, for the next quarter of a turn the curve gh would represent the motion of D, this curve being found by drawing the semicircle, shown on OY, with a diameter equal to aa_6, the distance to be moved through, and then proceeding as in Fig. 88. Transfer the ordinates thus found, at the points of division of Oh, to the line of motion aa_6 as indicated.

From the positions a, 1, 2, 3, etc., of the point a we can, knowing the length of the rod ab, and that the points b and d move in arcs of circles about c, determine the corresponding positions of the point d on which the cam is to act, marked d, 1, 2, 3, etc.

Now, having found a series of definite positions of the point d corresponding with a definite series of positions of the cam, we will concern ourselves only with its motion on an arc about the centre c, relative to the cam turning on the axis o. To find the pitch line of the cam, note that the relative motion of the cam and the point d moving about the centre c is the same whether we consider the point c as fixed and the cam to turn R.H. or whether we consider the cam as fixed and the lever

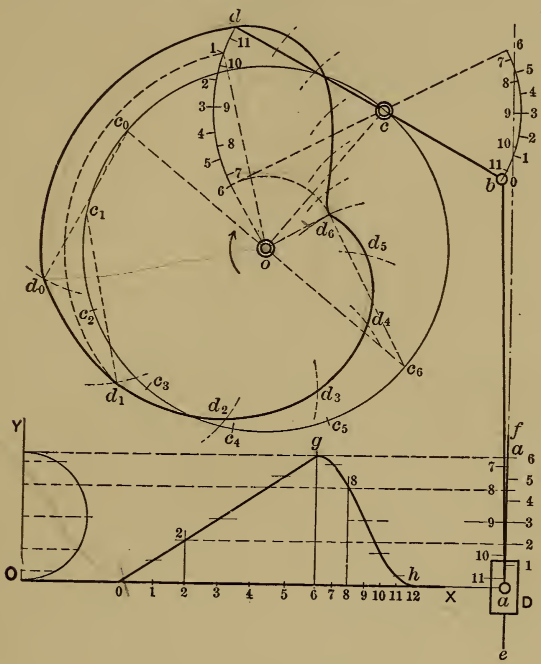


FIG. 91.

to revolve around the axis of the cam L.H., the point d at the same time having its proper rotation about c, as determined by the positions d, 1, 2, etc. Draw a circle through c with o as a centre, and divide it, proceeding in a left-handed direction, into parts corresponding with the divisions of the diagram. These points c_0, c_1, c_2, etc., would successively pass through c. When c_1 is at c the point d_1 would be at the position 1 on the arc through which the end d, of the lever bcd, moves, d_1 being located at a distance $d_1c_1 = dc$ from c_1, and at a distance $o1$ from o equal to the desired distance of the point d from the centre of the cam. In the same way the points d_2, d_3, etc., are found. Thus when the point c_6 is at c, d_6 will be at the point 6 on the arc through which the point d moves, d_6 being found by making $c_6d_6 = cd$, and $od_6 = o6$, the distance from o to the desired position of d. A smooth curve through the points d_0, d_1, d_2, etc., will be the pitch line of the cam. If greater accuracy is required, intermediate points should be found, or shorter intervals of rotation should be taken in those parts of the rotation where greater accuracy is desired.

It is interesting to notice in connection with Fig. 91 that, if the edge of a thin plate is shaped like the diagram, and the plate is then moved uniformly a distance Oh along OX (its edge acting on a), while the cam disc turns uniformly once right-handed, a pencil carried at d would trace the outline of the cam on the disc. The diagram could also be drawn upon a piece of paper moving uniformly along OX a distance Oh, by a pencil carried at the point a, and moving under the influence of a uniform right-handed rotation of the cam.

90. Positive-motion Cam.—When a cam actuates its follower equally well in both directions without external aid, as force-closure, it is called a *positive-motion cam*, the elements being so paired that only one motion is possible between them.

There are other means than the use of a groove for insuring the positive motion of the follower not open to the objection of binding the follower roller in its groove. Two rollers might be used, working on opposite sides of the same cam, and always situated diametrically opposite each other; in such a case, the original cam outline, that is, the outline passing through the centre of the rollers, must have a constant diameter equal to the distance between the centres of the rollers.

Such a cam is shown in Fig. 92, where the rollers A and B bear on opposite sides of the cam, and are carried by the frame CC. During the first $\frac{1}{4}$ turn of the cam in the direction of the arrow, the roller A moves to A_1 with a uniform motion; it rolls on the cam surface from s to s_1, while B rolls from s_2 to s_3; during the second $\frac{1}{4}$ turn A remains at A_1, the circular part of the cam s_1s_2 acting upon it, s_3s acting at the same time on B; during the third $\frac{1}{4}$ turn A_1 moves back to A under the action of s_2s_3, ss_1 acting on B; and during the last $\frac{1}{4}$ turn A remains stationary, s_3s acting upon it, while s_1s_2 acts on B.

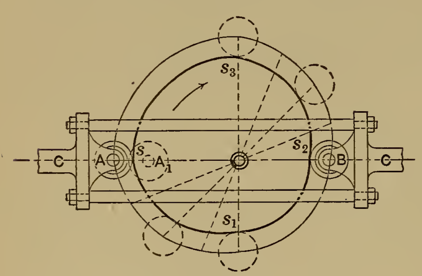


Fig. 92.

Two cams might be arranged side by side on the same shaft, and act on two connected rollers, one running on one, and the other on the other cam; this would render possible a more complicated motion than that shown in Fig. 92.

Two cams might also be used, as in Fig. 93, turning on separate shafts, but uniformly in relation to each other.

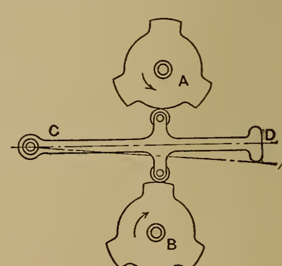


Fig. 93.

Here the cams are counterparts of each other, and have the sums of the radii from the centres of A and B to the centres of the rollers a constant. This arrangement has been used to operate the harnesses of looms, by connecting the end D of the lever CD to the harness frame.

91. It is often the case that only a few positions of the follower of a cam are fixed, and it is not particular what the intermediate motion is. In such a case, the outline of the cam may be constructed by passing arcs of circles through the fixed points of the cam in such a way as to make a smooth curve.

Fig. 94 shows such a cam as applied to the lever of a punching-machine. Here the cam turns right-handed; the left-hand end of the lever is raised by the action of the arc of the cam extending from d to c, thus bringing the punch, the slider of which is worked by the end of the lever to the right of the fulcrum e, down to the metal. A wheel is placed at c to lessen the friction during the punching; after the punching, the lever is lowered by the action of the arc cb. The part of the cam dab is made circular about the axis o, and for nearly one half a turn of the cam the lever remains at rest with the punch raised from the metal, thus allowing the workman time to adjust the plate for the next punching. The cam is often arranged to slide, by means of a key and keyway, on its shaft, and is brought under the lever, when required, by means of a treadle moved by the foot of the person operating the machine.

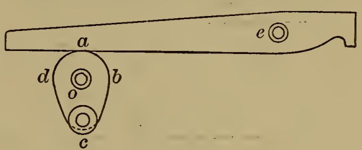


FIG. 94.

92. All cams thus far discussed have completed their action in one turn; by suitably shaping the follower, this action can be extended so as to require more than one turn, but the cams then become complicated, and are not much used on that account.

Fig. 95 shows a cam where two turns are necessary to complete its action. The follower has the elongated form shown at F in order that it may properly pass the intersection of the cam groove, which is the envelope of the follower. If the cam turns right-handed, there is a period of rest for the follower during the first $\frac{1}{2}$ turn of the cam, due to the circular groove ab; during the second $\frac{1}{2}$ turn the lever moves to the dotted position under the action of bc; cd then retains it for the third $\frac{1}{2}$ turn, and da returns it to its first position during the remaining $\frac{1}{2}$ turn.

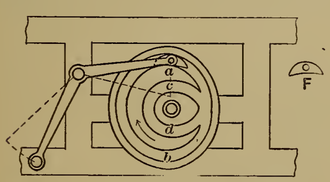


FIG. 95.

93. Two cams might be made, by means of a system of levers and rods, to act on the same point, the motions governed by each cam being in lines situated in one plane and making right angles with each other; the point so governed could be made to trace any plane curve within the limits of its movements.

If a plate cam is required to produce more than one double oscillation of a vibrating lever during one revolution, its edge would be formed into a corresponding number of waves; if a groove is used, the centre line of the groove would have the same series of waves.

94. Cylindrical Cams.—A cam may be made by cutting the proper groove around a cylinder; the motion of the follower would then be in a direction parallel to the axis of the cam. Such a cam, shown in Fig. 96 *A*, produces a motion similar to that given by the cam shown at *B*. Both forms are very extensively applied to actuate the feed mechanisms in some machine-tools; their period of action only occupies a small part of their rotation, and comes just before the tool is ready for a cut.

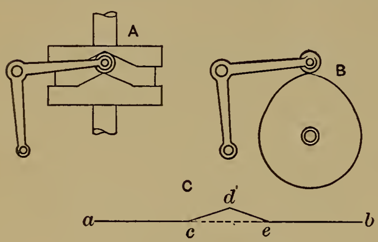


FIG. 96.

A cam like that shown in Fig. 96 *A* can be constructed by laying out, upon paper, its motion diagram (Fig. 96 *C*) on a line *ab*, equal in length to the circumference of the cam cylinder, and wrapping this diagram around the cylinder, taking care that the line *ab* is kept in a plane perpendicular to the axis of the cylinder. The centre line of the groove being thus determined, the groove can be made the envelope of the follower when it moves along this centre line.

A groove having a centre line *acdeb* (Fig. 96 *C*) might be cut in a flat plate which has a rectilinear motion along *ab*; a follower moving on a line perpendicular to *ab*, supplied with a roller working in the groove, would have a motion very nearly the same as that obtained by *A* and *B*. If the lines of motion in *A* and *B* were straight instead of curved, the roller in *C* would have exactly the same motion as in *A* and *B*. Cam grooves, cut in plates having a rectilinear motion, are often used in practice.

A cylindrical cam having a helical groove is shown in Fig. 97. The follower *F* is made to fit the groove sidewise, and is arranged to turn in the sliding rod, to which it gives motion in a line parallel with the axis of the cam. The guides for this rod are attached to the bearings of the cam, *A* and *B*, which form a part of the frame of the machine. A plan of the follower is shown at *G*: its elongated shape is necessary so that it may properly cross the junctures of the groove. In this cam there is a period of rest during one-half a turn of the cam at each end

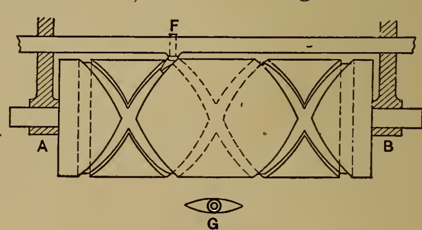


FIG. 97.

of the motion; the motion from one limit to the other is uniform, and consumes one and one-half uniform turns of the cam.

The cylinder (Fig. 97) may be increased in length, and the groove may be made of any desirable pitch; the period of rest can be reduced to zero, or increased to nearly one turn of the cam. A cylindrical cam, having a right- and a left-handed groove, is often used to produce a uniform reciprocating motion, the right- and left-handed threads or grooves passing into each other at the ends of the motion, so that there is no period of rest.

The period of rest in a cylindrical cam, like that shown in Fig. 97, can be prolonged through nearly two turns of the cylinder by means of the device shown in Fig. 98. A switch is placed at the junction of the right- and left-handed grooves with the circular groove, and it is provided that the switch shall be capable of turning a little in either direction upon its supporting pin, while the pin is capable of a slight longitudinal movement parallel with the axis of the cylinder This supporting pin is constantly urged to the right by a spring, shown in *A*, which acts on a slide carrying the pin; when in this position the space *a* between the switch and the circular part of the groove is too small to allow the follower to pass, and when the follower is in the position shown in *B*, the spring is compressed; then, if the follower moves on, the space behind it is closed, as the spring will tend to push the support to the right, and swing the switch on the follower as a fulcrum.

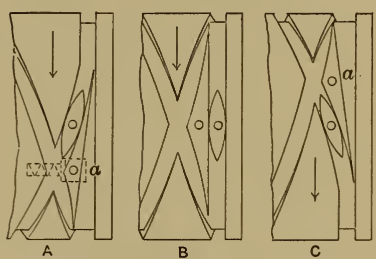


FIG. 98.

If the cam turns in the direction of the arrow, in *A* the shuttle-shaped follower is entering the circular portion of the groove, and leaves the switch in a position which will guide the follower into the circular groove when it again reaches the switch; in *B* the switch is pressed toward the left to allow the follower to pass. As motion continues, the support of the switch is pressed to the right, and the switch is thrown into the position shown in *C* ready to guide the shuttle into the returning groove. The period of rest in this case continues for about one and two-thirds turns of the cylinder.

95. Fig. 99 shows a method of drawing a cylindrical cam by means of the projection of its pitch line. Let the point p move uniformly to the right the distance pp_1 in $1\frac{1}{4}$ turns of the cylinder in the direction indicated; then let p remain stationary for a quarter-turn of the cylinder; then move to the left the distance p_1p with uniformly accelerated and retarded motion in one turn; and finally remain still for a half-turn. Intervals of rotation may be chosen as may be desirable. In the figure the surface of the cylinder is divided into spaces each subtending

45°. For the uniform motion to the right the line of motion parallel to the axis of the cylinder and equal to pp_1 is divided into *ten* equal parts, since there are ten 45° intervals in $1\frac{1}{4}$ turns. A smooth curve drawn through the points a_1, b_1, c_1, etc., the intersections of perpendiculars from the points a, b, c, etc., on the line of motion with the corresponding elements on the surface of the cylinder, will give the pitch line of the cam. For the motion to the left the line of motion is divided into eight spaces (since there are eight 45° intervals in one turn), and in proportion to the numbers 1, 3, 5, 7, 7, 5, 3, 1, and the intersections of perpendiculars from the ends of these spaces with the corresponding elements will give the points through which the pitch line is drawn. The pitch line for the two periods of rest will be parallel to the base of the cylinder as shown.

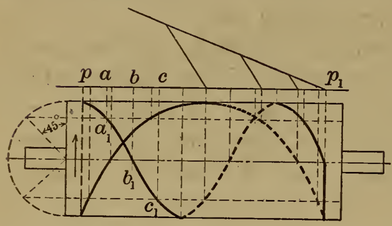


FIG. 99.

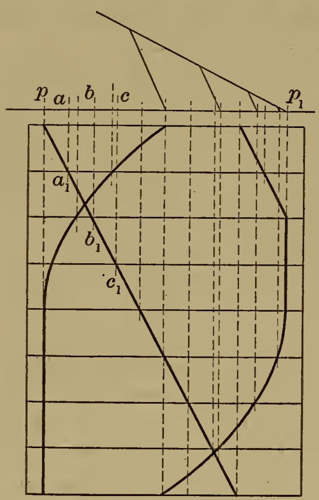


FIG. 100.

Fig. 100 is a development of Fig. 99, and could be plotted in a similar manner to that described for Fig. 99, as is indicated on the figure, excepting that the elements are spaced on the development of the cylinder, in this case becoming eight *equal* spaces. Wrapping this development around the cylinder, as was suggested in connection with Fig. 96, would give the pitch line as projected in Fig. 99.

If the path of the follower, in Fig. 97, is inclined to the axis of the cam, the groove would be cut in a cone, or hyperboloid, generated by revolving the line of action about the axis of the cam.

96. A conical cam might also be constructed to actuate a follower in a line perpendicular to its axis, and, by changing the position of the base of the cone relatively to the follower (the cam sliding along its axis), a variation in the motion could be obtained. A conical or cylindrical grooved cam might be made to actuate a roller in two directions radially and axially, such roller being supported by an arm moving on a spherical joint or its equivalent. A plate cam might also be arranged to turn slightly relative to its shaft, by means of a helically grooved sleeve and roller, and the relative times of the motions would thus be changed, an arrangement made use of in some governing devices.

CHAPTER VII.

LINKWORK.

97. **Linkage.**—Four pairs of elements may be combined, and this combination may take place in different ways. Suppose we have given the four parallel cylindric pairs, Fig. 101, each element of one pair rigidly joined to one element of another pair, we shall have an endless chain, or *linkage*, returning on itself.

Link.—The rigid body joining two elements of different pairs is called a *link*, a linkage being made up of a number of links. Fig. 101 may be taken as an example of a simple linkage, which consists of four pairs, each being a cylindrical pin fitting a corresponding eye, the axes of the pins being parallel. Here each link has a motion in a circle relative to its adjacent link; but every motion of any link must, according to its connection, be accompanied by alterations in the positions of the remaining links. Hence, if we consider one link of the chain to be fixed, the motions of the remaining links may be referred to it, and their relative motions determined.

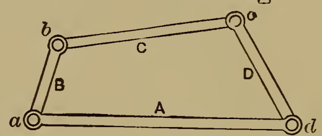


FIG. 101.

When in such a closed linkage we consider one of the links as fixed, we have a *mechanism*. Any link may be fixed, thus giving rise to as many mechanisms as the linkage has links. The fixed link forms a part of the frame of the machine, and may have a peculiar form.

The different parts of a linkage are named according to the motion they have in respect to the stationary link or frame.

Cranks, Levers, and Beams.—Links which turn or oscillate are called *cranks, levers,* or *beams;* the term *crank* being usually applied to links making complete turns, as the crank of a steam engine.

Connecting-rod; Coupling-rod.—The rigid link connecting the oscillating or rotating links is called by various names, as *connecting-rod, crank-rod, eccentric-rod, coupling-rod, parallel-rod,* etc., according to its location. It is attached to the pieces which it connects by pins or ball-and-socket joints, and maintains the distance between the centres of the pins or joints invariable. Hence its centre line is called *the line of connection,* and the centres of the pins are called the *connected points.*

Crank-arm.—In a turning piece the perpendicular let fall from the turning point upon its axis of rotation is called the *arm* or *crank-arm.*

98. Angular Velocity Ratio.—Before proceeding to the discussion of the different mechanisms that can be derived from the simple linkage shown in Fig. 101, we will determine the angular velocity ratios of the connected oscillating links. This may be found in two ways:

1° By reference to the instantaneous axis of the connecting link.

Let $ABCD$ (Fig. 102) represent the linkage, D being the fixed link, A and C oscillating about the points a and d respectively. Then o is the instantaneous axis about which B is revolving at the given instant. Produce bc to meet ad at e, and draw as, dr, and ot perpendicular to bc. By reference to the instantaneous axis, the l.v.'s of c and b, for the instant, are proportional to co and bo respectively, or

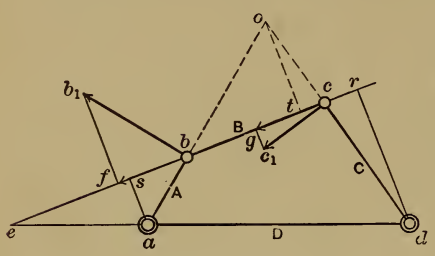


FIG. 102

$$\frac{\text{l.v. } c}{\text{l.v. } b}=\frac{co}{bo}.$$

But the a.v. of C is equal to the l.v. of c divided by the radius cd, and the a.v. of A is equal to the l.v. of b divided by ab; so we may write, noticing that the triangles oct and dcr are similar, as are also obt and abs,

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{\dfrac{\text{l.v. } c}{cd}}{\dfrac{\text{l.v. } b}{ab}}=\frac{\dfrac{oc}{cd}}{\dfrac{ob}{ab}}=\frac{oc}{cd}\times\frac{ab}{ob}=\frac{ot}{dr}\times\frac{as}{ot}=\frac{as}{dr},$$

and, by the similar triangles ase and dre,

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{as}{dr}=\frac{ae}{de}. \qquad (25)$$

2° By the resolution of velocities.

In Fig. 102 let bb_1 represent the l.v. of b; bf will be the component of this l.v. along bc, and cc_1 will therefore be the l.v. of c, $cg=bf$ being the component of the l.v. of c along bc;

$$\therefore \frac{\text{l.v. } c}{\text{l.v. } b}=\frac{cc_1}{bb_1},$$

and since a.v. equals l.v. divided by radius, and the triangles cc_1g and dcr are similar, as are also bb_1f and abs, we may write

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{cc_1}{cd}\div\frac{bb_1}{ab}=\frac{cg}{dr}\times\frac{as}{bf};$$

but the components cg and bf along bc are equal,

$$\therefore \frac{\text{a.v. } C}{\text{a.v. } A}=\frac{as}{dr}=\frac{ae}{de}. \qquad (26)$$

Thus the angular velocities of the connected oscillating links are to each other *inversely* as the segments into which the centre line of the connecting link divides the line of centres. Or, if the intersection of the line of centres and the connecting link is at an inconvenient distance, as will often happen, the rule may be stated by using the first ratio in equation (26): the a.v's of the connected oscillating links are to each other *inversely* as the perpendiculars from the centres of oscillation to the centre line of the connecting link.

This a.v. ratio of course varies for every relative position of the links; but if the perpendicular from the instantaneous axis to the centre line of the connecting link should fall at the intersection of the centre line of the connecting link and the line of centres, that is, in Fig. 102, if the points t and e should coincide, the a.v. ratio is essentially constant for slight movements in either direction. The same would be true should the points b and c be moving in lines parallel to each other.

99. Diagrams for Representing Changes in the L. V. Ratio or A. V. Ratio in any Linkage.—To obtain a clear knowledge of the change in velocity ratio in any linkage a diagram may be drawn where the abscissæ may represent successive positions of one of the oscillating links, and the ordinates represent the a.v. ratio of the oscillating links. A smooth curve through the points thus found would show clearly the fluctuations in the a.v. of one of the links relative to the other. A curve for l.v. ratio could be similarly plotted.

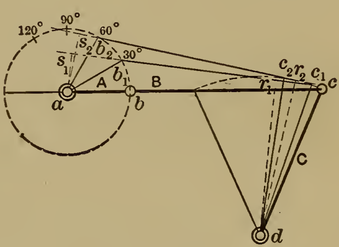


Fig. 103.

In the linkage shown in Fig. 103 let A turn uniformly L.H., to draw a curve to represent the ratio $\dfrac{\text{a.v. } C}{\text{a.v. } A}$ for a complete rotation of A. Take positions of A at intervals of 30° and draw perpendiculars from d and a to bc in each of its positions. The ratio of the two perpendiculars in each position will give the a.v. ratio: thus, starting with A as given in the figure, we have $\dfrac{\text{a.v. } C}{\text{a.v. } A}=0$; in the position ab_1c_1d we have $\dfrac{\text{a.v. } C}{\text{a.v. } A}=\dfrac{as_1}{dr_1}$; etc. Plotting these values as ordinates and the 30° positions of A as abscissæ will give the curve shown in Fig. 104.

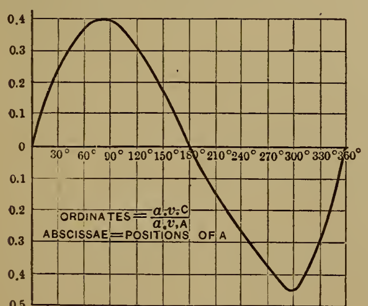


Fig. 104.

100. Crank and Rocker.—Let the link D (Fig. 105) be fixed, and suppose the crank A to revolve while the lever C oscillates about its axis d.

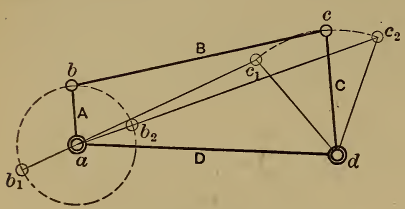


Fig. 105.

In order that this may occur, we must always have the conditions:

1°, $A+B+C>D$.
2°, $A+D+C>B$.
3°, $A+B-C<D$.
4°, $B-A+C>D$.

1° and 2° must hold in order that any motion shall be possible; 3° can be seen from the triangle ac_2d, in the extreme right position ab_2c_2d, which must not become a straight line; and 4° can be seen from the triangle ac_1d, in the left extreme position ab_1c_1d.

There are two points c_1 and c_2 in the path of c at which the motion of the lever is reversed, and it will be noticed that if the lever C is the driver, it cannot, unaided, drive the crank A, as a pull or a thrust on the rod B would only cause pressure on a, when c is at either c_1 or c_2. If A is the driver, this is not the case.

Dead Points.—The two points in the path of the crank at which it is impossible to start it by means of the connecting-rod alone are called *dead-points*.

The above form of linkage is applied in the beam engine as shown in Fig. 106, the link D being formed by the engine frame; corresponding parts are lettered the same as in Fig. 105. The instantaneous axis is, for the position shown, at o, and for the instant the l.v. of b is to the l.v. of c as ob is to oc, or as bf is to ce, the line ef, drawn parallel to bc, being made use of when the point o comes beyond the limits of the drawing.

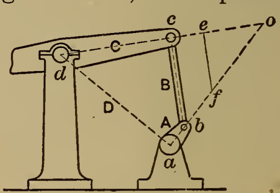


Fig. 106.

The angle through which the lever C (Fig. 105) swings can be calculated for known values of A, B, C, and D.

From the triangle ac_2d

$$\cos adc_2 = \frac{C^2+D^2-(B+A)^2}{2CD},$$

and from the triangle ac_1d

$$\cos adc_1 = \frac{C^2+D^2-(B-A)^2}{2CD},$$

and $$c_1dc_2 = adc_2 - adc_1.$$

Thus the two angles adc_2 and adc_1 can be calculated, and their difference will give the angle required.

If the link B is made stationary, the mechanism is similar, the only difference being in the relative lengths of the connecting-rod and stationary piece or frame.

101. Drag-link.—If the link A is made the stationary piece or frame (Fig. 107), the links B and C rotate about a and b respectively, that is, become cranks, and D becomes a connecting-rod. This mechanism is known as the *drag-link.*

In order that the cranks may make complete rotations, and that there may be no dead-points, the following conditions must hold:

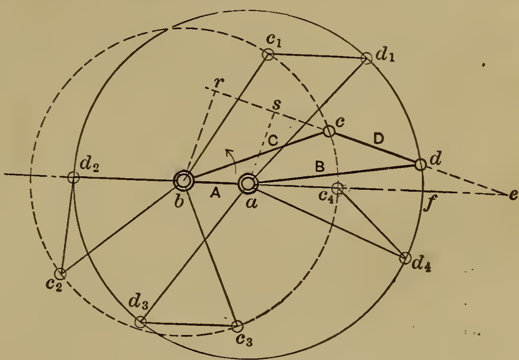


FIG. 107.

1° Each crank must be longer than the line of centres, which needs no explanation.

2° The link cd must be greater than the lesser segment c_4f and less than the greater segment c_4d_2, into which the diameter of the greater of the two crank circles is divided by the smaller circle. This may be expressed as follows:

$$cd > A + B - C \text{ (see triangle } ac_4d_4\text{);}$$
$$cd < B + C - A \text{ (see triangle } bc_2d_2\text{).}$$

Producing the line of the connecting-rod until it intersects the line of centres at e, and dropping the perpendiculars as and br upon it, we have

$$\frac{\text{a.v. } B}{\text{a.v. } C} = \frac{be}{ae} = \frac{br}{as}.$$

In the positions abc_1d_1 and abc_3d_3 when cd is parallel to the line of centres, the angular velocities of B and C are equal, as the perpendiculars br and as then become equal.

If C revolves left-handed and is considered the driver, it will be noticed that between the positions abc_3d_3 and abc_1d_1 the link B is gaining on C, and between abc_1d_1 and abc_3d_3 it is falling behind C. Therefore from abc_3d_3 to abc_1d_1 we have, as e falls to the right of A,

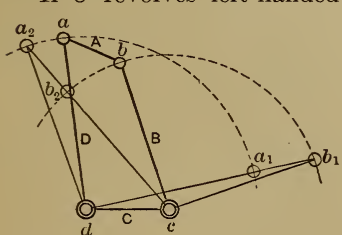


FIG. 108.

$$\frac{\text{a.v. } B}{\text{a.v. } C} = \frac{br}{as} > \text{unity},$$

while from abc_1d_1 to abc_3d_3, as e would fall to the left of A, we should have the a.v. ratio less than unity.

102. The Double Rocking Lever.—The remaining case is shown in Fig. 108, where C is the fixed link, and the levers B and D merely oscillate on their centres c and d; the extreme positions are shown at a_1b_1 and a_2b_2.

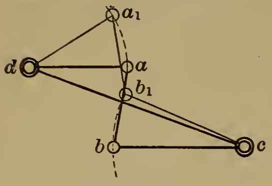


FIG. 109.

A modified form of this mechanism (Fig. 109) is used in mechanisms for tracing straight lines, commonly known as "parallel motions."

103. The following figures show a few applications of the preceding forms of linkwork: Fig. 110 shows a case of the *crank and rocker* as applied in wool-combing machinery. Here the crank ab turns uniformly on its axis a, while cd oscillates about d; both axes a and d are attached to the frame of the machine, which forms the fixed link ad. The connecting-rod cb is prolonged beyond b, and carries a comb e at its extremity, which takes a tuft of wool from the comb f and transfers it to the comb g, both combs f and g being attached to the frame of the machine. The full lines show the position of the links when the comb e is in the act of rising through the wool on f, thus detaching it, and the dotted lines show the position of the links when the comb e_1 is about to deposit the tuft of wool on g. The same combination inverted is used in some forms of wool-washing machines.

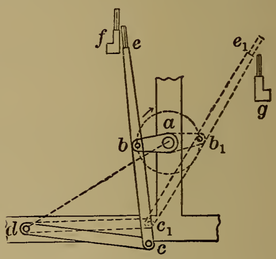


FIG. 110.

In Fig. 111, which represents Morgan's feathering paddle-wheel, an application of the *drag-link* is shown. Each float is attached to one end of a link D, and turns on a bearing in the paddle-wheel frame carried by the paddle-shaft; the lines joining these bearings with the axis of the paddle-shaft give the links B. To the other end of D the links C, which revolve about a fixed centre in the paddle-box, are attached. As the space for the links C is limited, the arrangement shown in the figure is used instead of having the links turn on one pin and located side by side. Here the link C is attached to a solid ring which rotates about the centre in the paddle-box, and the other links, corresponding to C, are attached to this ring, and have motions very near what they would have if they turned about the axis of the ring. The links are lettered the same as

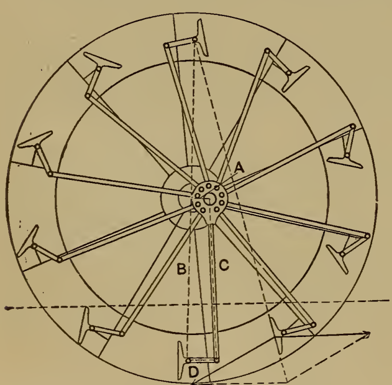


FIG. 111.

in Fig. 107, *A* being the fixed link, *B* and *C* the cranks, and *D* the connecting-rod.

If we suppose the paddle-wheel to have no slip, the proper angle for the floats to enter the water without disturbance can be found as follows: Take one of the floats, as shown just after entering the water, and, as its motion is made up of two components, one being due to the motion of the vessel while the other is due to the rotation of the wheel, draw the parallelogram of motions, and it will be found that the float while in the water should always have the line of its face pass through the highest point of the wheel.

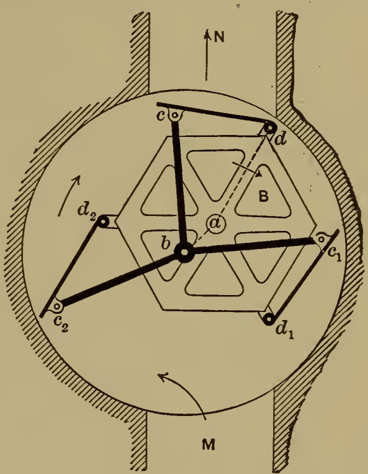


FIG. 112.

Another example of the *drag-link* mechanism is shown in Fig. 112, which represents a Lemielle ventilating-machine. The apparatus consists of a circular chamber with closed ends, having a passage *M* for the inlet, and one *N*, on the other side, for the discharge of air. A revolving drum *B* centred on the axis *a* carries a series of vanes *cd* free to turn on the axes *d*, and at the same time governed in their positions by the links *bc* attached to their free ends, and turning about a fixed centre *b* in the end of the chamber. It can easily be seen that, when the drum revolves in the direction indicated by the arrow, air will be drawn through the passage *M* and delivered to *N*; the vanes, opening out on passing the inlet and closing on passing the outlet, sweep the air before them.

104. Parallel-cranks.—If in Fig. 105 of the four-bar linkage, we make the opposite links equal, i.e., $A=C$ and $B=D$, the crank-chain becomes a parallelogram, as *ABCD* in the lower part of Fig. 113. The lever *C* then becomes a crank equal to *A*, and (if *D* is fixed and some means of passing the dead-points is provided) moves through the same angle as *A*. The nature of this mechanism is not changed by fixing any of the other links.

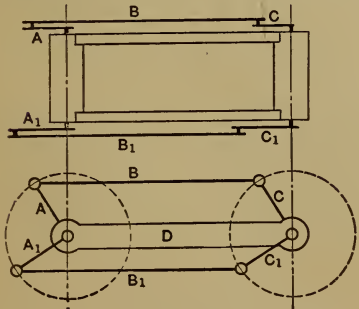


FIG. 113.

If, in the combination of two equal cranks, *A* and *C* (Fig. 113) with a connecting-rod *B*, equal in length to the distance between centres of cranks, the crank *A* rotates right-handed until it comes over D, thus bringing the four links in line with each other, it will be noticed that in this position (one of the dead-points) a further motion of A might cause C to turn either right-handed or left-handed. That is, for any given position of A, except the dead-points, there are two possible positions of C, which can be found by drawing an arc of radius B about the extremity of A as a centre, and noting where it cuts the path of the extremity of C. Thus a uniform right-handed rotation of A might cause a uniform right-handed rotation of C, or a variable left-handed rotation, as shown in Fig. 116. To prevent this change of motion, and to insure the passage of the cranks by their dead-points, two sets of equal cranks may be combined, as shown in Fig. 113; the angle between the two sets of cranks being commonly taken 90°, so that when one set of cranks is at a dead-point the other is in its best working position. Then a uniform rotation of the cranks A, A_1 will cause a uniform rotation of C, C_1, thus giving a uniform and continuous velocity ratio between the axes of the two sets of cranks. Locomotives with coupled drivers are familiar examples of this arrangement.

Fig. 114 shows another method of passing the dead-points where a third equal crank, C_1, is placed in the plane of the other two, and connected to them by links equal in length to the distance between its axis and the axes of the other cranks respectively.

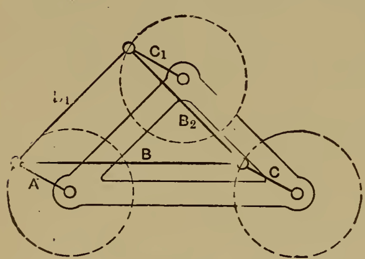


FIG. 114.

Two cranks at right angles to each other, and located in one plane, could be connected with two others also at right angles, and located in another plane parallel to the first, by means of two parallel connecting-rods sufficiently offset to enable them to clear each other in their motion, the distance between the two crank planes being made sufficient to admit of such an arrangement. This arrangement is practically of little value, especially when much force is to be transmitted, as offset-rods, unless made very heavy, are likely to bend and cause binding on the crank-pins.

It will be seen, on reference to Figs. 113 and 114, that the connecting-rods B, B_1, move in such a way that they are parallel to the line connecting the axes of the equal cranks which carry them, in all their positions, and also that all points in these rods move in circular paths of a radius equal to the length of the crank, i.e., the rods may be said to have circular translation (§ 25).

Parallel-rod.—The term *parallel-rod* or *coupling-rod* is used to designate the rods, such as B and B_1 (Fig. 113), employed to connect the driving axles of locomotives.

If two circular rings, *cef* and *dgh* (Fig. 115), are arranged to turn on their centres *b* and *a* respectively, with equal angular velocities, they may be connected by a series of equal parallel links, *cd*, *eg*, *fh*, etc., equal in length to the distance *ab* between the centres of the rings. These links would then have motions in circular paths, but would always remain parallel to *ab*, the link *A* being here fixed instead of *D* as in Fig. 113. This combination has been used in wire-rope machinery, where it is necessary that the wires of the rope be laid without having any twist put into them. As the rope in being drawn from the laying block does not turn, the individual wires also must not turn; this is accomplished by attaching the bobbins *E*, which carry the wires, to the parallel links *cd*, *eg*, etc., which have circular translation but no rotation, the laying block turning with the circular ring *dgh* on an axis through *a* perpendicular to the plane of the paper.

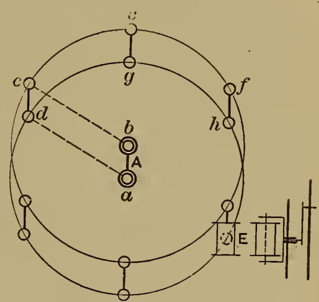


Fig. 115.

105. Anti-parallel Cranks.—It was stated in § 104 that, in the combination of two equal cranks with a connecting-rod, equal in length to the distance between centres of cranks, a right-handed rotation of one of the cranks might cause a left-handed rotation of the other. Fig. 116 shows such a case where a uniform right-handed rotation of *ab* causes a variable left-handed rotation of *cd*, when some method of passing the dead-points is provided. Now if we can provide these links, which are capable of two motions at their dead-points, or change positions, with pairs of elements so formed that they will give the required motion at these change points, the passage of the dead-points will be secured.

In order to do this, we must, by the principles of § 27, determine the axoids or centroids of the bodies to be paired. These bodies are, here, for example, the two cranks *A* and *C*, or otherwise the two links *B* and *D*. In the case of the general four-link chain, shown in Fig. 101, the centroids are very complicated figures; here they are made very simple by the equality of the opposite links. (The linkage taken in § 27 to illustrate a centroid was this linkage.) Remembering that the cranks are always to revolve in opposite directions, the centroids will be found to have the forms shown in Fig. 116.

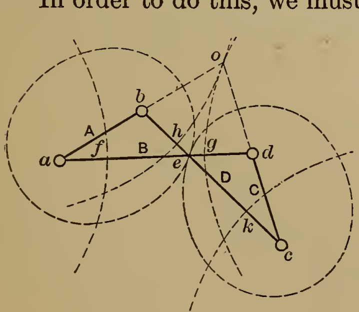


Fig. 116.

For the links *ab* and *cd*, the shorter pair, they are ellipses, having their foci at the ends, *a*, *b* and *c*, *d*, of the cranks, and their major axes equal in length to the links *bc* and *da*. The instantaneous centre moves backwards and forwards along the links *bc* and *da*, being always found at their intersection, as *e*. For *bc* and *da* the centroids are hyperbolas, their transverse axes *fg* and *hk* lying on the links themselves and being equal to $ab=cd$; their foci are the points *b*, *c* and *a*, *d*. The instantaneous centre traverses each branch of the curve to infinity, turning from $-\infty$ along the other branches. Thus the two ellipses or the two hyperbolas could replace the linkage. In § 58 it was found that two equal rolling ellipses were equivalent to this linkage, and also in § 61 the same was found to be true of two equal rolling hyperbolas.

If it be required to pair the two opposite links at their change points, a pair of elements must be employed in each case; such pairs need not, however, go further than correspond to the elements of the rolling conics in contact at the change positions. If the links chosen be the two shorter ones, *ab* and *cd*, these are the elements of the ellipses at the extremities of their major axes. By putting a pin and a gab (or open eye) at these points, as shown in Fig. 117 at *l* and *m*, l_1 and m_1, the mechanism becomes a closed chain; *l* gearing with l_1 causes the passage of one dead-point, and *m* gearing with m_1 causes the passage of the other.

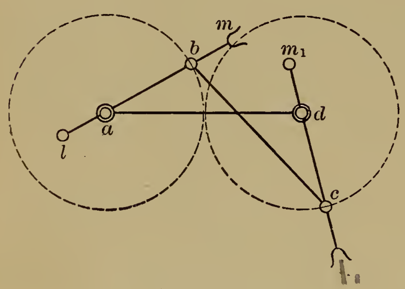


Fig. 117.

If the two longer links are to be paired instead of the two shorter ones, we have only to notice that the two vertexes *g* and *h*, and *f* and *k* touch each other in the change positions. By placing at these points a pin and corresponding gab (Fig. 118) we have again a pairing which effectually closes the chain. Thus we have two solutions of the problem before us. If it were desired, it would be possible to close the chain at one dead-point by one method, and at the other by another. A few teeth on the ellipses or hyperbolas might be used in place of the pin and gab.

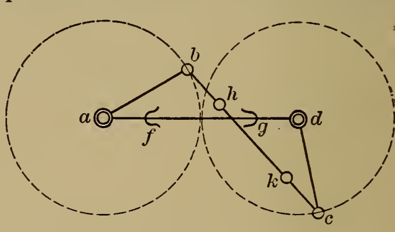


Fig. 118.

106. This linkage replaced by its centroids, the rolling ellipses, is used to give a quick-return motion on some forms of slotting-machines. In Fig. 119, if *ab* turns uniformly, R.H., the upward motion of the slide

E, operated by a crank de placed at an angle of $90°$ with cd, would begin in the position $abcd$ and end in the position ab_1c_1d, occupying a time proportional to the angle α; while the time occupied in the downward motion of E would be proportional to the angle β, or

$$\text{advance of } E : \text{return of } E = \beta : \alpha.$$

If the distance between the centres ad is given, and the ratio of advance to return, the angle bad is determined and the length of the crank ab ($=cd$) can be readily calculated; and from § 105 we know that a, b and c, d will be the foci of the ellipses and that ad ($=bc$) will be the length of the major axis of each.

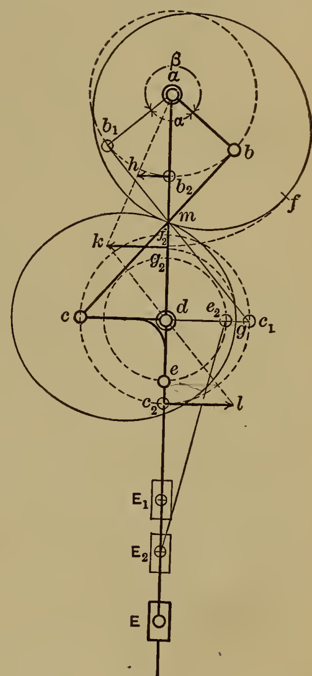


FIG. 119.

If the l.v. of the point b is known, the l.v. of the slide may be found for any position by the method in § 22, with the exception of the two positions in which the links ab, bc, and cd lie in the same straight line. To determine the l.v. at these two positions it is necessary to refer to the rolling ellipses. In the position ab_2c_2d the ellipses would be in contact at the point f_2g_2 and we should have l.v. of g_2 equal to the l.v. of the point f_2 in contact with it; therefore, if b_2h represents the l.v. of b_2, f_2k will be the l.v. of the point f_2 and so of g_2, and c_2l will be the l.v. of c_2. The l.v. of E for the position E_2 would be $c_2l \times \frac{de}{cd}$.

The a.v. ratio of the cranks ab and cd may be found for any position by the law deduced in § 98, excepting the two positions referred to in discussing the l.v. In the position $abcd$ we have

$$\frac{\text{a.v. } cd}{\text{a.v. } ab} = \frac{am}{dm},$$

but in the position ab_2c_2d, by referring to the rolling ellipses,

$$\frac{\text{a.v. } c_2d}{\text{a.v. } ab_2} = \frac{af_2}{dg_2},$$

that is, the a.v's are inversely as the radii of contact of the rolling surfaces.

To obtain a clear idea of the l.v. of the slide E as ab turns uniformly, a curve should be drawn having for ordinates the l.v. of the slide E, and for abscissas the corresponding angular positions of ab.

107. Slow Motion by Linkwork.—The simple linkage shown in Fig. 105 can, if properly proportioned, be made to produce a slow motion of one of the cranks. Such a combination is shown in Fig. 120, where two cranks A and C are arranged to turn on fixed centres and are connected by the link B. If the crank A is turned right-handed, the crank C will also turn right-handed, but with decreasing velocity, which will become zero when the crank A reaches position A_1, in line with the link B_1: any further motion of A will cause the link C to return toward its first position, its motion being slow at first and then gradually increasing. This type of motion is used in the Corliss valve-gear, as shown in Fig. 121. The linkage $abcd$, moving one of the exhaust-valves, will give to the crank cd a very slow motion, when c is near c_1, when the valve is closed, while between c and c_2, when the valve is opening or closing, the motion is much faster. The same is true for the admission-valves, as shown by the linkage $aefg$.

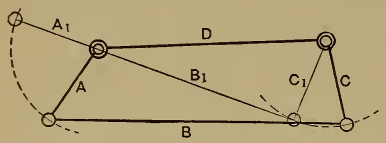


Fig. 120.

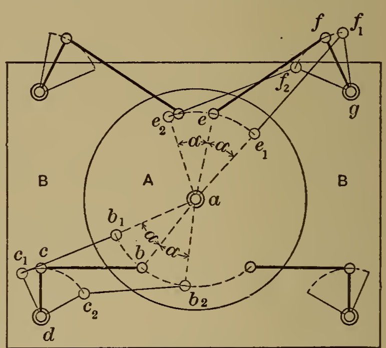


Fig. 121.

108. Forces Transmitted by Linkwork.—If we know the force applied at some point in a linkage, as the pull on the end of a lever, it is possible, by equating the moments of the forces acting around each axis of rotation, to determine the force resulting at some other point in the linkage, neglecting the losses due to friction. In the linkage in Fig. 122, let a force F act at the point a in the bell-crank lever abc and at right angles with ab; it will cause a pull F_1 at c in the direction of the link cd which will act on the point d of the lever def with the same intensity F_1 in the same direction. (The arrows in these figures represent the directions in which the forces may be considered to act, but do not represent the magnitudes of the forces unless otherwise stated.) Since the

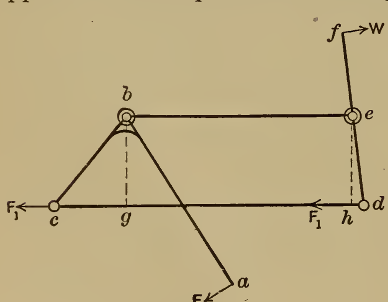


Fig. 122.

two forces F and F_1 acting around the axis b are balanced, their moments about b must be equal; therefore we have

$$F \times ab = F_1 \times bg \quad \text{and} \quad F_1 = \frac{F \times ab}{bg},$$

and the force W which would be exerted by the point f against a resistance acting at right angles with ef would be found by the relation

$$F_1 \times eh = W \times ef.$$

$$\therefore \quad W = \frac{F_1 \times eh}{ef} = F \times \frac{ab}{bg} \times \frac{eh}{ef}.$$

If the links are proportioned as in the figure, so that for a uniform R.H. motion of abc the lever def has a decreasing motion, we should find that for a given resistance, W, at f the force F required at a would diminish, its limit being zero at the moment when bc and cd come into line; or if we consider a constant force F at a, the force W which could be exerted by f would increase, its limit being infinity, which means that a very large force may be produced by such a linkage, when near this position of slowest motion, by the application of a relatively small force. This principle is used in many forms of riveting, shearing, and punching machinery.

109. Toggle-joint.—There are two forms of this joint shown in Figs. 123 and 124. In each case the point a is fixed and the point c

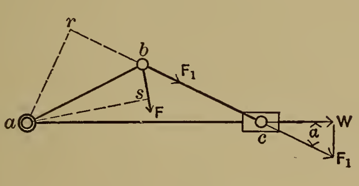


FIG. 123.

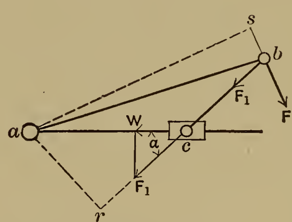


FIG. 124.

slides on the line ac. If a force F is applied at b in the direction indicated, its moment arm about a will be as, and the moment arm about a of the thrust F_1 along the connecting link bc will be ar; therefore

$$F \times as = F_1 \times ar \quad \text{and} \quad F_1 = F\frac{as}{ar}.$$

Using the parallelogram of forces at the point c as indicated,

$$F_1 \cos \alpha = W.$$

$$\therefore \quad W = F \times \frac{as}{ar} \times \cos \alpha.$$

As the links ab and bc come more and more into line, the distance ar becomes smaller, the component of the thrust along ac approaches nearer and nearer to the thrust on bc, and when the links are in line the thrust along bc is theoretically infinite, ar being then equal to zero.

Fig. 125 shows a metal-shearing machine, in which a slow motion and consequently an advantage in power is obtained by means of linkwork. Here the long lever A is formed by a continuation of the crank ab; the crank or lever C turning on d is connected to ab by the link bc, a and d being fixed centres carried by the frame of the machine, which forms the fourth link of the chain. The metal to be sheared is placed at S, and the power is applied at the end of the lever A. The operation of the machine can be easily understood from the figure. The links ab and bc, c moving nearly in a straight line, form a *toggle-joint*.

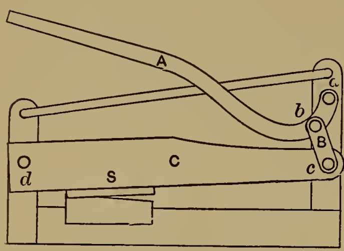


FIG. 125.

110. Double Oscillation by Linkwork.—If we combine the links dc, ce, and ie as shown in Fig. 126, d and i being fixed centres, with a crank ab (turning on the fixed centre a), and a connecting-rod bc, the lever ie can be made to make a double oscillation during one rotation of the crank ab. In order that this may properly occur, the extreme positions of the point c should be equidistant from the centre line de, and the extreme positions of e should be equidistant from the perpendicular dropped from the centre i on the line de. If we continue the link ie to f, and connect it with the links fg and gh, h being a fixed centre, and so chosen that the extreme positions of g are equidistant from the perpendicular dropped from h upon ig, the lever hm will make four oscillations for each rotation of the crank ab.

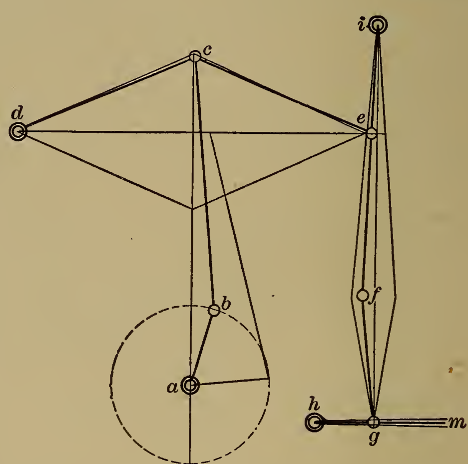


FIG. 126.

111. Linkwork with One Sliding Pair. Line of Connection in a Sliding Pair.—Returning again to the four-bar linkage of Fig. 105, we can substitute for the link C a small sector of an annular cylinder, and enclose this in a circular slot (Fig. 127) rigidly connected with the centre about which the crank A revolves. If the centre line of the slot be placed at a distance from d equal to the distance dc in Fig. 105, and if the link B is attached in the centre of the sector, the link B has exactly the same relative motions as it would have had had it been connected to the link C. The elementary link D and the sliding block C thus take the place of the links D and C of Fig. 105.

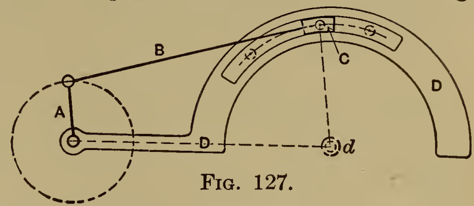


FIG. 127.

We can now, without introducing any constructive difficulties, make the radius of the slot (Fig. 127) of any required magnitude, the slot and the slider becoming flatter than before. If we make the radius infinite, and at the same time make the link D infinite, that is, make the distances cd and ad (Fig. 105) both infinite simultaneously, we shall obtain the mechanism shown in Fig. 128, which will be called the *sliding-block linkage*. We thus see that the line of connection in a sliding pair is a normal to the sliding surface from the axis of the pivot in the slider; and the same is true if a roller is substituted for the slide, these appliances replacing the equivalent link so far as the motion is concerned.

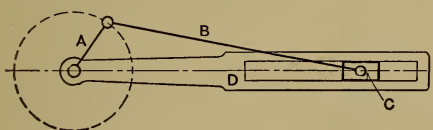


FIG. 128.

In Fig. 128 either of the four pieces A, B, C, or D may be fixed, thus giving rise to four mechanisms which will now be considered.

112. **The Sliding-block Linkage.**—Considering the link D (Fig. 128) as fixed, we obtain the mechanism commonly used in pumps and direct-acting steam engines. When employed in a steam engine, the block C, called the cross-head, is the driver and the crank A the follower; in a pump the reverse is the case.

Movement of Cross-head.—In Fig. 129 let ab represent the crank, bc the connecting-rod, and mn the path of the point c in the cross-head. The travel of the cross-head mn is equal to twice the length of the crank ab, and the distance of c from a varies between $B+A=an$ and $B-A=am$, A being the length of the crank, and B the length of the connecting-rod.

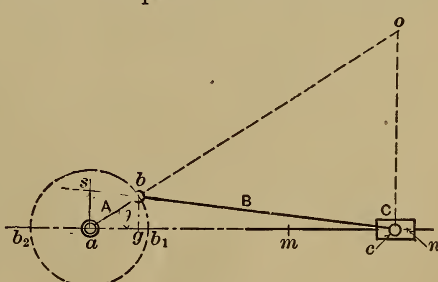


FIG. 129.

To find the distance the point c has moved from n, the beginning of its stroke or travel, let the angle made by the crank with the line an be represented by θ, and draw bg perpendicular to an. The movement of the cross-head from the beginning of its stroke is, for the angular motion θ of the crank,

$$cn = an - ac = an - (ag + gc).$$

From the right triangle bcg,

$$gc = \sqrt{\overline{bc}^2 - \overline{bg}^2}.$$

Hence

$$cn = an - ab \cos\theta - \sqrt{\overline{bc}^2 - \overline{ab}^2 \sin^2\theta}$$

$$= A + B - A\cos\theta - \sqrt{B^2 - A^2\sin^2\theta}. \quad . \quad . \quad . \quad . \quad (27)$$

$$= A(1 - \cos\theta) + B\left\{1 - \sqrt{1 - \frac{A^2}{B^2}\sin^2\theta}\right\}. \quad . \quad . \quad . \quad (28)$$

If the length of the connecting-rod has a certain relation to that of the crank, A being the length of the crank, and lA that of the rod, we have, substituting lA for B in equation (27)

$$cn = A(1 + l - \cos\theta - \sqrt{l^2 - \sin^2\theta}). \quad . \quad . \quad . \quad . \quad (29)$$

The motion may be represented graphically by plotting a curve, where the ordinates represent successive values of cn, and the abscissas represent angular positions of the crank ab. Fig. 130 shows the curve for the linkage given in Fig. 132.

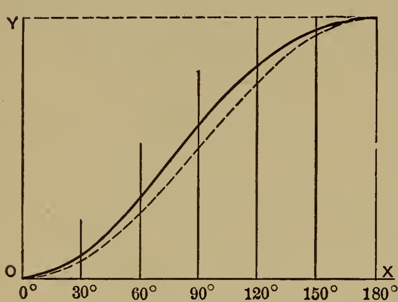


FIG. 130.

L.V. Ratio.—In Fig. 129 continue the line of the connecting-rod to s, and draw the line as through a and perpendicular to am. The instantaneous centre of the rod bc is at o, found by drawing the lines bo and co perpendicular to the lines of motion for the instant of the points b and c respectively.

As the l.v's of the points b and c are proportional to their distances from the instantaneous centre o (§ 23), we have

$$\frac{\text{l.v. of } c}{\text{l.v. of } b} = \frac{oc}{ob} = \frac{as}{ab} = \frac{as}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

as the triangles abs and obc are similar.

From the similar triangles cas and cbg we have

$$\frac{as}{bg} = \frac{ac}{gc}, \quad \text{or} \quad as = bg\frac{ac}{gc} = bg\frac{ag + gc}{gc};$$

$$as = \frac{A\sin\theta\{A\cos\theta + \sqrt{B^2 - A^2\sin^2\theta}\}}{\sqrt{B^2 - A^2\sin^2\theta}}.$$

Substituting in (30), we have

$$\frac{\text{l.v. of } c}{\text{l.v. of } b}=\frac{as}{A}=\sin\theta+\frac{A\sin\theta\cos\theta}{\sqrt{B^2-A^2\sin^2\theta}}. \quad . \quad . \quad . \quad . \quad (31)$$

The velocity of b being constant, that of c can be found by equation (31).

This same result may be obtained by another method. Since velocity if variable may be expressed by the equation $v=\frac{ds}{dt}$, we may find the l.v. of c by differentiating equation (27), where $cn=s$ and where the angle θ must be expressed in terms of the a.v., α, of the crank A, and of the time t. Writing equation (27) in this form gives

$$s=A+B-A\cos\alpha t-\sqrt{B^2-A^2\sin^2\alpha t}. \quad . \quad . \quad . \quad . \quad (32)$$

$$\therefore \text{ l.v. } c=\frac{ds}{dt}=\alpha A\sin\alpha t+\frac{\alpha A^2\sin\alpha t\cos\alpha t}{\sqrt{B^2-A^2\sin^2\alpha t}}.$$

But l.v. $b=\alpha A$;

$$\therefore \frac{\text{l.v. } c}{\text{l.v. } b}=\sin\alpha t+\frac{A\sin\alpha t\cos\alpha t}{\sqrt{B^2-A^2\sin^2\alpha t}}. \quad . \quad . \quad . \quad . \quad (33)$$

When $\theta=90°$, $as=A$, and the velocities of c and b are equal. To find other values for θ, when the velocities c and b are equal, we have, from equation (31),

$$\frac{\text{l.v. of } c}{\text{l.v. of } b}=1=\sin\theta+\frac{A\sin\theta\cos\theta}{\sqrt{B^2-A^2\sin^2\theta}};$$

$$(1-\sin\theta)\sqrt{B^2-A^2\sin^2\theta}=A\sin\theta\sqrt{1-\sin^2\theta};$$

$$\sqrt{B^2-A^2\sin^2\theta}=\frac{A\sin\theta\sqrt{1-\sin^2\theta}}{1-\sin\theta}.$$

Squaring, $$B^2-A^2\sin^2\theta=\frac{A^2\sin^2\theta(1+\sin\theta)}{1-\sin\theta}.$$

Solving, for $\sin\theta$, we have

$$\sin\theta=\frac{B}{4A^2}\{-B\pm\sqrt{8A^2+B^2}\}.$$

The l.v. ratio between b and c may be shown graphically, using coordinate axes, the ordinates representing the ratio and the abscissas representing angular positions of the crank ab. Fig. 131 shows the curve for the linkage given in Fig. 132.

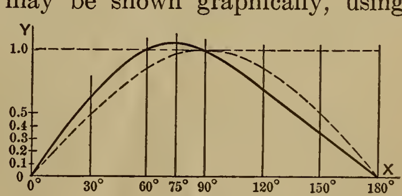


FIG. 131.

Fig. 132 illustrates other methods of showing the l.v. ratio. In this figure the constant l.v. of b

is represented by the crank length $ab=A$. From equation (30) we have

$$\frac{\text{l.v. } c}{\text{l.v. } b}=\frac{as}{ab}.$$

Therefore, if we lay off on the line ab, which shows the crank position, the distance $at=as$, and repeat this construction for a sufficient number of crank positions, we shall obtain the full curve ata, where the intercept at on the crank line shows the velocity of c, ab being the constant velocity of b. A similar curve would be found for the crank positions below the line ma. Similarly we might obtain the full curve nt_1m by laying off on the successive perpendiculars drawn through the point c

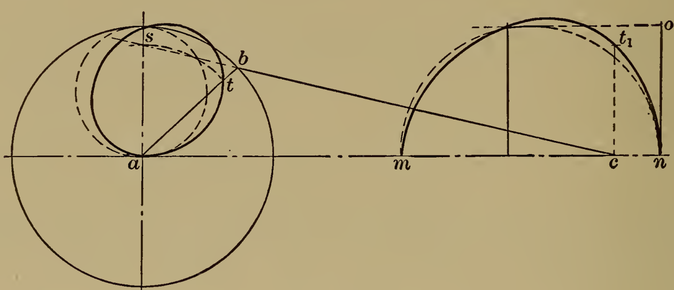


FIG. 132.

the corresponding distances as; then the ordinates of this curve drawn through the cross-head position would give the velocity of c at that position, the ordinate $no=ab$ representing the constant l.v. of b.

If the length of the connecting-rod B (Fig. 129) is made infinite, the motion of the point c will be equal to the projection of the motion of b on the diameter b_1b_2, and will be, therefore, simple harmonic motion. For any angle θ this motion will be

$$b_1g=ab(1-\cos\theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Referring to equation (28), the motion of c when the connecting-rod is finite varies from harmonic motion by the quantity $B\left(1-\sqrt{1-\frac{A^2}{B^2}\sin^2\theta}\right)$, which approaches zero as a limit as B is made greater.

The connecting-rod is rarely made more than six or seven times as long as the crank, as the additional space required is not compensated for by the slightly nearer approach to harmonic motion. The dotted curve in Fig. 130 would be the curve for harmonic motion.

The l.v. of c, if B were infinite, would be found by resolving the l.v. of b into two components, one vertical and one horizontal (Fig. 133), the horizontal component being the l.v. along the rod bc if it were infinite and thus equal to the l.v. of c. From this we have

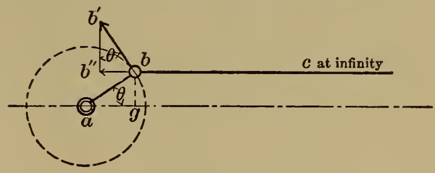


FIG. 133.

$$\frac{\text{l.v. } c}{\text{l.v. } b}=\frac{bb''}{bb'}=\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Comparing this with equation (31), the l.v. ratio when B is finite varies from harmonic by the amount $\dfrac{A\sin\theta\cos\theta}{\sqrt{B^2-A^2\sin^2\theta}}$, which approaches zero as a limit as B is made greater. The dotted curves in Figs. 131 and 132 would show the l.v. of c if the motion were harmonic. In Fig. 132 the dotted curves will be found to be circular.

If, in Fig. 128, we consider the crank A as the driver, it can always produce reciprocating motion in the block C; but if C is the driver, it cannot produce continuous circular motion unless some means of pass-

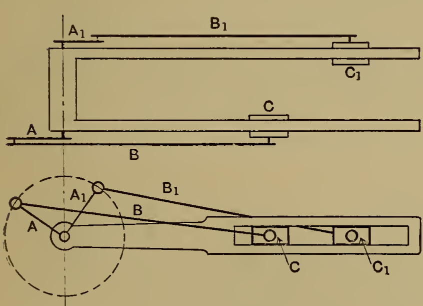


FIG. 134.

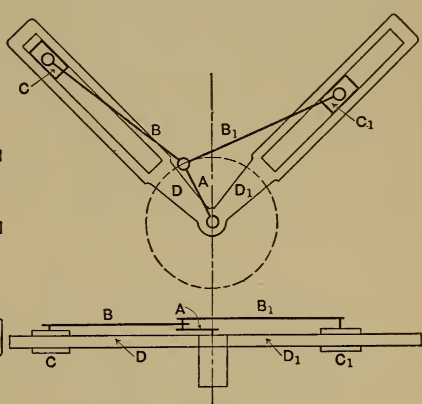


FIG. 135.

ing the dead-points be devised. This is usually accomplished in steam engines by attaching to the crank-shaft a heavy fly-wheel, the momentum of which carries the crank by the dead-points. The impossibility of starting at the dead-points still remains.

To obviate this difficulty two crank and connecting-rod mechanisms may be combined, as shown in Fig. 134, where the cranks are placed at right angles to each other and joined by a shaft. This combination is employed in locomotives, and in hoisting and marine engines, one crank being very near its best position to be acted on by the rod while the other is at a dead-point.

Fig. 135 shows another method of passing the dead-points sometimes used in marine engines. Here the two connecting-rods B and B_1 are located in parallel planes and act upon the same crank A. By suitably forming the ends of the rods, they might be located in the same plane.

113. The Swinging-block Linkage. — If we consider the link B (Fig. 128) as fixed, we obtain the *swinging-block linkage* commonly used in oscillating engines. Such a mechanism is shown in Fig. 136, from which we obtain the mechanism shown in Fig. 137 by inverting the pair CD, this having no effect on the relative motions of C and D (§ 39). Here A represents the crank, D the piston-rod, and C, the swinging slide or block, the cylinder.

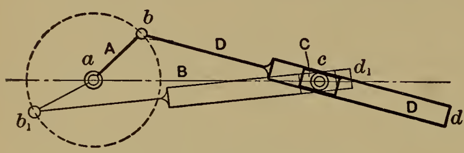


Fig. 136.

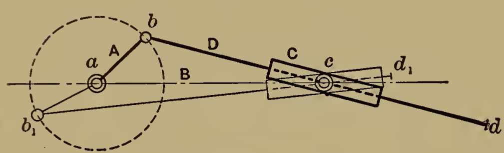


Fig. 137.

In Fig. 138, which represents an oscillating engine, three relations may be studied: 1° the motion of the piston D relative to the cylinder

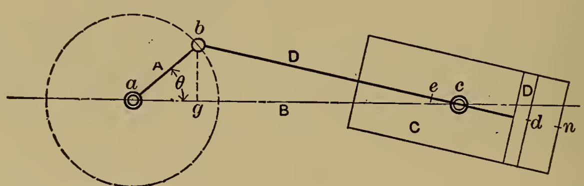


Fig. 138.

C; 2° the ratio of the l.v. of the piston relative to the cylinder and the l.v. of b; and 3° the a.v. ratio of the cylinder, about the axis c of the trunnions which support it, to the crank A.

1° To find the distance dn (Fig. 138) which the piston has moved from the beginning of its stroke, for a given angle $bac=\theta$. Let e be the point on the piston-rod D which is coincident with c when A and D are in the same line; then ce will be equal to the motion of the piston dn. From the figure we have

$$dn=ec=bc-be=bc-(B-A).$$

But

$$bc=\sqrt{A^2+B^2-2AB\cos\theta};$$

$$\therefore\ dn=ec=\sqrt{A^2+B^2-2AB\cos\theta}-B+A. \quad . \quad . \quad . \quad . \quad (36)$$

2° To find the l.v. of the piston in the cylinder. In Fig. 139 let bb' represent the l.v. of b around a; then the component bb'' along the piston-rod would be the desired l.v. of the piston relative to the cylinder. The ratio of the l.v. of piston in the cylinder to the l.v. of the crank-pin b may be found by reference to the instantaneous axis of the link bd. In the link bd the direction of the motion of b will be along the line bb', and the direction of the point on the link which coincides at the instant with the axis c of the trunnions which support the cylinder will be along the line bd; therefore the instantaneous axis of bd will be found at o, the intersection of the lines bo and co perpendicular respectively to bb' and bc.

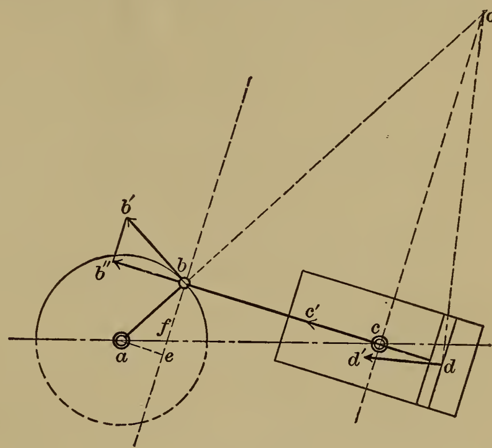


Fig. 139.

$$\therefore \frac{\text{l.v. of piston relative to cylinder}}{\text{l.v. of crank-pin } b} = \frac{co}{bo} = \frac{bb''}{bb'},$$

since the triangle obc is similar to $bb'b''$.

To find the actual l.v. of any point in the piston-rod or piston, as d, we have

$$\frac{\text{l.v. of } d}{\text{l.v. of } b} = \frac{do}{bo},$$

and its direction of motion would be along dd' perpendicular to do.

3° To find the ratio of the a.v. of the cylinder about the axis c, and the a.v. of the crank ab. To determine this ratio, by using the law deduced in § 98 for a.v. ratio, it will be necessary to draw the centre lines of the two infinite links which have been replaced by the sliding pair. These lines must be perpendicular to the centre line of the sliding pair (§ 111) and will be be and co (Fig. 139). Since ac is the line of centres, be will be the centre line of the infinite connecting link. Applying the law for a.v. ratio,

$$\frac{\text{a.v. cylinder}}{\text{a.v. } ab} = \frac{ae}{bc} = \frac{af}{cf}.$$

114. Quick-return Motion using the Swinging-block Linkage.—If in Fig. 136 or 137 the piece C is made long, and the link D is reduced to a sliding block, we have a linkage which may be drawn as shown in Fig. 140, where a uniform rotation of A will cause an oscillation of the link C. If A turns R.H. through the angle α, the link C and the connected tool-slide E will travel to the right; and while A turns through the angle β, C and E move to the left; thus we have

$$\frac{\text{time of advance of } E}{\text{time of return of } E}=\frac{\alpha}{\beta}.$$

If we have given the desired time-ratio of advance to return; the line of centres, ac; the centre, a; and the length of the crank, ab; to locate the centre c draw the crank-pin circle bb_1 and make the angle b_1ac equal to $\frac{1}{2}\beta$, where $\frac{\alpha}{\beta}=\frac{\text{advance}}{\text{return}}$. The tangent b_1c to the crank-pin circle at b_1 will give the desired centre c.

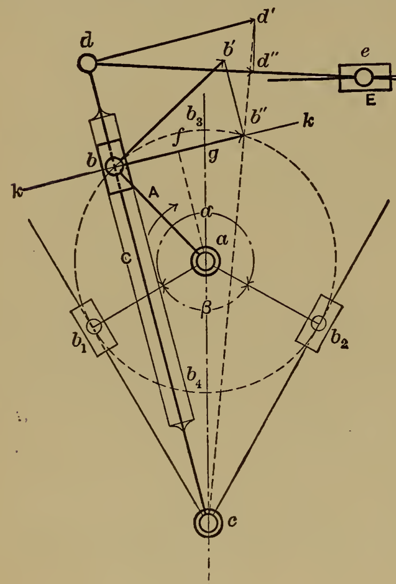


Fig. 140.

To determine the l.v. of the slide E at any moment, given the l.v. of b around a, let bb' represent the l.v. of b around a, then bb'' will be the l.v. b around c; dd' will be the l.v. of d around c, and ee' will represent the l.v. of the slide E.

To find the ratio $\frac{\text{a.v. of } C}{\text{a.v. of } A}$ we have, as in Fig. 139, the sliding pair replacing two infinite links, the centre lines of which must be perpendicular to the centre line of the sliding pair. Therefore the centre line of the infinite connecting link would be the line bk through the crank-pin b perpendicular to the centre line cd of the sliding pair. This would give

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{af}{bc}=\frac{ag}{cg}.$$

When the crank is in the positions ab_1 and ab_2 it will be seen that the centre line of the infinite connecting link would pass through a, giving $\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{0}{b_1c}$, or the link C has no angular motion, as would have been evident from its position. From the position b_1 the a.v. of C increases to a maximum, for the forward stroke, at the position b_3; the maximum on the return stroke being at b_4. In the former position

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{ab_3}{cb_3},$$

and in the latter position

$$\frac{\text{a.v. } C}{\text{a.v. } A}=\frac{ab_4}{cb_4}.$$

115. The Turning-block Linkage.—If the link A, Fig. 128, is considered as fixed, we shall have the linkage shown in Fig. 141, where B is a crank turning uniformly about b, which on so turning will cause D to make complete rotations about a, but with a variable motion.

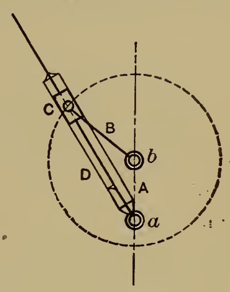


Fig. 141.

Whitworth Quick Return is the name given to the linkage when it is used as a quick-return motion, as in Fig. 142. If the crank bc (Fig. 142) turns uniformly R.H. from the position c_1 to the position c_2, the slide e will travel from its extreme position at the right to the end of its stroke at the left; and while bc turns from c_2 to c_1, the slide e returns:

$$\therefore \frac{\text{time of advance of } e}{\text{time of return of } e}=\frac{\alpha}{\beta}.$$

To locate the centre a, given the time-ratio of advance to return; the line of centres, the axis b and the crank bc; make the angle c_1ba equal to $\frac{1}{2}\beta$, where $\frac{\alpha}{\beta}=\frac{\text{advance}}{\text{return}}$, and draw c_1a through c_1 perpendicular to the line of centres; the point a is the axis of the link ad. If the stroke of the slide e is not on a line passing through a, but below it, as is commonly the case, the time-ratio of advance to return would be somewhat different from the above.

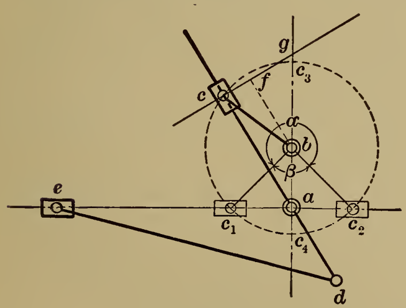


Fig. 142.

The l.v. of the slide e may be found by the same method as that used in the swinging-block linkage.

For the a.v. ratio we have

$$\frac{\text{a.v. } ad}{\text{a.v. } bc}=\frac{bf}{ac}=\frac{bg}{ag},$$

the line cg being the centre line of the infinite connecting link. This ratio is unity at the two positions c_1 and c_2, the centre line of the connecting link being parallel to the line of centres. From the position c_1

the ratio diminishes until c is at c_3, when it becomes $\frac{bc_3}{ac_3}$, which is its minimum value; it then increases to unity at c_2, still further increasing until at c_4 it has its maximum value $\frac{bc_4}{ac_4}$.

For the development of this linkage as practically used see § 118, Fig. 151.

116. By fixing the block C (Fig. 128) we obtain the fourth form of the linkage having one sliding pair, as shown in Fig. 143. The link

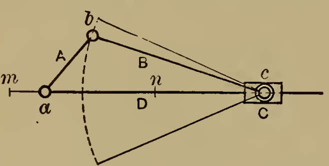


FIG. 143.

B now swings about a fixed axis in C, and the slide D moves rectilinearly to and fro in the block C, which is now the frame; the link A, now a connecting-rod, has a complex motion made up of a combined oscillation and rotation. If the link A is so expanded that it can be caused to make complete rotations relative to the axis a, the link D would have a reciprocation relative to the block C, the stroke of D being twice ab. Such a development is shown in Fig. 152, § 118.

117. **The Isosceles Sliding-block Linkage.**—If we make the length of the connecting-rod B (Fig. 128) equal to that of the crank A, we shall obtain the linkage abc (Fig. 144). Here, if we consider ab as the driver, and c to start from the position c_1, it will be found that when the crank ab is at an angle of 90° with ac_1, the path of c, the block c is directly over a, and any further rotation of ab will only cause a similar rotation of bc. In order to cause c to continue in its path when ab reaches the 90° position, it will be necessary to find the centroids of B and D, and apply the principles of § 105.

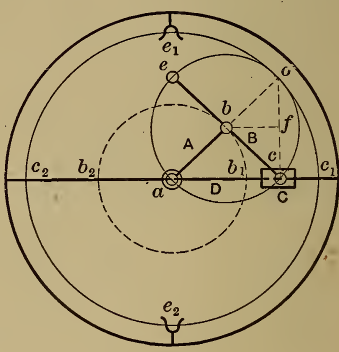


FIG. 144.

If we assume the piece D as fixed, the centroid of B will be the light circle $c_1e_1c_2e_2$, the point b moving in the circular path b_1bb_2, and c in the rectilinear path c_1cc_2. The trace of the surface for bc which by rolling on the centroid of bc would give the same motion to bc as the linkage, would be found, by the method of § 27, to be the smaller circle $coea$. Now if we continue the line cb to e, making $be=bc$, the point e will be found at e_1 and e_2 when ab and bc are in line; and if we supply the centroids with pairs of elements which will be in contact at e_1 and e_2, these pairs will cause the point c to travel by a. The motion of c will then be four times the length of the crank ab.

Since the point c in the sliding block is always under the point o, and since o is always in the continuation of ab, if ab turns uniformly the block C will have simple harmonic motion. This can also be shown as follows:

$$\frac{\text{l.v. } c}{\text{l.v. } b}=\frac{co}{bo}=\frac{co}{\frac{1}{2}ao}=2 \sin \theta.$$

$$\therefore \text{ l.v. } c=2\times\text{l.v. } b\times \sin \theta,$$

or the l.v. c is of the same nature as that illustrated by Fig. 133, but twice as fast, assuming the same length of crank and the same velocity of crank-pin.

The four mechanisms of the linkage with one sliding pair here become two only, the first of which has just been discussed. The same mechanism is obtained whether we consider the block C or the slot in D to be stationary. The second case is when the link ab is fixed; or the same mechanism would result by fixing the link bc.

If we fix the link ab, the link bc, rotating about b, would cause the link ac_1 to turn about a, and if the centroids are used the small circle coe, containing bc, turning uniformly around b, would cause the large circle c_1oe_1, containing ac_1, to make complete rotations about a, the a.v. ratio being

$$\frac{\text{a.v. } D}{\text{a.v. } B}=\frac{bo}{ao}=\frac{1}{2},$$

the radius of the smaller circle being one half that of the larger.

The a.v. ratio can also be shown from the linkage. In the given position the centre line of the infinite connecting link will be co; therefore

$$\frac{\text{a.v. } ac_1}{\text{a.v. } bc}=\frac{bf}{ac}=\frac{bo}{ao}=\frac{1}{2}.$$

It is interesting to note that the path of the point e relative to ac_1 is a diameter of the circle $c_1e_1c_2e_2$ at right angles to c_1c_2, and if we replace D by a disc having two grooves at right angles to each other, intersecting at a, at the same time supplying e with a sliding block similar to c, the disc D will make one revolution while ce makes two revolutions. Thus ce can be considered as a wheel of two teeth rolling inside of another, D, of four teeth: in such a case, the blocks c are usually made cylindrical, and roll in the grooves so as to reduce the friction. Three grooves might be made in the disc intersecting at a, and making angles of 60° with each other; the circle ce would then need to be supplied with three rollers spaced equidistant on its circumference; the relative motion of the disc and bc would be the same as before.

118. Expansion of Elements in the Linkages with One Sliding Pair.—So far we have not concerned ourselves with the diameter of the cylindric pairs in these mechanisms, as alterations in the diameters would not affect the relative motions. Also a change in shape or size of the links will not alter the relative motions, so long as the centre lines of the elementary links remain unchanged, and yet such change may make the action of the linkage possible. Since these enlargements of the elements of the cylindric pairs sometimes conceal the real nature of the mechanism and cause much indistinctness, it will be well to consider a few cases here.

We will consider first the *sliding-block linkage,* shown in Fig. 128.

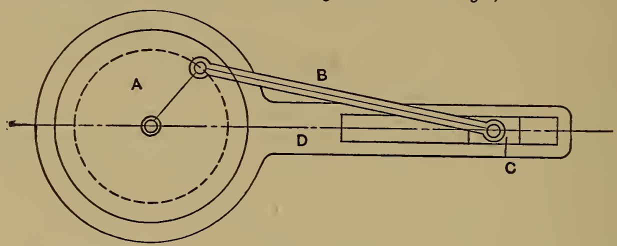


FIG. 145.

Each of its four links is more or less closely connected with its three cylindric pairs, and their forms are therefore dependent upon the relative sizes of the latter, although this size does not affect the nature of their relative motion. Evidently we do not alter the combination kinematically, if we increase the diameter of the crank-shaft so as to include the crank-pin as shown in Fig. 145, where the different links are lettered the same as in Fig. 128. The open cylinder of D must be enlarged to the same extent as the shaft, so that the pair is still closed.

This arrangement is used in practice, in some slotting and shearing machines; to work a short-stroke pump from the end of an engine shaft; and in other cases where a short crank forms one piece with its own shaft.

If we expand the crank-pin until it includes the shaft, as shown in Fig. 146, we obtain the common *eccentric and rod,* which can be seen to differ only in form from the common *crank and connecting-rod.* This mechanism is much used to operate the valve motions in steam engines, where it is necessary to obtain a reciprocating motion, often less than the diameter of the engine shaft. The part of the rod B

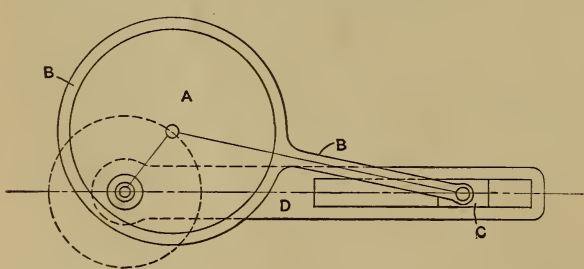


FIG. 146.

which encloses A is called the *eccentric-strap,* and is made in two parts,

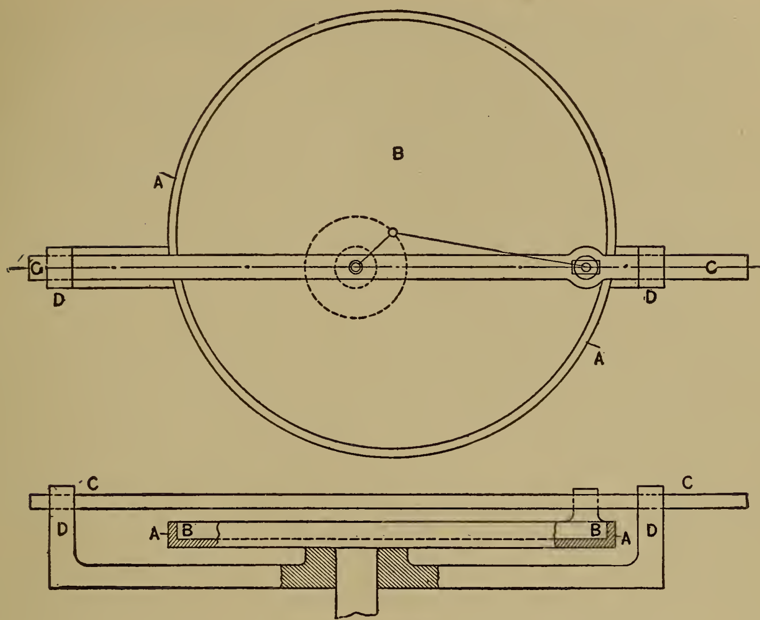


Fig. 147.

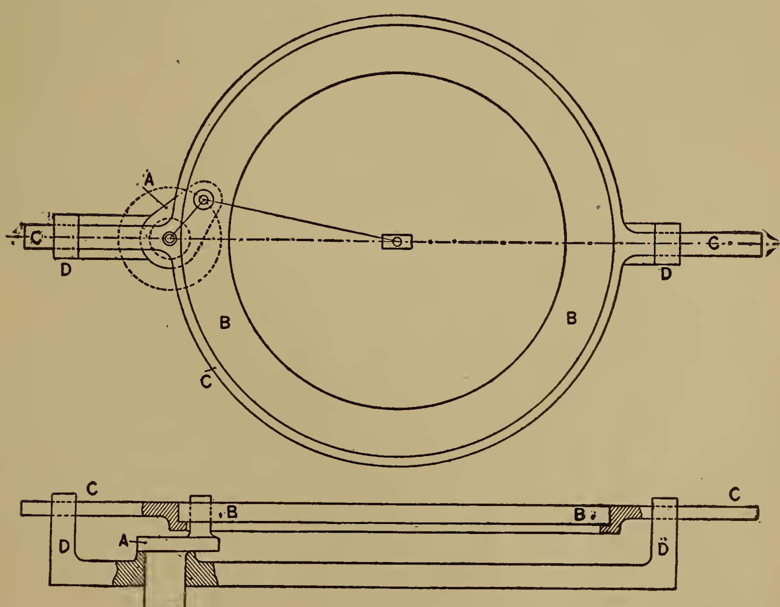


Fig. 148.

and separate from B, the *eccentric-rod*, which is usually bolted to one of these parts; the cylindrical pair is also so shaped as to allow no axial motion of B on A.

If the crank-pin is still further expanded until it includes the cross-head pin, we shall obtain the arrangement shown in Fig. 147. In this case, the element of the cylindric pair which belongs to the crank A has been inverted, and thus made open (§ 39). The rod B becomes an eccentric disc which swings about the bearing in the piece C, and is always in contact with the hollow disc A, carried by the shaft turning in D.

If, instead of enlarging the crank-pin to include the cross-head pin, we enlarge the latter to include the former, the arrangement shown in Fig. 148 is obtained. The rod B is again an eccentric disc or annular ring; but it now oscillates in a ring forming part of the piece C, while the crank-pin drives it by internal contact. In order to make the relations of these expansions more clear, fine light lines have been drawn in each case, showing the elementary links. The above exhausts all the practicable combinations of the three cylindric pairs.

In Fig. 148 we can replace the link B by an annular ring containing the crank-pin and oscillating in a corresponding annular groove in the piece C. So long as we keep the centre of this ring the same as that of B, we have not altered the mechanism, and as the motion of the ring is

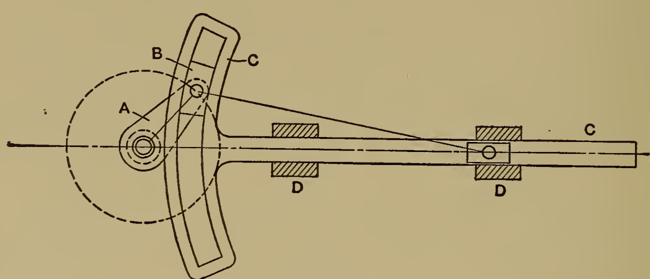


Fig. 149.

merely oscillatory, we need only use a sector of it, and enough of the annular groove to admit of sufficient motion of the sector in its swing. Fig. 149 represents the arrangement altered in this way, the different parts being lettered the same as in Figs. 148 and 128; B is still the connecting-rod, and its motion as a link in the chain remains the same as before, and is completely restrained; the shape of the sector always fixes the length of the connecting-rod. This mechanism is made use of in the Stevenson and Gooch reversing-gears for locomotives, and in other places; the chains are not there simple, but compound.

The mechanism shown in Fig. 150, which sometimes occurs in slotting and metal-punching machines, is another illustration of pin expansion. The whole forms a sliding-block linkage; the link B is formed essentially as in Fig. 149, but here the profiles against which it works are concave on both sides of the crank-pin, the upper profile being of large, and the lower of very small, radius, but both forming part of the block C. The work is done when the block C is moving downwards, and the small radius profile being then in use, the friction is reduced. In this case the block C is so enclosed by the slide D that the profiles representing the cross-head pin lie entirely within the sliding pair, an illustration of how the method of expansion can be applied to the fourth or sliding pair.

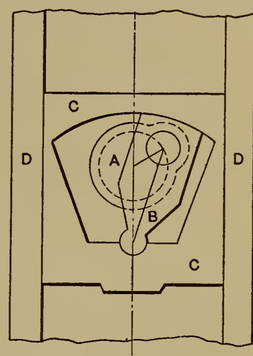


Fig. 150.

To make possible the action of the Whitworth Quick Return, § 115, Fig. 142, the axis b may be expanded until the axis a can be placed upon it, or a shaft passed through it if the crank ad is required on the other side of the frame. Fig. 151 shows the expansion where A is the enlarged axis b and is part of the frame of the machine; the crank bc has become a spur-gear B, turning on A, and driven uniformly by a small pinion below it. The crank-pin works in a block C turning on the face of B and fitting in a slot in the crank D, the expansion of the link ad, pivoted on a pin a provided for it in the frame A. While the gear B turns so that the axis of the crank-pin moves through the larger angle c_1bc_2, the slide E will have its slow motion or advance, the return occurring while the gear moves through the smaller angle c_2bc_1.

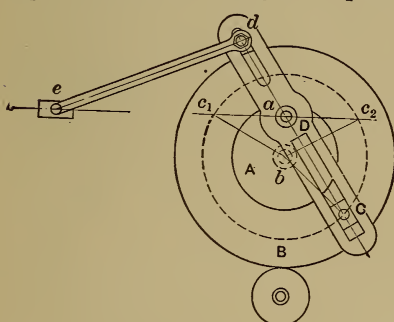


Fig. 151.

A development of the fourth form of the linkage with one slide, mentioned in § 116, Fig. 143, is shown in Fig. 152. The connecting link ab is expanded into a worm-wheel A, which may be made to rotate about the axis a by a worm keyed to the shaft D. The worm and wheel are kept in contact by a piece which supports the bearing of A, hangs from the shaft D, and confines the worm between its bearings. A rotation of the shaft D will turn A, causing a reciproca-

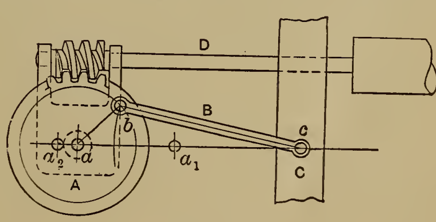


Fig. 152.

tion of the axis a, and consequently of the driving shaft D, through a distance a_1a_2 equal to twice ab.

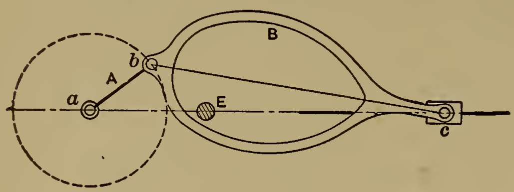


Fig. 153.

A change in the shape of an elementary link frequently permits motions to take place which are not otherwise possible. In Fig. 153, for example, a complete rotation of A to cause a reciprocation of C would be possible with the open rod B moving around the fixed shaft E, but not with elementary link bc, shown by a light line.

119. Linkwork with Two Sliding Pairs.—If we apply the principle of § 111 to Fig. 149, and allow the length of the link B to become infinite, the slot in the slide C will become straight and at right angles to the sliding pair C, and the connecting block B becomes a prismatic slide.

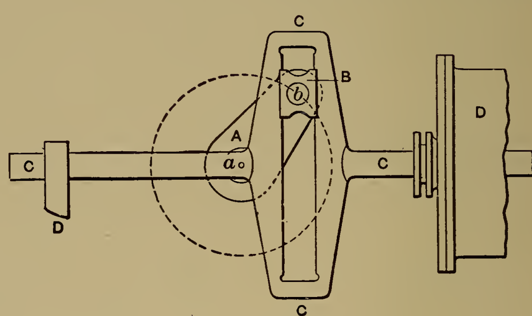


Fig. 154.

Such a mechanism is shown in Fig. 154, the different parts being lettered the same as in Fig. 149. The block C now consists of two sliding pairs at right angles to each other, and the connecting-rod is infinite in length. Here, as in the linkage with one slide, since there are four elementary links, four mechanisms would result. It will be seen, however, that two are identical, leaving three distinct mechanisms. Fig. 155 corresponds with Fig. 154 without the expansion of the cross C, which would be required to make the motion possible, and will be more convenient to use in studying the relative motion.

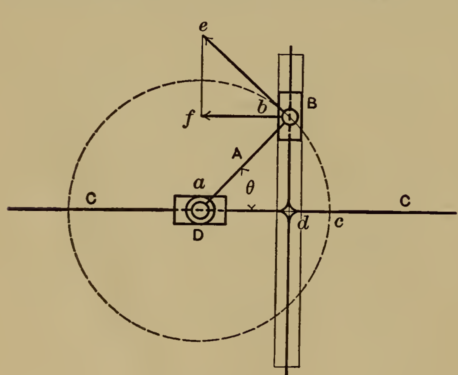


Fig. 155.

120. This mechanism with the link D fixed is often used, and is known under various names, as *crank and slotted cross-head, crank with an infinite connecting-rod, harmonic motion,* etc.

If in Fig. 155 we assume the block D fixed, and give a uniform rotation to A, we shall have a reciprocation of the cross C through D while the block B slides up and down on the cross, giving to it a simple harmonic motion. If the bloc B were fixed instead of D, the cross C would have exactly the same form of motion as when D was fixed, only it would be up and down through the slide B.

To determine the motion of C for any angular motion of A, let the crank A turn through the angle θ, Fig. 155; the distance cd through which the cross has moved will be

$$cd = ac - ad = ab(1 - \cos\theta), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

which was found in § 112, equation (34), as the equation for simple harmonic motion.

For the l.v. ratio, from Fig. 155,

$$\frac{\text{l.v. of cross } C}{\text{l.v of crank-pin } b} = \frac{bf}{be} = \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

(see equation (35), § 112).

Fig. 156 shows a combination of an eccentric circular disc A, and a sliding piece C, moving through fixed guides, one of which is shown at D. A uniform rotation of A about the axis a will give harmonic motion to C. This can be shown by noticing that the distance which C has moved from its highest position is

$$cd = ef = ab(1 - \cos\theta),$$

which is the equation for simple harmonic motion where $2ab$ is the stroke of the slide.

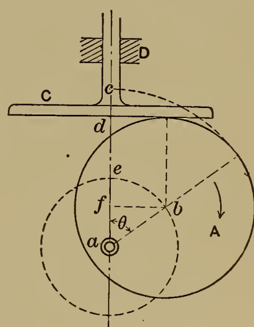


Fig. 156.

This mechanism can also be found by an expansion of the crank-pin b, Fig. 154, until it includes the shaft a, the slot in C being correspondingly enlarged, and then after turning the figure through 90°, omitting the lower part of the cross C, allowing A and C to be paired by force-closure.

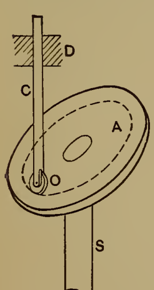


Fig. 157.

The Swash-plate.—The apparatus shown in Fig. 157, known as a *swash-plate*, consists of an elliptical plate A set obliquely upon the shaft S, which by its rotation causes a sliding bar C to move up and down, in a line parallel to the axis of the shaft, in the guides D, the friction between the end of the bar and the plate being lessened by a small roller O. When a roller is used, the motion of the bar C is approximately harmonic—the smaller the roller the closer the approximation. If a point is used in place of the roller, the motion is harmonic, which can be shown as follows:

Since the bar C remains always parallel to the axis of the shaft, the path of the point O, projected upon an imaginary plane through the lowest position of O and perpendicular to the shaft S, will be a circle, and the actual path of O on the plate A will be an ellipse.

In Fig. 158 let eba represent the angular inclination of the plate to the axis of the shaft, ab the axis of the shaft, eof the actual path of the point o on the plate, and the dotted circle erd the projection of this path upon a plane through e (the lowest position of o) perpendicular to the axis ab.

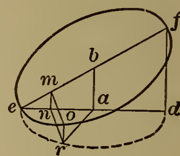


Fig. 158.

Draw om perpendicular to ef, or perpendicular to the plane erd, and rn perpendicular to ed, the diameter of the circle erd. Join mn, and suppose the plate to rotate through an angle $ear=\theta$, and thus to carry the point o through a vertical distance equal to or.

Then

$$\begin{aligned} or=mn=ab\times\frac{en}{ea} & \quad \left(\text{as } \frac{mn}{ab}=\frac{en}{ea}\right) \\ =ab\left(\frac{ea-an}{ea}\right) & \\ =ab\left(1-\frac{an}{ea}\right) & \\ =ab(1-\cos\theta), & \end{aligned}$$

or the same formula as was derived in the case of harmonic motion. In this case ab represents the length of the equivalent crank, and is equal in length to one-half of the stroke of the rod C.

121. If instead of fixing the block D or the block B, Fig. 155, we assume the link A to be fixed, we obtain the second form of the mechanism. This is shown in Fig. 159, where the axes a and b are fixed and the blocks B and D are free to turn, while the cross C will slide through them, being forced at the same time to revolve. If D turns uniformly, B will also turn uniformly; for if D turns through any angle θ as shown, the centre lines of the slots in the cross C must occupy the position shown by the light lines, and the angle abc has diminished by the same amount as bac has increased or B has turned through the same angle as D. It is

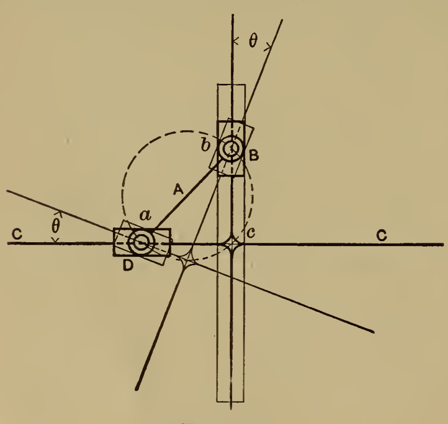


Fig. 159.

interesting to notice that the piece C has an eccentric revolution, the intersection c of the centre lines moving in a circular path with ab as its diameter.

Oldham's Coupling.—The mechanism shown in Fig. 160, known as *Oldham's coupling*, is an interesting example of the above mechanism. Its object is to connect two parallel shafts placed a short distance apart so as to communicate a uniform rotation from one to the other.

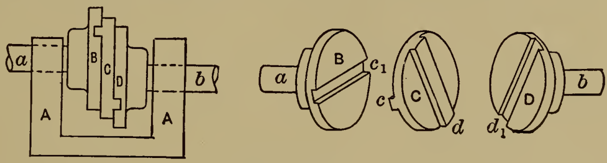


FIG. 160.

The bearings for the two shafts a and b are in the piece A, which takes the place of the crank A (Fig. 159); the pieces B, C, and D (Fig. 160) take the places of those similarly lettered in Fig. 159, and are drawn separated at the right of the figure to make their construction clearer. The piece C has two diametrical slides c and d placed on its opposite sides and at right angles to each other. The grooves c_1 and d_1, in the pieces B and D respectively, fit the corresponding slides similarly lettered on C. Fig. 160 is an expansion of the elementary links in Fig. 159.

122. If in the linkage with two slides, Fig. 155, we fix the cross C, we

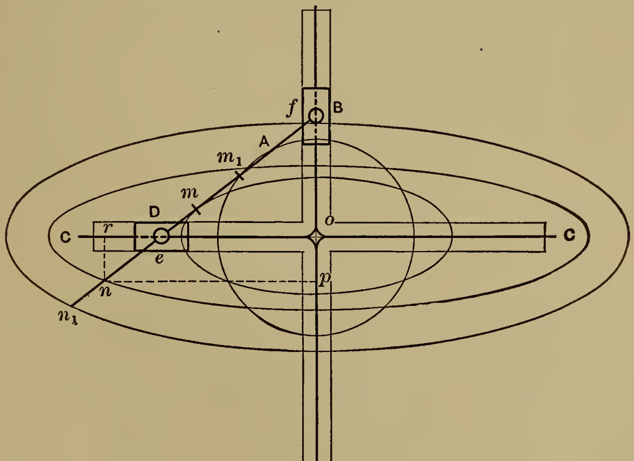


FIG. 161.

have the third mechanism. This is shown in Fig. 161, where, if we replace the cross by grooves, the blocks B and D may slide in their respective grooves, with the result that any point on ef, between e and f

or on ef produced, will trace an ellipse. The point m will trace an ellipse of which the semi-major axis is fm and the semi-minor axis is em. The point n will trace an ellipse, of which the semi-major and semi-minor axes are fn and en respectively. The linkage thus arranged is called an *elliptic trammel.* All ellipses traced by points on ef beyond e have the difference of their semi-major and semi-minor axes equal to ef. All ellipses traced by points between e and f have the sum of the semi-major and semi-minor axes equal to ef. The point m_1 half-way between e and f traces a circle with a diameter ef (see Fig. 144). In the elliptic trammel the ellipse is usually traced by a point outside of e; e and f are made so that their distance apart is adjustable and they are set one half the difference of the major and minor axes apart.

An ellipse can be readily drawn by taking a card one corner of which shall represent the tracing-point n. Points corresponding to the desired positions of e and f (Fig. 161) are then marked on the edge of the card, and by placing these points in successive positions on lines at right angles with each other, corresponding to the slots in which the blocks in Fig. 161 move, and marking the successive positions of n, will give a series of points on the required ellipse.

To prove that the point n moves on an ellipse, let $np=x$ and $nr=y$; nf (semi-major axis) $=a$ and ne (semi-minor axis) $=b$. The equation for an ellipse referred to the centre as the origin is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

In Fig. 161 we have

$$\frac{x}{a}=\frac{np}{nf} \quad \text{and} \quad \frac{y}{b}=\frac{nr}{ne},$$

and, since the triangles ern and npf are similar,

$$\frac{\overline{np}^2}{\overline{nf}^2}+\frac{\overline{nr}^2}{\overline{ne}^2}=\frac{\overline{np}^2}{\overline{nf}^2}+\frac{\overline{fp}^2}{\overline{nf}^2}=\frac{\overline{nf}^2}{\overline{nf}^2}=1,$$

or

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \qquad \text{. . . . . (39)}$$

and the point n moves on an ellipse.

Now in drawing an ellipse, the paper is fixed, and the pencil is moved over it; but in turning an ellipse in a lathe, the tool, which has the same position as the pencil, is fixed, and the piece to be turned should have such a motion as would compel the tool to cut ellipses.

This is accomplished in the *elliptic chuck,* Fig. 162, the points f and e, which correspond with the points f and e (Fig. 161), being fixed, and the tool acting at the point n, Fig. 162, corresponding with the tracing-point n in Fig. 161. The sliding blocks BB and DD, as well as the points f, e, and n, correspond with the parts similarly lettered in Fig. 161, but in this case ef is fixed instead of C.

In the form of elliptic chuck shown in Fig. 162, A represents the headstock casting, which furnishes bearings for the spindle E, the trace of the axis of which is shown at e, and a fastening for the piece GG, which provides a centre f through which the centre line of the slot $B'B'$ always passes. The face-plate C, to which the material to be turned

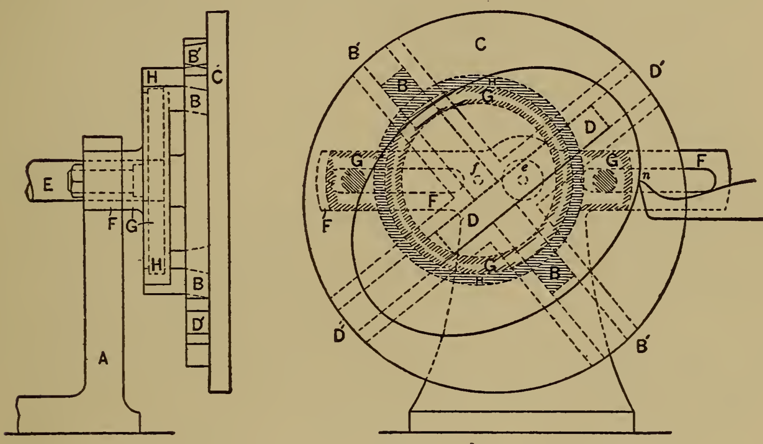


FIG. 162.

is attached, is furnished with two straight grooves at right angles to each other. The lathe-spindle E is provided at its front end with a straight arm DD, the centre line of which passes through its axis e. This arm fits the groove $D'D'$ and communicates the rotation of the spindle to the face-plate C, at the same time always compelling the centre line of the slot to pass through e.

The piece GG, consisting of an annular ring with two projecting lugs, is arranged to slide in the line fe, and is held in place by bolts fastening it to slotted projections on the headstock casting FF. This annular ring allows the spindle to turn within it and carries the strap HH, which can turn upon it but can have no axial motion, as will be seen from its construction. Two projections BB, the centre line of which passes through f, the centre of the ring, are formed on the strap HH, and are fitted to the dovetailed groove $B'B'$ in the piece C; thus the connection between the pieces G and C is such that the centre line of the groove $B'B'$ always passes through the point f, and no motion along the axis of the spindle is allowed in C.

The above is then an arrangement where the plate C has a motion such that the centre lines of the two grooves always pass through the points e and f. The distance ef, equal to the difference between the semi-axes of the ellipse, can be adjusted at will, and various ellipses may be turned. For example, if the axes of the ellipse are 6″ and 4″, the distance *ef* is 1″.

123. In the linkages with one slide, the centre line of the sliding pair may not pass through the point *a*, as in Fig. 128. This will give rise to another series of mechanisms, somewhat similar to those described.

In the sliding-block linkage, Fig. 163, the motion of the block *C* from one end of its stroke to the other will require a motion of *less* than 180° of the crank *ab* in one direction, and *more* than 180° in the other. Slight differences would also occur in the l.v. ratio, but the same laws apply, the instantaneous centre being at *o* and

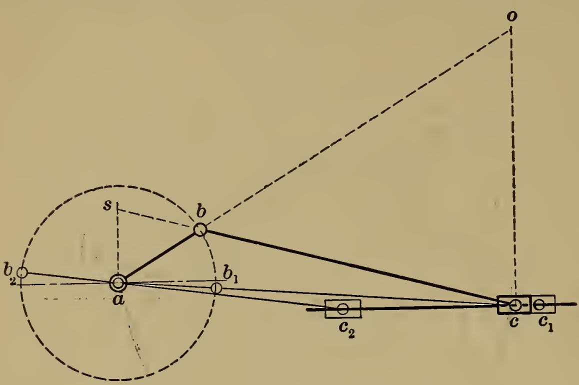


FIG. 163.

$$\frac{\text{l.v. } c}{\text{l.v. } b}=\frac{co}{bo}=\frac{as}{ab}.$$

In the swinging-block linkage the mechanism could be arranged as in Fig. 164. In the position shown in the figure

$$\frac{\text{a.v. } cd}{\text{a.v. } ab}=\frac{ae}{cf},$$

since *be* is the centre line of the infinite connecting link. To prove that this is so, let *bg* represent the l.v. of *b* around *a*. Resolve this into two components, one of sliding along *cd* and the other of rotation of *b* around *c*. This will give *bh* as the l.v. of *b* around *c*. We may then write

$$\frac{\text{a.v. } cd}{\text{a.v. } ab}=\frac{\dfrac{\text{l.v. } b \text{ around } c}{bc}}{\dfrac{\text{l.v. } b \text{ around } a}{ab}}=\frac{\dfrac{bh}{bc}}{\dfrac{bg}{ab}},$$

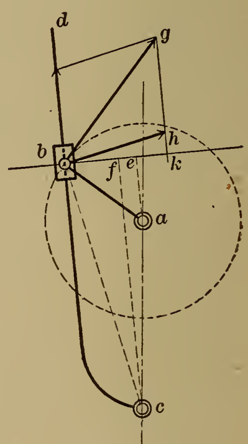


FIG. 164.

but by the similar triangles *bhk* and *cbf*, and *bgk* and *abe*,

$$\frac{bh}{bc}=\frac{bk}{cf} \quad \text{and} \quad \frac{bg}{ab}=\frac{bk}{ae};$$

$$\therefore \frac{\text{a.v. } cd}{\text{a.v. } ab}=\frac{bk}{cf}\times\frac{ae}{bk}=\frac{ae}{cf}.$$

Similarly in the linkages with two slides the arms of the cross C may not be at right angles, in which case a new series of mechanisms will result, one example being shown by Fig. 165.

In Fig. 165 let be represent the l.v. of b around a, then bf will be the l.v. of the cross C. If the crank coincides with the centre line of the slot in C, the l.v. of C, b_1f_1 at ab_1, will be greater than the l.v. of the crank-pin, b_1e_1. If ab turns uniformly, the motion of C will be found to be harmonic, but the length of stroke is greater than twice ab.

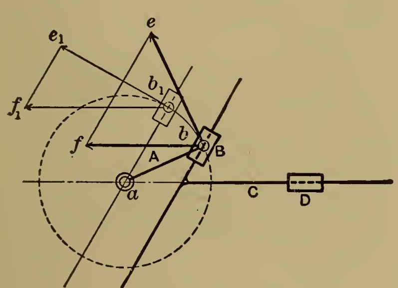


FIG. 165.

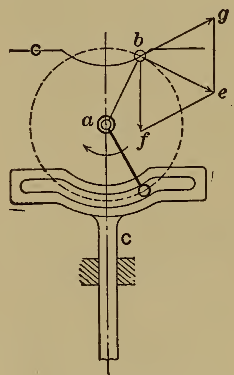


FIG. 166.

Other forms of the linkage with two slides occur, as in Fig. 166, where a slight period of rest is desired for the piece C at the end of each downward stroke. To find the l.v. of C at any point, as when the crank is at ab, it is necessary to resolve the l.v. of b, represented by be, into two components, one, bf, in the direction of motion of the piece C, and the other, bg, tangent to the centre line of the slot in C.

124. The Conic Four-bar Linkage.—If the axes of the four cylindric pairs (Fig. 101) of the four-bar linkage are not parallel, but have a common point of intersection at a finite distance, the chain remains movable and also closed (Fig. 167). The lengths of the different links will now be measured on the surface of a sphere whose centre is at the point of intersection of the axes. The axoids will no longer be cylinders, but cones, as all the instantaneous axes must pass through the common point of intersection of the pin axes.

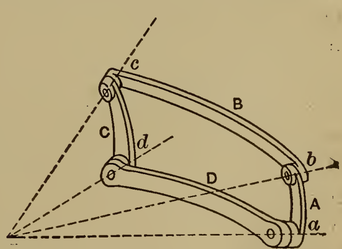


FIG. 167.

The different forms of the cylindric linkage repeat themselves in the conical one, but with certain differences in their relations. The principal difference is in the relative lengths of the links, which would vary if they were measured upon spherical surfaces of different radii, the links being necessarily located at different distances from the centre of the sphere in order that they may pass each other in their motions. The ratio, however, between the length of a link and its radius remains constant for all values of the radius, and these ratios are merely the values of the circular measures of the angles subtended by the links. In place of the link lengths, we can consider the relative magnitudes of these angles, which can be also designated by the letters A, B, C, and D.

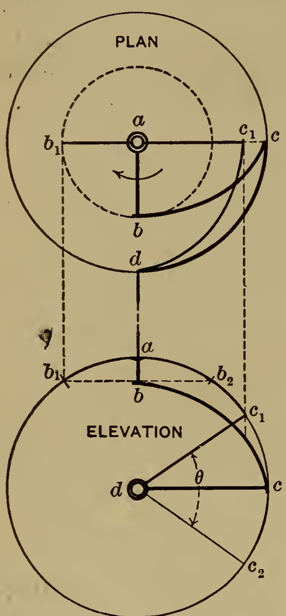


FIG. 168.

The alterations in the lengths of the links will now be represented by corresponding angular changes. The infinitely long link corresponds to an angle of 90°, as this gives motion on a great circle which corresponds to straight-line motion in the cylindric linkages.

Fig. 168 shows plan and elevation of a conic four-bar linkage *abcd*, the link *ab* turning about *a*, and, for a complete turn, causing an oscillation of the link *cd* about *d* through the angle θ, shown in the elevation. In the figure each of the links *bc* and *cd* subtends 90°, while the link *ab* subtends about 30°. Varying the angles which the links subtend will, of course, vary the relative motions of *ab* and *cd*.

125. **Hooke's Joint.**—If in Fig. 168 each of the links *ab*, *bc*, and *cd* is made to subtend an angle of 90°, we shall find that *ab* and *cd* will each make complete rotations. This mechanism, known as a *Hooke's joint*, is represented by Fig. 169; *a* and *d* are the two intersecting shafts, and the links *ab* and *cd*, fast to the shafts *a* and *d* respectively, subtend 90°, while the connecting link *bc* also subtends 90°.

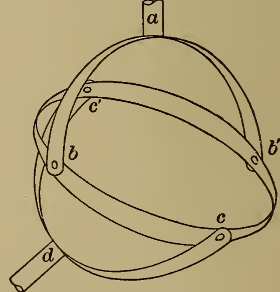


FIG. 169.

In order to make the apparatus stronger and stiffer, two sets of links are used, and the link *cb* is continued around as shown, thus giving an annular ring joining the ends of the double links *cdc′* and *bab′*. This ring is sometimes replaced by a sphere into which the pins *c*, *b*, *c′*, and

b' are fitted, or by a rectangular cross with arms of a circular section working in the circular holes at b, c, c', and b'. Or, the arms bab_1 and cac_1 may be paired with grooves cut in a sphere in planes passing through the centre of the sphere and at right angles to each other. Such forms of Hooke's joint are now on the market and much used.

Relative Motion of the two Connected Shafts.—Given the angular motion of ab, to find the angle through which cd turns. Fig. 170 shows a plan and elevation of a Hooke's joint, so drawn that the axis a is perpendicular to the plane of elevation. If the link ab is turned through an angle θ, it will be projected in the position ab_1. The path of the point c will be on a great circle in a plane perpendicular to the axis d, which will appear in the elevation as the ellipse bce. The point c will then move to c_1, found by making the angle b_1ac_1 equal to 90°, for the link bc subtends 90°, and since the radius from b to the centre of the sphere is always parallel to the plane of elevation, its projection and that of the radius from c will always be at right angles. The projected position of the linkage after turning a through the angle θ will be ab_1c_1d. To find the true angle through which the link cd and the shaft d have turned, swing the ellipse bce with the axis d, until d is perpendicular to the plane of elevation, when the points c and c_1 will be found at c' and c_1', respectively, giving the angle $c_1'ac'=\phi$ as the true angle through which the axis d has turned. Or the arm dc_1 may be revolved until shaft d is perpendicular to the horizontal plane, giving $c_1bc_1'=\phi$, as shown in the plan.

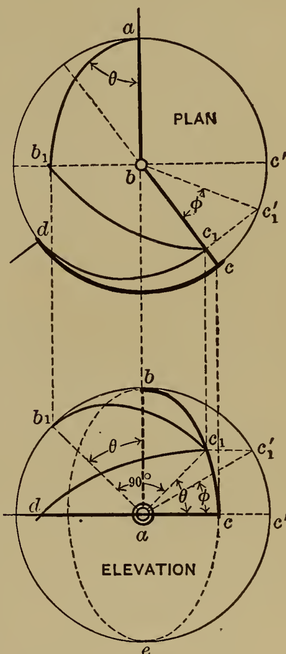


Fig. 170.

It is evident from the above that two intersecting shafts connected by a single Hooke's joint cannot have uniform motions. If, however, two joints are used to connect two parallel or intersecting shafts, they may be so arranged that they will have uniform motions.

Double Hooke's Joint.—*Two parallel or intersecting shafts may be connected by a double Hooke's joint and have uniform motions, provided that the intermediate shaft makes equal angles with the connected shafts, and that the links on the intermediate shaft are in the same plane.* Fig. 171 gives a plan and elevation of two shafts so connected, and the position after turning through an angle θ. It is evident that one joint just neutralizes the effect of the other.

The term *universal joint* is often used to designate the above-described mechanism.

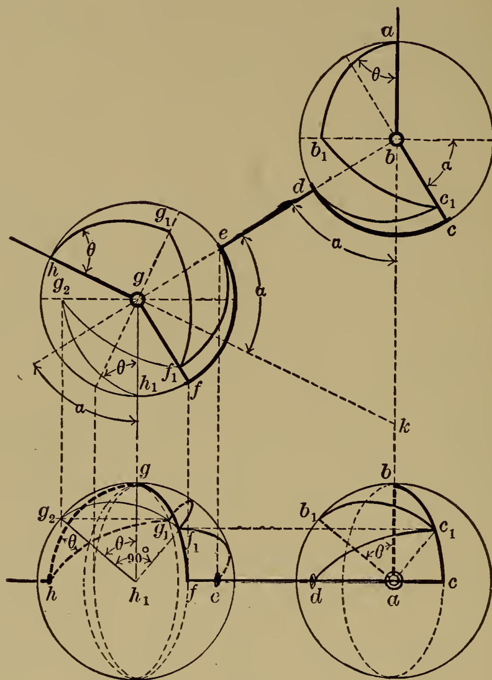


FIG. 171.

Angular Velocity Ratio in a Single Hooke's Joint.—Fig. 172 reproduces the elevation given in Fig. 170, which shows the angles θ and ϕ through which a and d move respectively. The angle $eac=\alpha$ will be the true angle between the planes in which the paths of the points b and c lie; to find the angle ϕ analytically in terms of θ and α, we have, from Fig. 172,

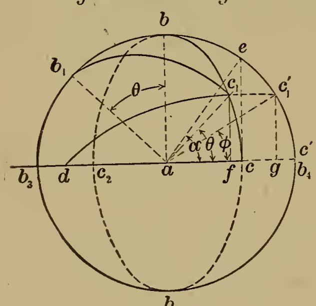


FIG. 172.

$$\tan\phi=\frac{c_1'g}{ag}=\frac{c_1f}{ag};\quad \tan\theta=\frac{c_1f}{af}.$$

$$\therefore\ \frac{\tan\phi}{\tan\theta}=\frac{c_1f}{ag}\times\frac{af}{c_1f}=\frac{af}{ag}=\frac{ac}{ac'}=\frac{ac}{ae}=\cos\alpha;$$

$$\therefore\ \tan\phi=\tan\theta\cos\alpha.\quad\ldots\ldots\ldots\quad(40)$$

To obtain the velocity ratio, we must differentiate equation (40), remembering that $\cos\alpha$ is a constant; then

$$\frac{d\phi}{d\theta}=\frac{\sec^2\theta}{\sec^2\phi}\cos\alpha=\frac{1+\tan^2\theta}{1+\tan^2\phi}\cos\alpha.\quad\ldots\ldots\quad(41)$$

If we eliminate ϕ and θ from equation (41), by use of equation (40) we shall obtain

$$\frac{d\phi}{d\theta}=\frac{\cos\alpha}{1-\sin^2\theta\sin^2\alpha} \quad \ldots \ldots \ldots \quad (42)$$

$$=\frac{1-\cos^2\phi\sin^2\alpha}{\cos\alpha}. \quad \ldots \ldots \ldots \quad (43)$$

Assume ab and cd the starting positions of the arms ab and cd respectively; then equations (42) and (43) will have minimum values when $\sin\theta=0$ and $\cos\phi=1$; this will happen when θ and ϕ are 0° and 180°, giving $\frac{d\phi}{d\theta}=\cos\alpha$ in both cases. Thus the minimum velocity ratio occurs when the driving arm is at ab and ab_2, the corresponding positions of the following arm being cd and c_2d. Maximum values occur when $\sin\theta=1$ and $\cos\phi=0$; then $\frac{d\phi}{d\theta}=\frac{1}{\cos\alpha}$, which will happen when θ and ϕ are 90° and 270°, the corresponding positions of the driving arm being ab_3 and ab_4.

Hence in one rotation of the driving shaft the velocity ratio varies twice between the limits $\frac{1}{\cos\alpha}$ and $\cos\alpha$; and between these points there are four positions where the value is unity.

If the angle α increases, the variation in the angular velocity ratio of the two connected shafts also increases; and when this variation becomes too great to be admissible in any case, other arrangements must be employed.

CHAPTER VIII.

PARALLEL MOTIONS.—STRAIGHT-LINE MOTIONS.

A PARALLEL MOTION is a linkage designed to guide a reciprocating piece either exactly or approximately in a straight line, in order to avoid the friction arising from the use of straight guides. Some parallel motions are exact, that is, they guide the reciprocating piece in an exact straight line; others, which occur more frequently, are approximate, and are usually designed so that the middle and two extreme positions of the guided point shall be in one straight line, while at the same time care is taken that the intermediate positions deviate as little as possible from that line.

126. Peaucellier's Straight-line Motion.—Fig. 173 shows a linkage, invented by M. Peaucellier, for describing an exact straight line within the limits of its motion.

It consists of eight links joined at their ends. Four of these links, A, B, C, and D, are equal to each other and form a cell; the two equal links E and F connect the opposite points of the cell a and e with the fixed centre of motion d; the link $G = \frac{1}{2}bd$ oscillates on the fixed centre c, cd thus forming the fixed link equal in length to G.

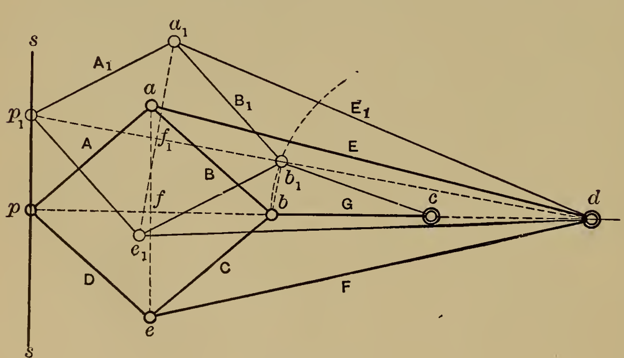


FIG. 173.

If now the linkage be moved within the limits possible by its construction (that is, until the links B and G, and C and G come into line on opposite sides of the centre line of motion cd), the cell will open and close; the points a and e will describe circular arcs about d, and b about c. Finally, the point p will describe a straight line ss perpendicular to the line of centres cd.

To prove this, move the linkage into some other position, as $p_1a_1b_1cd$. (It is to be noticed that since the links A and C, B and D, and E and F, always form isosceles triangles with a common base, a straight line from p to d will always pass through b.) If the line traced by the point p is a straight line, the angle p_1pd will be 90°. The angle bb_1d *is* 90°, since $bc=cd=b_1c$; therefore the triangles p_1pd and bb_1d would be similar right triangles, and we should have

$$\frac{pd}{p_1d}=\frac{b_1d}{bd}.$$

To prove that ss is a straight line it is necessary to show that the above relation exists in the different positions of the linkage. In Fig. 173

$$E^2=\overline{af}^2+(bf+bd)^2;$$
$$B^2=\overline{af}^2+\overline{bf}^2;$$
$$\therefore\ E^2-B^2=2(bf)(bd)+\overline{bd}^2=bd(bd+2bf).$$

But, since the links A and B are equal, the triangle pab is isosceles and the base $pb=2bf$.

$$\therefore\ E^2-B^2=(bd)(pd). \quad \dots\dots \quad (44)$$

By the same process, when the linkage is in any other position, as $p_1a_1b_1cd$, we should have

$$E_1{}^2-B_1{}^2=(b_1d)(p_1d). \quad \dots\dots \quad (45)$$

Equatin equations (44) and (45),

$$(bd)(pd)=(b_1d)(p_1d),$$

or

$$\frac{pd}{p_1d}=\frac{b_1d}{bd},$$

which proves that the path of the point p is on the straight line ss.

If the relation between the links cd and bc be taken different from that shown (Fig. 173), the points b and p, sometimes called the poles of the cell, will be found to describe circular arcs whose centres are on the line passing through c and d; in the case shown, one of these circular arcs has a radius infinity.

127. Scott Russell's Straight-line Motion.—This motion, suggested by Mr. Scott Russell, is an application of the *isosceles sliding-block linkage*, § 117, shown in Fig. 144.

It is made up of the links ab and pc, Fig. 174. The link ab, centred at a, is joined to the middle point b of the link pc, and ab, bc, and pb are taken equal to each other; and the point c is constrained to move in the straight line ac by means of the sliding block. In this case the motion of the sliding block c is slight, as the entire motion of p is seldom taken as great as cp.

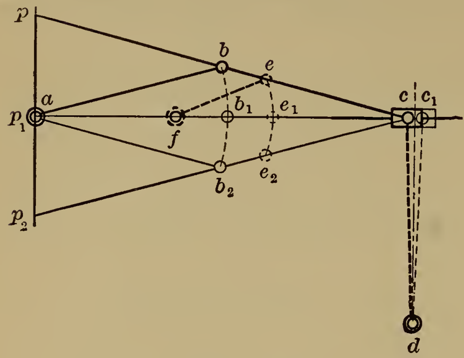


Fig. 174.

To show that the point p describes a straight line pp_1p_2 perpendicular to ac through a, a semicircle may be drawn through p and c with b as a centre; it will also pass through a so that pac will be a right angle; therefore the point p is on ap, which is true for all positions of p.

The point a should be located in the middle of the path or stroke of p. The motion of c may then be found by the equation

$$cc_1 = cp - \sqrt{\overline{cp}^2 - \overline{ap}^2},$$

where ap is the half-stroke of p.

Approximate straight-line motions somewhat resembling the preceding may be obtained by guiding the link cp entirely by oscillating links, instead of by a link and slide.

1° In the link cp (Fig. 174) choose a convenient point e whose mean position is e_1, and whose extreme positions are e and e_2. Through these three points pass a circular arc, ee_1e_2, the centre of which f will be found on the line ac. Join e and f by a link ef, and the two links ab and ef will so guide pe that the mean and extreme positions of p will be found on the line pp_2, provided suitable pairs are supplied to cause passage by the central position.

2° The point c may be made to move very nearly in a straight line cc_1 by means of a link cd centred on a perpendicular erected at the middle point of the path of c. The longer this link the nearer the path of c will approach a straight line.

This straight-line motion has been applied in a form of small stationary engines, commonly known as *grasshopper engines*, where cbp (Fig. 174), extended beyond p, forms the beam of the engine, its right-hand end being supported by the link cd. The piston-rod is attached, by means of a cross-head, to the point p, which describes a straight line, and the connecting-rod is attached to a point in the line cp produced, both piston-rod and connecting-rod passing downward from cp. In this case it will be noticed that the pressure on the fulcrum c, of the beam, is equal to the difference of the pressures on the cross-head pin and crank-pin instead of the sum, as in the ordinary form of beam engine.

In this second form of motion it is not always convenient to place the point a in the line of motion pp_2, and it is often located on one side, as shown in Fig. 175.

The proportions of the different links which will cause the point p to be nearly on the straight line at the extreme positions and at the middle may be found as follows:

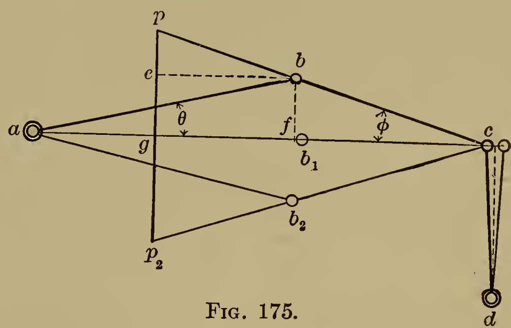


FIG. 175.

Let pg be one half the stroke of the point p, and let the angle $bac=\theta$, and $bca=pbe=\phi$. In this extreme position we may write

$$ag=af-fg=af-be$$
$$=ab\cos\theta-pb\cos\phi$$
$$=ab\left(1-2\sin^2\frac{\theta}{2}\right)-pb\left(1-2\sin^2\frac{\phi}{2}\right).$$

But if the links are taken long enough, so that for a given stroke the angles θ and ϕ are small, then $\sin\theta=\theta$, nearly, and $\sin\phi=\phi$, nearly, and

$$ag=ab\left(1-\frac{\theta^2}{2}\right)-pb\left(1-\frac{\phi^2}{2}\right)$$
$$=ab-pb-ab\frac{\theta^2}{2}+pb\frac{\phi^2}{2}. \quad \dots\dots \quad (46)$$

If the linkage is now placed in its mid-position,

$$ag=ab-pb. \quad \dots\dots \quad (47)$$

Equating equations (46) and (47),

$$ab\frac{\theta^2}{2}=pb\frac{\phi^2}{2},$$

or

$$\frac{ab}{pb}=\frac{\phi^2}{\theta^2}. \quad \dots\dots \quad (48)$$

But in the triangle abc

$$\frac{ab}{bc}=\frac{\sin\phi}{\sin\theta}=\frac{\phi}{\theta}, \text{ nearly;}$$

$$\therefore \quad \frac{ab}{pb}=\frac{\overline{ab}^2}{\overline{bc}^2}, \quad \text{or} \quad (ab)(pb)=\overline{bc}^2. \quad \dots\dots \quad (49)$$

Hence the links must be so proportioned that bc is a mean proportional between ab and pb, which also holds true when the path of p falls to the left of a instead of between a and c.

As an example of the case where the path of the guided point falls to the left of a we have the straight-line motion of the Thompson steam-engine indicator, Fig. 181.

128. Watt's Parallel Motion.—This motion is an application of the modified form of the *double rocking lever* (Fig. 109).

Fig. 176 shows such a motion; here the two links ad and bc connected by the link ab oscillate on the fixed centres d and c, and any point, as p, in the connecting link ab will describe a complex curve. If the point p be properly chosen, a double-looped curve will be obtained, two parts of which are nearly straight lines. In designing such a motion it is customary to use only a portion ef of one of the approximate straight lines, and to so proportion the different links that the extreme and middle points e, f, and p shall be on a line perpendicular to the centre lines of the levers ad and bc in their middle positions, when they should be taken parallel to each other.

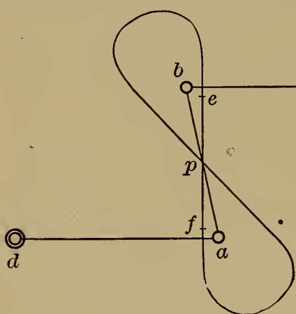


FIG. 176.

The linkage is shown in its mid-position by $dabc$, Fig. 177, and in the upper extreme position by da_1b_1c, where pp_1 is to be one half the stroke of p. Given the positions of the links ab and dc when in their mid-position, the axes c and d, the line of stroke ss, and the length of the stroke desired; to find the points a and b, giving the link ab, and to prove that the point p, where ab crosses ss, will be found on the line ss when it is moved up (or down) one half the given stroke. Lay off on ss from the points g and h, where the links ad and bc cross the line

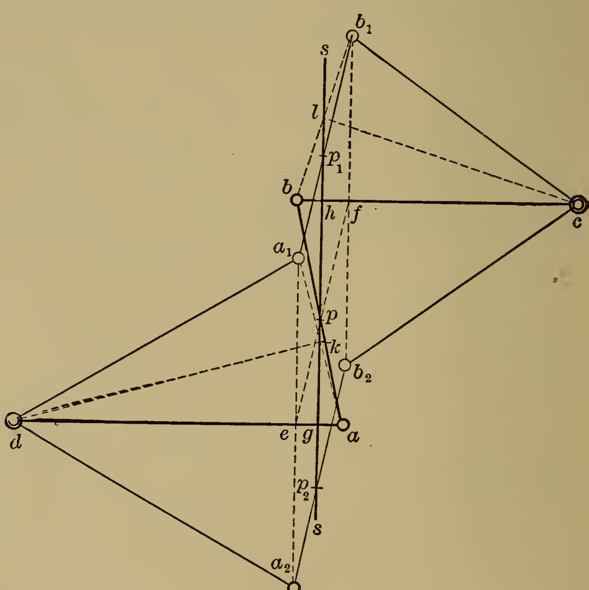


FIG. 177.

ss, *one quarter* of the stroke, giving the points k and l; connect these points with the axes d and c respectively; draw the lines aka_1 and blb_1 perpendicular to dk and cl respectively, making $aa_1=2ak$ and $bb_1=2bl$; then if the link centred at d were ad, it could swing to a_1d, and similarly bc could swing to b_1c. By construction $kg=\frac{1}{4}$ stroke, and $aa_1=2ak$; therefore $a_1e=\frac{1}{2}$ stroke. Similarly $b_1f=\frac{1}{2}$ stroke, which would make the figure ea_1b_1f a parallelogram, and a_1b_1 would equal ef. But ef is equal to ab, since $bh=hf$ and $ag=ge$. Therefore if the linkage is $dabc$, it can occupy the position da_1b_1c; and since $ap=ep=a_1p_1$, and $pp_1=ea_1=\frac{1}{2}$ the stroke, the point p will be at p_1 and $\frac{1}{2}$ the stroke above p.

To calculate the lengths of the links, given dg, ch, and gh, and the length of stroke S. Since the chord aa_1 of the arc through which a would move is bisected at right angles by the line dk,

$$\overline{gk}^2=(ag)(dg)=\frac{S^2}{16}.$$

$$\therefore\ ag=\frac{S^2}{16dg}\quad\text{and}\quad ad=dg+\frac{S^2}{16dg}.$$

Similarly
$$bc=ch+\frac{S^2}{16ch};$$
$$ab=[\overline{gh}^2+(ag+bh)^2]^{\frac{1}{2}}$$
$$=\left[\overline{gh}^2+\left(\frac{S^2}{16dg}+\frac{S^2}{16ch}\right)^2\right]^{\frac{1}{2}}.$$

To determine the position of the point p we have from the figure

$$ap:bp=ag:bh. \qquad (50)$$

$$\therefore\ ap:bp=\frac{S^2}{16dg}:\frac{S^2}{16ch}=ch:dg,$$

from which

$$ap:ab=ch:ch+dg,$$

or

$$bp:ab=dg:ch+dg,$$

from which the position of the guided point p can be calculated. If, as is very often the case, $ad=bc$, then

$$ad=bc=dg+\frac{S^2}{16dg},$$

and
$$bp:ab=dg:2dg,$$
or
$$bp=\tfrac{1}{2}ab,$$

and the point p is thus at the middle of the link ab.

This parallel motion may be arranged as shown in Fig. 178, where the centres c and d are on the same side of the line of motion. The graphical solution is the same as in Fig. 177, with the result that p is found where ab extended crosses the line of stroke ss, and, as before, it can be shown that if p is moved up one half the given stroke, it will be found on the line of stroke ss.

In Fig. 177, letting the angle $ada_1=\theta$ and $bcb_1=\phi$, we have, from equation (50),

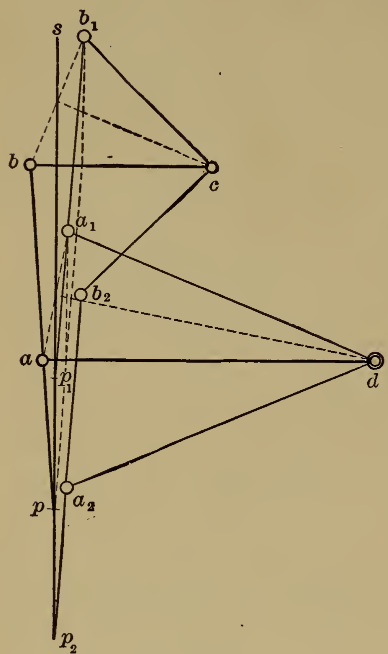


FIG. 178.

$$\frac{ap}{bp}=\frac{ag}{bh}=\frac{ae}{bf}$$

$$=\frac{ad(1-\cos\theta)}{bc(1-\cos\phi)}=\frac{ad\,2\sin^2\frac{\theta}{2}}{bc\,2\sin^2\frac{\phi}{2}},$$

which may be written

$$\frac{ap}{bp}=\frac{bc}{ad}\times\frac{\overline{ad}^2\sin^2\frac{\theta}{2}}{\overline{bc}^2\sin^2\frac{\phi}{2}}.$$

But $ad\sin\theta=bc\sin\phi$; and since the angles θ or ϕ would rarely exceed 20°, we may assume that $ad\sin\frac{\theta}{2}=bc\sin\frac{\phi}{2}$.

$$\therefore\ \frac{ap}{bp}=\frac{bc}{ad},\text{ nearly}, \quad . \quad . \quad . \quad (51)$$

or the segments of the link are inversely proportional to the lengths of the nearer levers, which is the rule usually employed when the extreme positions can vary a very little from the straight line. When the levers are equal this rule is exact.

129. The Pantograph.—The pantograph is a four-bar linkage so arranged as to form a parallelogram $abcd$, Fig. 179. Fixing some point in the linkage, as e, certain other points, as f, g, and h, will move parallel and similar to each other over any path either straight or curved. *These points, as f, g, and h, must lie on the same straight line passing through the fixed point e, and their motions will then be proportional to their distances from the fixed point.* To prove

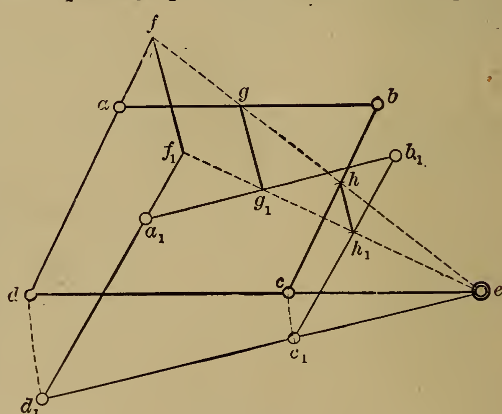


FIG. 179.

that this is so, move the point f to any other position, as f_1; the linkage will then be found to occupy the position $a_1b_1c_1d_1$. Connect f_1 with e;

then h_1, where f_1e crosses the link b_1c_1, can be proved to be the same distance from c_1 that h is from c, and the line hh_1 will be parallel to ff_1.

In the original position, since fd is parallel to hc, we may write

$$\frac{fd}{hc}=\frac{de}{ce}=\frac{fe}{he}.$$

In the second position, since f_1d_1 is parallel to h_1c_1 and since f_1e is drawn a straight line, we have

$$\frac{f_1d_1}{h_1c_1}=\frac{d_1e}{c_1e}=\frac{f_1e}{h_1e}.$$

Now in these equations $\frac{de}{ce}=\frac{d_1e}{c_1e}$; therefore $\frac{fd}{hc}=\frac{f_1d_1}{h_1c_1}$; but $fd=f_1d_1$, which gives $hc=h_1c_1$, which proves that the point h has moved to h_1. Also $\frac{fe}{he}=\frac{f_1e}{h_1e}$, from which it follows that ff_1 is parallel to hh_1, and

$$\frac{ff_1}{hh_1}=\frac{fe}{he}=\frac{de}{ce},$$

or the motions are proportional to the distances of the points f and h from e.

To connect two points, as a and b, Fig. 180, by a pantograph, so that their motions shall be parallel and similar and in a given ratio, we have, first, that the fixed point c must be on the straight line ab continued, and so located that ac is to bc as the desired ratio of the motion of a to b. After locating c, an infinite number of pantographs might be drawn. Care must be taken that the links are so proportioned as to allow the desired magnitude and direction of motion.

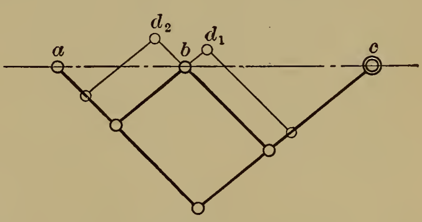


Fig. 180.

It is interesting to note that if b were the fixed point, a and c would move in opposite directions. It can be shown as before that their motions would be parallel and as ab is to bc.

The pantograph is often used to reduce or enlarge drawings, for it is evident that similar curves may be traced as well as straight lines. Also pantographs are used to increase or reduce motion in some definite proportion, as in the indicator rig on an engine where the motion of the cross-head is reduced proportionally to the desired length of the indicator diagram. When the points, as f and h (Fig. 179), are required to move in parallel straight lines it is not always necessary to employ a complete parallelogram, provided the mechanism is such that the points f and h are properly guided. Such a case is shown in Fig. 181, which is a diagram of the mechanism for moving the pencil on a Thompson steam-engine indicator. The pencil at f, which traces the diagram on a paper carried by an oscillating drum, is guided by a Scott-Russell straight-line motion $abcd$ so that it moves nearly in a straight line ss parallel to the axis of the drum, and to the centre line of the cylinder tt. It must also be arranged that the motion of the pencil f always bears the same relation to the motion of the piston of the indicator on the line tt. To secure this draw a line from f to d and note the point e where it crosses the line tt: e will be a point on the piston-rod, which

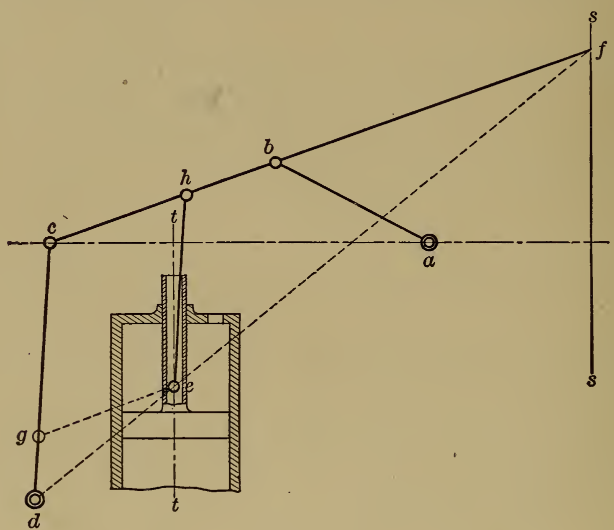


FIG. 181.

rod is guided in an exact straight line by the cylinder. If now the link eh is added so that its centre line is parallel to cd, we should have, assuming f to move on an exact straight line, the motion of f parallel to the motion of e and in a constant ratio as $cf:ch$ or as $df:de$. This can be seen by supposing the link eg to be added, which completes the pantograph $dgehcf$. If eg were added, the link ab could not be used, as the linkage $abcdf$ does not give an exact straight-line motion to f. For constructive reasons the link eg is omitted; a ball joint is located at e which moves in an exact straight line, and the point f is guided by the Scott-Russell motion, the error in the motion being very slight indeed.

Slides are often substituted, in the manner just explained, for links of a pantograph, and exact reductions are thereby obtained. In Fig. 182 the points f and h are made to move on the parallel lines mm and nn respectively. Suppose it is desired to have the point h move $\frac{1}{3}$ as much as f. Draw the line fhe and lay off the point e so that $eh:ef=1\cdot3$; draw a line, as cd, and locate a point d upon it which when connected to f with a link df will move nearly an equal distance to the right and left of the line ef and above and below the line mm for the known motion of f. Draw ch through h and parallel to df. The linkage $echdf$ will accomplish the result required. The dotted link ah may be added to complete the pantograph, and the slide h may then be removed or not as desired. The figure also shows how a point g may be made to move in the opposite direction to f in the same ratio as h but on the line n_1n_1, the equivalent pantograph being drawn dotted. The link ed is shown in its extreme position to the left by heavy lines and to the right by light lines.

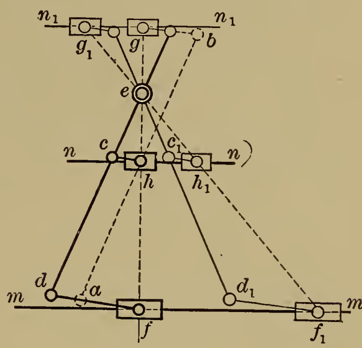


FIG. 182.

130. Applications of Watt's Parallel Motion. — Watt's parallel motion has been much used in beam engines, and it is generally necessary to arrange so that more than one point can be guided, which is accomplished by a pantograph attachment.

In Fig. 183 a parallel motion is arranged to guide three points p, p_1, and p_2 in parallel straight lines. The case chosen is that of a compound condensing beam engine, where P_2 is the piston-rod of the low-pressure cylinder, P_1 that of the high-pressure cylinder, and P the pump-rod, all of which should move in parallel straight lines, perpendicular to the centre line of the beam in its middle position.

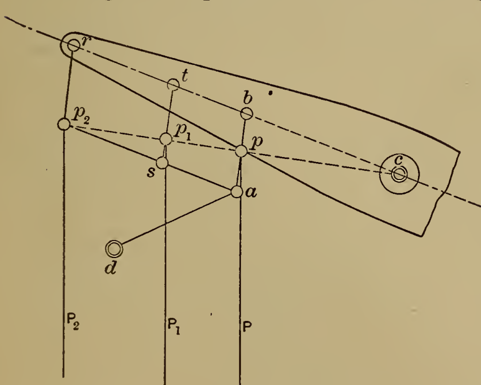


FIG. 183.

The fundamental linkage $dabc$ is arranged to guide the point p as required; then adding the parallelograms $astb$ and ap_2rb, placing the links st and p_2r so that they pass through the points p_1 and p_2 respectively, found by drawing the straight line cp and noting points p_1 and p_2 where it intersects lines P_1 and P_2, we obtain the complete linkage. The links are arranged in two sets, and the rods are carried between them; the links da are also placed outside of the links p_2a. When the point p falls within the beam a double pump-rod must be used. The linkage is shown in its extreme upper position to render its construction clearer.

The various links are usually designated as follows: *cr* the *main beam, ad* the *radius-bar* or *bridle*, p_2r the *main link, ab* the *back link*, and p_2a the *parallel bar*, connecting the main and back links.

In order to proportion the linkage so that the point p_2 shall fall at the end of the link rp_2 we have, by similar triangles *cbp* and crp_2,

$$cb : bp = cr : rp_2 = cr : ab.$$

$$\therefore \quad cr = \frac{ab \times cb}{bp}.$$

The relative stroke S of the point p_2 and s of the point p are expressed by the equation

$$S : s = cp_2 : cp = cr : cb.$$

If we denote by M and N the lengths of the perpendiculars dropped from c to the lines of motion P_2 and P respectively, then

$$S : s = M : N$$

and

$$S = s\frac{M}{N}; \quad s = S\frac{N}{M}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A)}$$

The problem will generally be, given the centres of the main beam c and bridle d, the stroke S of the point p_2, and the paths of the guided points p, p_1, and p_2, to find the remaining parts. The strokes of the guided points can be found from equation (A) and then the method of § 128, Fig. 177, can be applied.

131. Roberts's Approximate Straight-line Motion.—This might also be called the W straight-line motion, and is shown in Fig. 184. It consists of a rigid triangular frame *abp* forming an isosceles triangle on *ab*, the points *a* and *b* being guided by links $ad = bc = bp$, oscillating on the centres *d* and *c* respectively, which are on the line of motion *dc*.

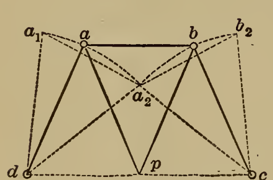


Fig. 184.

To lay out the motion, let *dc* be the straight line of the stroke along which the guided point *p* is to move approximately, and *p* be the middle point of that line. Draw two equal isosceles triangles, *dap* and *cbp*; join *ab*, which must equal $dp = pc$. Then *abp* is the rigid triangular frame, *p* the guided point, and *d* and *c* are the centres of the two links. The extreme positions when *p* is at *d* and *c* are shown at da_1a_2 and ca_2b_2, the point a_2 being common to both. The length of each side of the triangle, as $ap = da$, should not be less than 1.186 *dp*, since in this case the points ca_2a_1 and da_2b_2 lie in straight lines.

It may be made as much greater as the space will permit, and the greater it is the more accurate will the motion be. The intermediate positions between *dp* and *cp* vary somewhat from the line *dc*.

132. Tchebicheff's Approximate Straight-line Motion.—Fig. 185 shows another close approximation to a straight-line motion invented by Prof. Tchebicheff of St. Petersburg. It is an application of the *double rocking lever* (Fig. 109) with the levers crossed, *cd* being the fixed link.

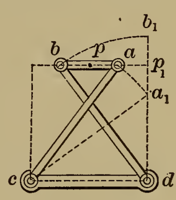


FIG. 185

The links are made in the following proportion: If $cd=4$, then $ac=bd=5$ and $ab=2$. The guided point p is located midway between a and b on the link ab and is distant from cd an amount equal to $\sqrt{5^2-3^2}=4$. When the point p moves to p_1, directly over d, $dp_1=db_1-b_1p_1=5-1=4$. Thus the middle and extreme positions of p, as shown, are in line, but the intermediate positions will be found to deviate slightly from the straight line. To render the range of motion shown on the figure possible the links *ac* and *bd* would need to be offset.

133. Parallel Motion by Means of Four-bar Linkage.—The parallel crank mechanism, § 104, Fig. 105, is very often used to produce parallel motions. The common parallel ruler, consisting of two parallel straight-edges connected by two equal and parallelly-placed links is a familiar

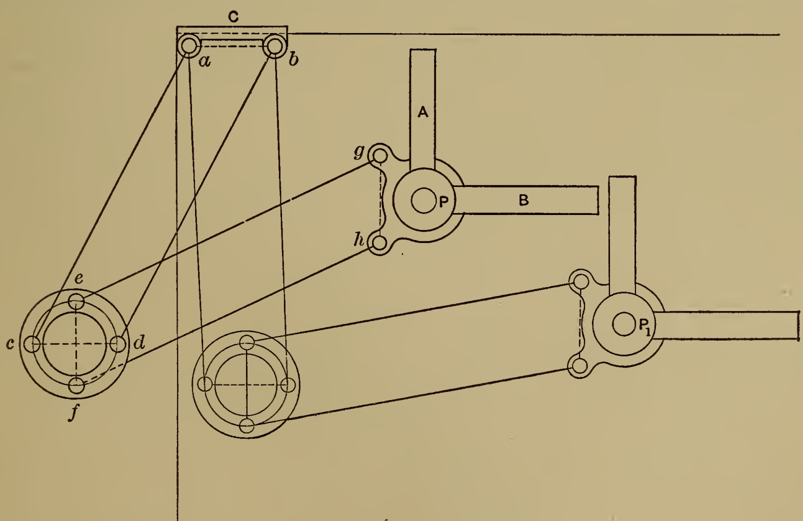


FIG. 186.

example of such application. A double parallel crank mechanism is applied in the Universal Drafting-machine, now extensively used in place of T square and triangles. Its essential features are shown in

Fig. 186. The clamp C is made fast to the upper left-hand edge of the drawing-board and supports the first linkage *abdc*. The ring *cedf* carries the second linkage *efhg*, guiding the head P. The two combined scales and straight-edges A and B, fixed at right angles to each other, are arranged to swivel on P, and by means of a graduated circle and clamp-nut may be set at any desired angle, the device thus serving as a protractor. The fine lines show how the linkages appear when the head is moved to P_1, and it is easily seen that the straight-edges will always be guided into parallel positions.

134. **Parallel Motion by Cords.**—Cords, wire ropes, or small steel wires are frequently used to compel the motion of long narrow carriages or sliders into parallel positions. In Fig. 187 the slider R has at either end the double-grooved wheels E and F. A cord attached to the hook A passes vertically downward under F, across over E, and downward to the hook C. A similarly arranged cord starts from B, passes around E and F, using the remaining grooves, and is made fast to hook D. On moving the slider downward it will be seen that for a motion of 1″ the wheel F will give out 1″ of the rope from A and take up 1″ of rope from D, which is only possible when E takes up 1″ from C and gives out 1″ to B. Thus the slider R is constrained to move into parallel positions. In practice turnbuckles or other means are provided to keep the cords taut.

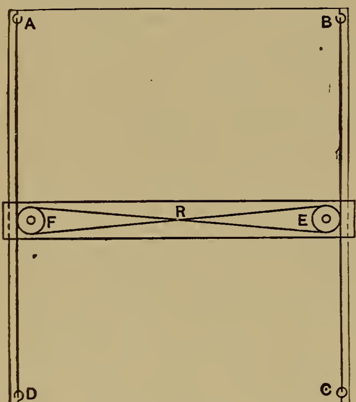


Fig. 187.

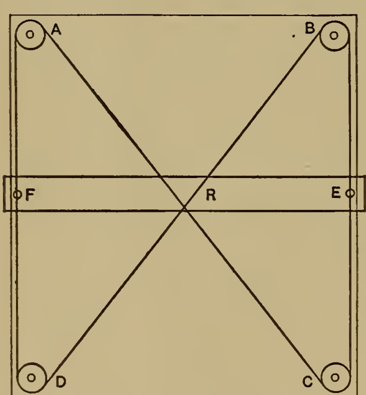


Fig. 188.

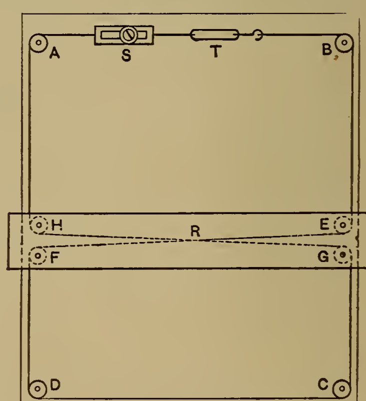


Fig. 189.

Figs. 188 and 189 show two other arrangements which will accomplish the same purpose. In Fig. 188, sometimes applied to guide straight-edges on drawing-boards, the cords or wires cross on the back side of the board where the four guide-wheels are located and the straight-edge R is guided by special fastenings E and F, passing around the edges of and under the board. By making one of these fastenings movable the straight-edge may be adjusted. Fig. 189 shows a similar device that might be applied on a drawing-board. Here the wires are on the front of the board and are arranged to pass under the straight-edge in a suitable groove. The turnbuckle T serves to keep the wires taut, and the slotted link S allows adjustment.

The device shown in Fig. 187 is often known as a *squaring-band* and is applied in spinning-mules and in some forms of travelling cranes.

CHAPTER IX.

INTERMITTENT LINKWORK.—INTERMITTENT MOTION.

A reciprocating motion in one piece may cause an intermittent circular or rectilinear motion in another piece. It may be arranged that one half of the reciprocating movement is suppressed and that the other half always produces motion in the same direction, giving the *ratchet-wheel;* or the reciprocating piece may act on opposite sides of a toothed wheel alternately, and allow the teeth to pass one at a time for each half reciprocation, giving the different forms of *escapements* as applied in timepieces.

135. **Ratchet-wheel.** — A wheel, provided with suitably shaped pins or teeth, receiving an intermittent circular motion from some vibrating or reciprocating piece, is called a *ratchet-wheel.*

In Fig. 190 A represents the ratchet-wheel turning upon the shaft a; C is an oscillating lever carrying the *detent, click,* or *catch* B, which acts on the teeth of the wheel. The whole forms the three-bar linkage acb. When the arm C moves left-handed, the *click* B will push the wheel A before it through a space dependent upon the motion of C. When the arm moves back, the *click* will slide over the points of the teeth, and will be ready to push the wheel on its forward motion as before; in any case, the *click* is held against the wheel either by its weight or the action of a spring. In order that the arm C may produce motion in the wheel A, its oscillation must be at least sufficient to cause the wheel to advance one tooth.

It is often the case that the wheel A must be prevented from moving backward on the return of the click B. In such a case a fixed *pawl, click,* or *detent,* similar to B, turning on a fixed pin, is arranged to bear on the wheel, it being held in place by its weight or a spring. Fig. 190 might be taken to represent a retaining-pawl, in which case ac is a fixed link and the click B would prevent any right-handed motion of the wheel A. Fig. 191 shows a retaining-pawl which would prevent rotation of the wheel A in either direction; such pawls are often used to retain pieces in definite adjusted positions.

If the diameter of the wheel A (Fig. 190) be increased indefinitely, it will become a rack which would then receive an intermittent translation on the vibration of the arm C: a retaining-pawl might be required in this case also to prevent a backward motion of the rack.

A click may be arranged to push, as in Fig. 190, or to pull, as in Fig.

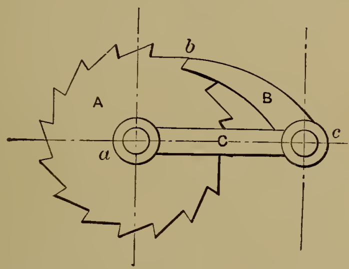


FIG. 190.

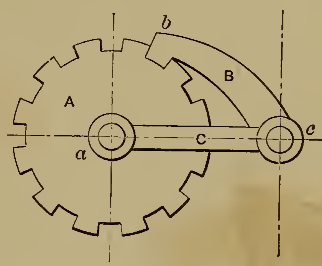


FIG. 191.

197. In order that a click or pawl may retain its hold on the tooth of a ratchet-wheel, the common normal to the acting surfaces of the click and tooth, or pawl and tooth, must pass *inside* of the axis of a pushing click or pawl, as shown on the lowest click, Fig. 192, and *outside* the axis of the pulling click or pawl; the normal might pass through the axis, but the pawl would be more securely held if the normal is located according to the above rule, which also secures the easy falling of the pawl over the points of the teeth. It is sometimes necessary, or more convenient, to place the click-actuating lever on an axis different from that of the ratchet-wheel; in such a case care must be taken that in all positions of the click the common normal occupies the proper position; it will generally be sufficient to consider only the extreme positions of the pawl in any case. Since when the lever vibrates on the axis of the wheel, the common normal always makes the same angle with it in all positions, thus securing a good bearing of the pawl on the tooth, it is best to use this construction when practicable.

The effective stroke of a click or pawl is the space through which the ratchet-wheel is driven for each forward stroke of the arm. The total stroke of the arm should exceed the effective stroke by an amount sufficient to allow the click to fall freely into place.

A common example of the application of the click and ratchet-wheel may be seen in several forms of ratchet-drills used to drill metals by hand. As examples of the retaining-pawl and wheel we have capstans and windlasses, where it is applied to prevent the recoil of the drum or barrel, for which purpose it is also applied in clocks.

It is sometimes desirable to hold a drum at shorter intervals than would correspond to the movement of one tooth of the ratchet-wheel; in such a case several equal pawls may be used. Fig. 192 shows a case where three pawls were used, all attached by pins c, c_1, c_2 to the fixed piece C, and so proportioned that they come into action alternately. Thus, when the wheel A has moved an amount corresponding to one-third of a tooth, the pawl B_1 will be in contact with the tooth b_1; after the next one-third movement, B_2 will be in contact with b_2; then after the remaining one-third movement, B will come into contact with the tooth under b; and so on. This arrangement enables us to obtain a slight motion and at the same time use comparatively large and strong teeth on the wheel in place of small weak ones. The piece C might also be used as a driving arm, and the wheel could then be moved through a space less than that of a tooth. The three pawls might be made of different lengths and placed side by side on one pin, as c_1, in which case a wide wheel would be necessary: the number of pawls required would be fixed by the conditions in each case.

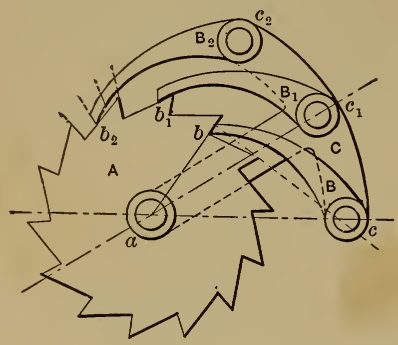


FIG. 192.

136. Reversible Click or Pawl.—The usual form of the teeth of a ratchet-wheel is that given in Fig. 192, which only admits of motion in one direction; but in *feed mechanisms*, such as those in use on shapers and planers, it is often necessary to make use of a click and ratchet-wheel that will drive in either direction. Such an arrangement is shown in Fig. 193, where the wheel A has radial teeth, and the click, which is made symmetrical, can occupy either of the positions B or B', thus giving to A a right- or a left-handed motion. In order that the click B may be held firmly against the ratchet-wheel A in all positions of the arm C, its axis c, after passing through the arm, is provided with a small triangular piece (shown dotted); this piece turning with B has a flat-ended presser, always urged upward by a spring (also shown dotted) bearing against the lower angle opposite B, thus urging the click toward the wheel; a similar action takes place when the click is in the dotted position B'. When the click is placed in line with the arm C, it is held in position by the side of the triangle parallel to the face of the click; thus this simple contrivance serves to hold the click so as to drive in either direction, and also to retain it in position when thrown out of gear.

As for different classes of work a change in the "feed" is desired, we must arrange that the motion of the ratchet-wheel A (Fig. 193), which produces the feed, can be adjusted. This is often done by changing the swing of the arm C, which is usually actuated by a rod attached at its free end. The other end of the rod is attached to a vibrating lever which has a definite angular movement at the proper time for the feed to occur, and is provided with a T slot in which the pivot for the rod can be adjusted by means of a thumb-screw and nut. By varying the distance of the nut from the centre of motion of the lever, the swing of the arm C can be regulated; to reverse the feed, it occurring in the same position as before, the click must be reversed and the nut moved to the other side of the centre of swing of the lever.

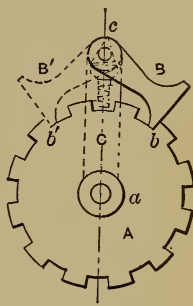


FIG. 193.

Figs. 194 and 195 show other methods of adjusting the motion of the ratchet-wheel. In Fig. 194, which shows a form of feed mechanism used by Sir J. Whitworth in his planing-machines, C is an arm carrying the click B, and swinging loosely on the shaft a fixed to the ratchet-wheel A. The wheel E, also turning loosely on the shaft a, and placed just behind the arm C, has a definite angular motion sufficient to produce

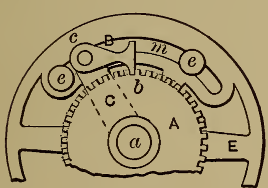


FIG. 194.

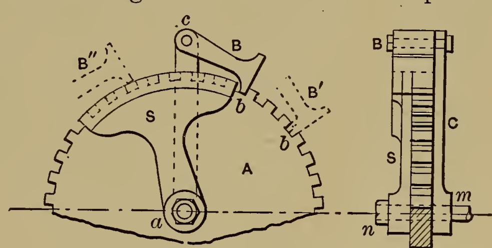


FIG. 195.

the coarsest feed desired; its concentric slot m is provided with two adjustable pins ee, held in place by nuts at their back ends, and enclosing the lever C, but not of sufficient length to reach the click B. When the pins are placed at the ends of the slot, no motion will occur in the arm C; but when e and e are placed as near as possible to each other, confining the arm C between them, all of the motion of E will be given to the arm C, thus producing the greatest feed; any other positions of the pins will give motions between the above limits, and the adjustment may be made to suit each case.

Fig. 195 shows another method of adjusting the motion of the ratchet-wheel A. The stationary shaft a, made fast to the frame of the machine at m, carries the vibrating arm C, ratchet-wheel A, and adjustable shield S; the two former turn loosely on the shaft, while the latter is made fast to it by means of a nut n, the hole in S being made smaller than that in

A, to provide a shoulder against which S is held by the nut. The arm C carries a pawl B of a thickness equal to that of the wheel plus that of the shield S; the extreme positions of this pawl are shown by dotted lines at B' and B''. The teeth of the wheel A may be made of such shape as to gear with another wheel operating the feed mechanism; or another wheel, gearing with the feed mechanism, might be made fast to the back of A, if more convenient: in the latter case, the arm C would be placed back of this second wheel.

If we suppose the lever in its extreme left position, the click will be at B'' resting upon the face of the shield S, which projects beyond the points of the teeth of A; and in the right-handed motion of the lever the click will be carried by the shield S until it reaches the position B, where it will leave the shield and come in contact with the tooth b, which it will push to b' in the remainder of the swing. In the backward swing of the lever the click will be drawn over the teeth of the wheel and face of the shield to the position B''. In the position of the shield shown in the figure a feed corresponding to three teeth of the wheel A is produced; by turning the shield to the left one, two, or three teeth, a feed of four, five, or six teeth might be obtained; while, by turning it to the right, the feed could be diminished, the shield S being usually made large enough to consume the entire swing of the arm C. This form of feed mechanism is often used in slotting-machines, and in such cases, as well as in Figs. 194 and 195, the click is usually held to its work by gravity.

137. Double-acting Click.—This device consists of two clicks making alternate strokes, so as to produce a nearly continuous motion of the ratchet-wheel which they drive, that motion being intermittent only at the instant of reversal of the movement of the clicks. In Fig. 196 the clicks act

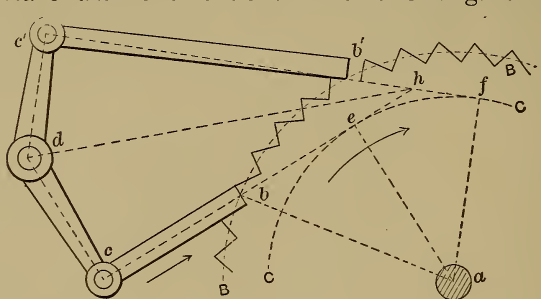


FIG. 196.

by pushing, and in Fig. 197 by pulling; the former arrangement is generally best adapted to cases where much strength is required, as in windlasses.

Each single stroke of the click-arms cdc' (Fig. 196) advances the ratchet-wheel through one-half of its pitch or some multiple of its half-pitch. To make this evident, suppose that the double click is to advance the ratchet-wheel one tooth for each double stroke of the click-arms, the arms being shown in their mid-stroke position in the figure. Now when the click *bc* is beginning its forward stroke, the click *b′c′* has just completed its forward stroke and is begining its backward stroke; during the forward stroke of *bc* the ratchet-wheel will be advanced one-half a tooth; the click *b′c′*, being at the same time drawn back one-half a tooth, will fall into position ready to drive its tooth in the remaining single stroke of the click-arms, which are made equal in length. By the same reasoning it may be seen that the wheel can be moved ahead some whole number of teeth for each double stroke of the click-arms.

In Fig. 196 let the axis *a* and dimensions of the ratchet-wheel be given, also its pitch circle *BB*, which is located half-way between the tips and roots of the teeth. Draw any convenient radius *ab*, and from it lay off the angle *bae* equal to the mean obliquity of action of the clicks, that is, the angle that the lines of action of the clicks at mid-stroke are to make with the tangent to the pitch circle through the points of action. On *ae* let fall the perpendicular *be*, and with the radius *ae* describe the circle *CC*: this is the *base circle*, to which the lines of action of the clicks should be tangent. Lay off the angle *eaf* equal to *an odd number of times the half-pitch angle*, and through the points *e* and *f*, on the base circle, draw two tangents cutting each other in *h*. Draw *hd* bisecting the angle at *h*, and choose any convenient point in it, as *d*, for the centre of the rocking shaft, to carry the click-arms. From *d* let fall the perpendiculars *dc* and *dc′* on the tangents *hec* and *fhc′* respectively; then *c* and *c′* will be the positions of the click-pins, and *dc* and *dc′* the centre lines of the click-arms at mid-stroke. Let *b* and *b′* be the points where *ce* and *c′f* cut the pitch circle; then *cb* and *c′b′* will be the lengths of the clicks. The *effective stroke* of each click will be equal to half the pitch as measured on the base circle *CC* (or some whole number of times this half-pitch), and the total stroke must be enough greater to make the clicks clear the teeth and drop well into place.

In Fig. 197 the clicks pull instead of push, the obliquity of action is zero, and the base circle and pitch circle become one, the points *b*, *e*, and *b′*, *f* (Fig. 196) becoming *e* and *f* (Fig. 197). In all other respects the construction is the same as when the clicks act by pushing, and the different points are lettered the same as in Fig. 196.

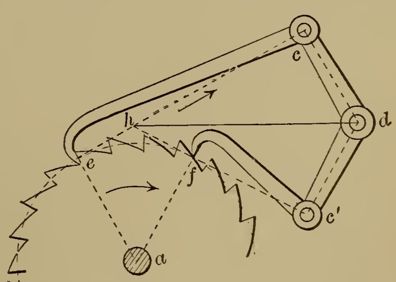


Fig. 197.

Since springs are liable to lose their elasticity or become broken after being in use some time, it is often desirable to get along without applying them to keep clicks in position.

Fig. 198 shows in elevation a mechanism where no springs are required to keep the clicks in place, it being used in some forms of lawn-mowers to connect the wheels to the revolving cutter when the mower is pushed forward, and to allow a free backward motion of the mower while the cutter still revolves. The ratchet A is usually made on the inside of the wheels carrying the mower, and the piece C, turning on the same axis as A, carries the three equidistant pawls or clicks B, shaped to move in the cavities provided for them. In any position of C, at least one of the clicks will be held in contact with A by the action of gravity, and any motion of A in the direction of the arrow will be given to the piece C. Here the ratchet-wheel drives the click, ac being the actuated click-lever.

FIG. 198.

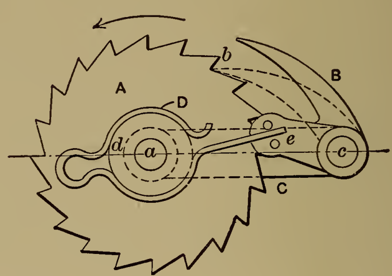


FIG. 199.

The piece C is sometimes attached to a roller by means of the shaft a; then any left-handed motion of C will be given to A, while the right-handed motion will simply cause the clicks to slide over the teeth of A. The clicks B are usually held in place by a cap attached to C.

Fig. 199 shows a form of click which is always thrown into action when a left-handed rotation is given to its arm C, while any motion of the wheel A left-handed will immediately throw the click out of action. The wheel A carries a projecting hub d, over which a spring D is fitted so as to move with slight friction. One end of this spring passes between two pins, e, placed upon an arm attached to the click B. When the arm C is turned left-handed, the wheel A and the spring D being stationary, the click B will be thrown toward the wheel by the action of the spring on the pin e. The motion of the wheel A will be equal to that of the arm C, minus the motion of C necessary to throw the click into gear. Similarly, when A turns left-handed, the click B is thrown out of gear. This mechanism is employed in some forms of spinning-mules to actuate the spindles when winding on the spun yarn.

138. Friction-catch.—Various forms of catches depending upon friction are often used in place of clicks; these catches usually act upon the face of the wheel or in a suitably formed groove cut in the face.

Friction-catches have the advantage of being noiseless and allowing any motion of the wheel, as they can take hold at any point; they have the disadvantage, however, of slipping when worn, and of getting out of order.

Fig. 200 shows a friction-catch *B* working in a V-shaped groove in the wheel *A*, as shown in section *A'B'*. Here *B* acts as a retaining-click, and prevents any right-handed motion of *A*; its face is circular in outline, the centre being located at *d*, a little above the axis *c*. A similarly shaped catch might be used in place of an actuating click to cause motion of *A*.

Fig. 201 shows four catches like *B* (Fig. 200) applied to drive an annular ring *A* in the direction indicated by the arrow. When the piece *c* is turned right-handed, the catches *B* are thrown against the

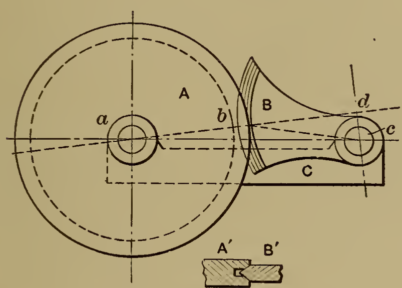


Fig. 200.

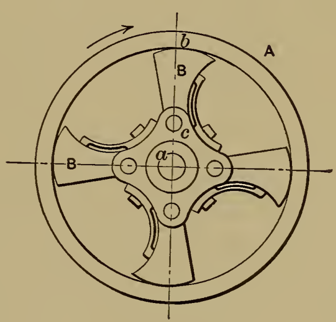


Fig. 201.

inside *b* of the annular ring by means of the four springs shown; on stopping the motion of *c*, the pieces *B* are pushed, by the action of *b*, toward the springs which slightly press them against the ring and hold them in readiness to again grip when *c* moves right-handed. Thus an oscillation of the piece *c* might cause continuous rotation of the wheel *A*, provided a fly-wheel were applied to *A* to keep it going while *c* was being moved back. The annular ring *A* is fast to a disc carried by the shaft *a*; the piece *c* turning loosely on *a* has a collar to keep it in position lengthwise of the shaft.

The nipping-lever shown in Fig. 202 is another application of the friction-catch. A loose ring *C* surrounds the wheel *A*; a friction-catch *B* having a hollow face works in a pocket in the ring and is pivoted at *c*. On applying a force at the end of the catch *B* in the direction of the arrow, the hollow face of the catch will "nip" the wheel at *b*, and cause the ring to bear tightly against the left-hand part of the circumference of the wheel; the friction thus set up will cause the catch, ring, and wheel to move together as one piece. The greater the pull applied at the end of the catch the greater will be the friction, as the friction is proportional to the pressure; thus the amount of friction developed will depend upon the resistance to motion of A. Upon reversing the force at the end of the catch, the hollow face of the catch will be drawn away from the face of A, and the rounding top part of the catch, coming in contact with the top of the cavity in the ring, will cause the ring to slide back upon the disc. An upward motion of the click end will again cause the wheel A to move forward, and thus the action is the same as in a ratchet and wheel.

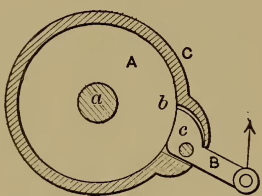


Fig. 202.

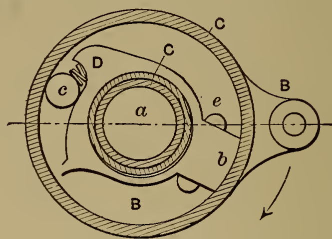


Fig. 203.

Fig. 203 shows, in section, a device which has been applied to actuate sewing-machines in place of the common crank. Two such mechanisms were used, one to rotate the shaft of the machine on a downward tip of the treadle, while the other acted during the upward tip, the treadle-rods being attached to the projections of the pieces B. The mechanism shown in the figure acts upon the shaft during the downward motion of the projection B as shown by the arrow.

The piece C, containing an annular groove, is made fast to the shaft a, the sides of this groove being turned circular and concentric with the shaft. The piece B, having a projecting hub fitting loosely on the inner surface of the groove in C, is placed over the open groove, and is held in place by a collar on the shaft. The hub on the piece B, and the piece C, are shown in section. The friction-catch D, working in the groove, is fitted over the hub of B, the hole in D being elongated in the direction ab so that D can move slightly upon the hub and between the two pins e fast in the piece B. A cylindrical roller c is placed in the wedge-shaped space between the outer side of the groove and the piece D, a spring always actuating this roller in a direction opposite to that of the arrow, or towards the narrower part of the space.

Now when the piece B is turned in the direction of the arrow by a downward stroke of the treadle-rod, it will move the piece D with it by means of the pins e; at the same time the roller c will move into the narrow part of the wedge-shaped space between C and D, and cause binding between the pieces D and C at b and at the surface of the roller. The friction at b thus set up will cause the motion of D to be given to C. On the upward motion of the projection B the roller will be moved to the large part of its space by the action of the piece C revolving with the shaft combined with that of the backward movement of D, thus releasing the pressure at b and allowing C to move freely onward. The other catch would be made just the reverse of this one, and would act on an upward movement of the treadle-rod.

Another form of friction catch, sometimes used in gang saws to secure the advance of the timber for each stroke of the saw, and called the *silent feed*, is shown in Fig. 204.

The saddle-block B, which rests upon the outer rim of the annular wheel A, carries the lever C turning upon the pin c. The block D, which fits the inner rim of the wheel, is carried by the lever C, and is securely held to its lower end by the pin d on which D can freely turn. When the pieces occupy the positions shown in the figure, a small space exists between the piece D and the inside of the rim A.

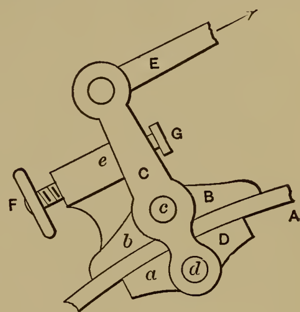


FIG. 204.

The upper end of the lever C has a reciprocating motion imparted to it by means of the rod E. The oscillation of the lever about the pin c is limited by the stops e and G carried by the saddle-block B. When the rod E is moved in the direction indicated by the arrow, the lever turning on c will cause the block D to approach B, and thus nip the rim at a and b; and any further motion of C will be given to the wheel A. On moving E in the opposite direction the grip will first be loosened, and the lever striking against the stop e will cause the combination to slide freely back on the rim A. The amount of movement given to the wheel can be regulated by changing the stroke of the rod E by an arrangement similar to that described in connection with the reversible click, § 136. The stop G can be adjusted by means of the screw F so as to prevent the oscillation of the lever upon its centre c, thus throwing the grip out of action. The saddle-block B then merely slides back and forth on the rim, the action being the same as that obtained by throwing the ordinary click out of gear.

139. **Masked Wheels.**—It is sometimes required that certain strokes of the click-actuating lever shall remain inoperative upon the ratchet-wheel. Such arrangements are made use of in numbering-machines where it is desired to print the same number twice in succession; they are called *masked wheels*.

Fig. 205, taken from a model, illustrates the action of a masked wheel; the pin-wheel D represents the first ratchet-wheel, and is fast to the axis a; the second wheel A has its teeth arranged in pairs, every alternate tooth being cut deeper, and it turns loosely on the axis a.

The click B is so made that one of its acting surfaces, i, bears against the pins e of the wheel D, while the other, g, is placed so as to clear the pins and yet bear upon the teeth of A, the wheel A being located so as to admit of this.

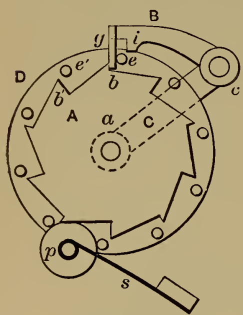


Fig. 205.

If now we suppose the lever C to vibrate through an angle sufficient to move either wheel along one tooth, both having the same number, it will be noticed that when the projecting piece g is resting in a shallow tooth of the wheel A, the acting surface i will be retained too far from the axis to act upon the tooth e, and thus this vibration of the lever will have no effect upon the pin-wheel D, while when the piece g rests in a deep tooth, as b', the click will be allowed to drop so as to bring the surface i into action with the pin e'.

In the figure the click B has just pushed the tooth e' into its present position, the projection g having rested in the deep tooth b' of the wheel A; on moving back, g has slipped into the shallow tooth b, and thus the next stroke of the lever and click will remain inoperative on the wheel D, which advances but one tooth for every two complete oscillations of the lever C.

Both wheels should be provided with retaining-pawls, one of which, p, is shown. This form of pawl, consisting of a roller p turning about an axis carried by the spring s, attached to the frame carrying the mechanism, is often used in connection with pin-wheels, as by rolling between the teeth it always retains them in the same position relative to the axis of the roller; a triangular-pointed pawl which also passes between the pins is sometimes used in place of the roller.

The pins of the wheel D might be replaced by teeth so made that their points would be just inside of the bottoms of the shallow teeth of A; a wide pawl would then be used, and when it rested in a shallow tooth of A it would remain inoperative on D, while when it rested in a deep tooth it would come in contact with the adjacent tooth of D and push it along.

So long as the click B and the wheels have the proper relative motion it makes no difference which we consider as fixed, as the action will be the same whether we consider the axis of the wheels as fixed and the click to move, or the click to be fixed and the axis to have the proper relative motion in regard to it. The latter method is made use of in some forms of numbering-machines.

140. Fig. 206 shows the mechanism of a "counter" used to record the number of double strokes made by a pump; the revolutions made by a steam engine, paddle, propeller, or other shaft, etc. Two views are given in the figure, which represents a counter capable of recording revolutions from 1 to 999; if it is desired to record higher numbers, it will only be necessary to add more wheels, such as A. A plate, having a long slot or series of openings opposite the figures 000, is placed over the wheels, thus only allowing the numbers to be visible as they come under the slot or openings.

The number wheels A, A_1, A_2, are arranged to turn loosely side by side upon the small shaft a, and are provided with a series of ten teeth cut into one side of their faces, while upon the other side a single notch is cut opposite the zero tooth on the first side, it having the same depth and contour. This single notch can be omitted on the last wheel A_2. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are printed upon the faces of the wheels in proper relative positions to the teeth t.

Two arms C are arranged to vibrate upon the shaft a of the number wheels, and carry at their outer ends the pin c, on which a series of clicks, b, b_1, and b_2, are arranged, collars placed between them serving to keep them in position on the pin. The arms are made to vibrate through an angle sufficient to advance the wheels one tooth, i.e., one-tenth of a turn; their position after advancing a tooth is shown by dotted lines in the side view. A common method of obtaining this vibration is to attach a rod at r, one end of the pin c, this rod to be so attached at its other end to the machine as to cause the required backward and forward vibrations of the lever C for each double stroke or revolution that the counter is to record.

The click b is narrow, and works upon the toothed edge of the first wheel A, advancing it one tooth for every double stroke of the arm c. The remaining clicks b_1 and b_2 are made broad, and work on the toothed edges of A_1 and A_2, as well as on the notched rims of A and A_1, respectively. When the notches n and m come under the clicks b_1 and b_2 the clicks will be allowed to fall and act on the toothed parts of A_1 and A_2; but in any other positions of the notches the clicks will remain inoperative upon the wheels, simply riding upon the smooth rims of A and A_1, which keep the clicks out of action. Each wheel is provided with a retaining-spring s to keep it in proper position.

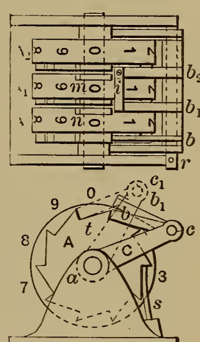


FIG. 206.

Having placed the wheels in the position shown in the figure, the reading being 000, the action is as follows: The click b moves the wheel A along one tooth for each double stroke of the arm C, the clicks b_1 and b_2 remaining inoperative on A_1 and A_2; on the figure 9 reaching the slot, or the position now occupied by 0, the notch n will allow the click b_1 to fall into the tooth 1 of the wheel A_1, and the next forward stroke of the arm will advance both the wheels A and A_1, giving the reading 10; the notch n having now moved along, the click b_1 will remain inoperative until the reading is 19, when b_1 will again come into action and advance A_1 one tooth, giving the reading 20; and so on up to 90, when the notch m comes under the click b_2. To prevent the click b_2 from acting on the next forward stroke of the arm, which would make the reading 101 instead of 91, as it should be, a small strip i is fastened firmly to the end of the click b_2, its free end resting upon the click b_1. This strip prevents the click b_2 from acting until the click b_1 falls, which occurs when the reading is 99; on the next forward stroke the clicks b_1 and b_2 act, thus giving the reading 100. As the strip merely rests upon b_1, it cannot prevent its action at any time. If another wheel were added, its click would require a strip resting on the end of b_2. A substitute for these strips might be obtained by making the wheel A fast to the shaft a, and allowing the remaining wheels to turn loose upon it, thin discs, having the same contour as the notched edge of the wheel A, being placed between the wheels A_1 A_2, A_2 A_3, etc., and made fast to the shaft, the notches all being placed opposite n; thus the edges of the discs would keep the clicks b_2, b_3, etc., out of action, except when the figure 9 of the wheel A is opposite the slot, and the notches m, etc., are in proper position. A simpler form of counter will be described in § 141.

Intermittent Motion.

So far we have considered a uniform reciprocating motion in one piece, as giving an intermittent circular or rectilinear motion to another, the click being the driver and the wheel the follower.

141. It is often required that a uniform circular motion of the driver shall produce an intermittent circular or rectilinear motion of the follower. The following examples will give some solutions of the problem:

Fig. 207 shows a combination by which the toothed wheel A is moved in the direction of the arrow, one tooth for every complete turn of the shaft d, the pawl B retaining the wheel in position when the tooth t on the shaft d is out of action. The stationary link adc forms the frame, and provides bearings for the shafts d and a, and a pin c for the pawl B. The arm e, placed by the side of the tooth upon the shaft, is arranged to clear the wheel A in its motion, and to lift the pawl B at the time when the tooth t comes into action with the wheel, and to drop the pawl when the

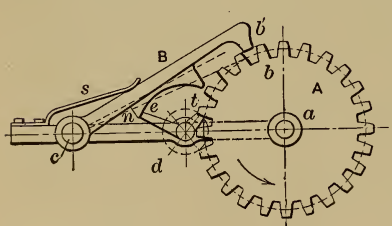


Fig. 207.

action of *t* ceases, i.e., when the wheel has been advanced one tooth. This is accomplished by attaching the piece *n* to the pawl, its contour in the raised position of the pawl being an arc of a circle about the centre of the shaft *d*; its length is arranged to suit the above requirements. When the tooth *t* comes in contact with the wheel, the arm *e*, striking the piece *n*, raises the pawl (which is held in position by the spring *s*), and retains it in the raised position until the tooth *t* is ready to leave the wheel, when *e*, passing off from the end of *n*, allows the pawl to drop.

In Fig. 208 the wheel *A* makes one-third of a revolution for every turn of the wheel *bc*, its period of rest being about one-half the period of revolution of *bc*. If we suppose *A* the follower, and to turn right-handed while the driver *bc* turns left-handed, one of the round pins *b* is just about to push ahead the long tooth of *A*, the circular retaining sector *c* being in such a position as to follow a right-handed motion of *A*. The first pin slides down the long tooth, and the other pins pass into and gear with the teeth *b′*, the last pin passing off on the long tooth *e*, when the sector *c* will come in contact with the arc *c′*, and retain the wheel *A* until the wheel *bc* again reaches its present position.

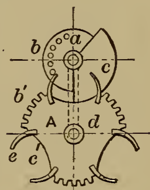


Fig. 208.

Geneva Stop.—In Fig. 209 the wheel *A* makes one-sixth of a revolution for one turn of the driver *ac*, the pin *b* working in the slots *b′* causing the motion of *A*; while the circular portion *c* of the driver, coming in contact with the corresponding circular hollows *c′*, retains *A* in position when the tooth *b* is out of action. The wheel *a* is cut away just back of the pin *b* to provide clearance for the wheel *A* in its motion. If we close up one of the slots, as *b′*, it will be found that the shaft *a* can only make a little over five and one-half revolutions in either direction before the pin *b* will strike the closed slot. This mechanism, when so modified, has been applied to watches to prevent overwinding, and is called the *Geneva stop*, the wheel *a* being attached to the spring-shaft so as to turn with it, while *A* turns on an axis *d* in the spring-barrel. The number of slots in *A* depends upon the number of times it is desired to turn the spring-shaft.

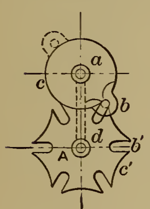


Fig. 209.

By placing another pin opposite *b* in the wheel *ac*, as shown by dotted lines, and providing the necessary clearance, the wheel *A* could be moved through one-sixth of a turn for every half turn of *ac*.

A simple type of counter extensively used on water-meters is shown in Fig. 210. It consists of a series of wheels *A*, *B*, *C*, mounted side by side and turning loosely on the shaft *S*; or the first wheel to right may be fast to the shaft and all the remaining wheels loose upon it. Each wheel is numbered on its face as in Fig. 206, and it is provided, as shown, that the middle row of figures appears in a suitable slot in the face of the counter. The first wheel *A* is attached to the worm-wheel *E*, having 20 teeth and driven by the worm *F* geared to turn twice for one turn of the counter driving shaft.

On a parallel shaft *T* loose pinions *D* are arranged between each pair of wheels. Each pinion is supplied with six teeth on its left side extending over a little more than one-half its face and with three teeth, each alternate tooth being cut away, for the remainder of the face, as clearly shown in the sectional elevations. The middle elevation (Fig. 210) shows a view of the wheel *B* from the right of the line *ab* with the pinion *D* sectioned on the line *cd*. The right elevation shows a view of

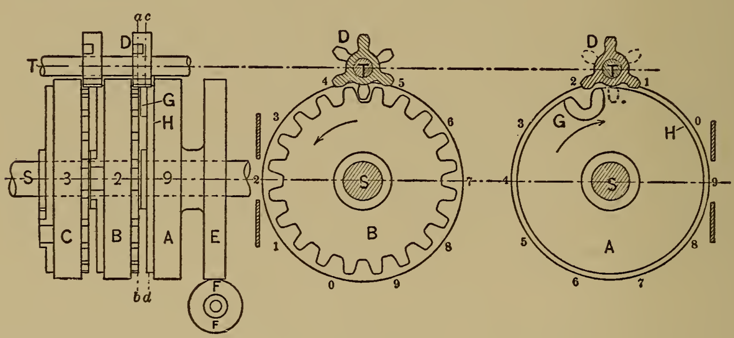
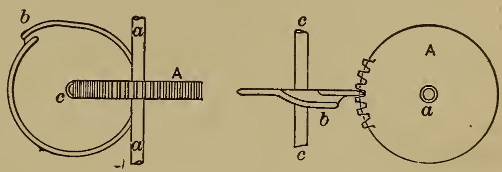


Fig. 210.

the wheel *A* from the left of the line *ab* with the pinion *D* sectioned on the line *cd*. The first wheel *A*, and all others except the last, at the left, have on their left sides a double tooth *G*, which is arranged to come in contact with the six-tooth portion of the pinion; the space between these teeth is extended through the brass plate, whose periphery *H* acts as a stop for the three-tooth portion of the pinion, as clearly shown in the figure to the right. Similarly on the right side of each wheel, except the first, is placed a wheel of 20 teeth gearing with the six-tooth part of the pinion, as shown in the middle figure. When the digit 9 on any wheel, except the one at the left, comes under the slot, the double tooth *G* is ready to come in contact with the pinion; as the digit 9 passes under the slot the tooth *G* starts the pinion, which is then free to make one-third of a turn and again become locked by the periphery *H*. Thus any wheel to the left receives one-tenth of a turn for every passage of the digit 9 on the wheel to its right. In the figure the reading 329 will change to 330 on the passing of the digit 9. This counter can be made to record oscillations by supplying its actuating shaft with a ten-tooth ratchet, arranged with a click to move one tooth for each double oscillation.

Figs. 211A and 211B show two methods of advancing the wheels *A* through a space corresponding to one tooth during a small part of a revolution of the shafts *c*; in this case the shafts are at right angles to each other. In Fig. 211A a raised circular ring with a small spiral part *b* attached to a disc is made use of; the circular part of the ring retains the wheel in position, while the spiral part gives it its motion. In Fig. 211B the disc carried by the shaft *cc* has a part of its edge bent helically at *b*; this helical part gives motion to the wheel, and the remaining part of the disc edge retains the wheel in position. By using a regular spiral, in Fig. 211A, and one turn of a helix, in Fig. 211B, the wheels *A* could be made to move uniformly through the space of one tooth during a uniform revolution of the shafts *c*.



211A. FIG. 211B.

In Fig. 212 the wheel *A* is arranged to turn the wheel *B*, on a shaft at right angles to that of *A*, through one-half a turn while it turns one-sixth of a turn, and to lock *B* during the remaining five-sixths of the turn.

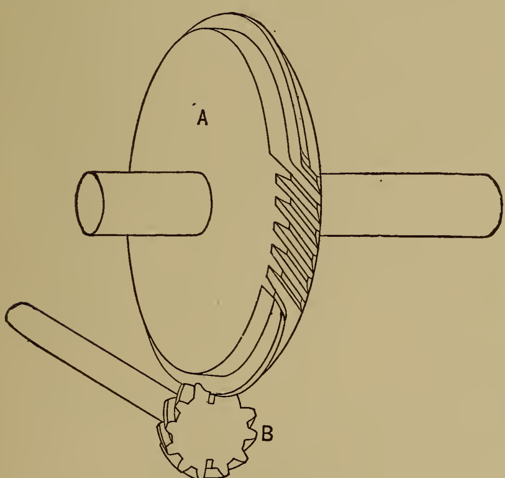


FIG. 212.

The Star Wheel. — In Fig. 213 the wheel *A*, commonly known as the *star wheel*, turns through a space corresponding to one tooth for each revolution of the arm carrying the pin *b* and turning on the shaft *c*. The pin *b* is often stationary, and the star wheel is moved past it; the action is then evidently the same, as the pin and wheel have the same relative motion in regard to each other during the time of action. The star wheel is often used on moving parts of machines to actuate some feed mechanism, as may be seen in cylinder-boring machines on the facing attachment, and in spinning-machinery.

FIG. 213.

142. Cam and Slotted Sliding Bar.—Fig. 214 shows an equilateral triangle *abc*, formed by three circular arcs, whose centres are at *a*, *b*, and *c*, the whole turning about the axis *a*, and producing an intermittent motion in the slotted piece *B*. The width of the slot is equal to the radius of the three circular arcs composing the three equal sides of the triangular cam *A*, and therefore the cam will always bear against both sides of the groove.

If we imagine the cam to start from the position shown in Fig. 215 when *b* is at 1, the slotted piece *B* will remain at rest while *b* moves from 1 to 2 (one-sixth of the circle 1, 2 . . . 6), the cam edge *bc* merely sliding over the lower side of the slot. When *b* moves from 2 to 3, i.e., from the position of *A*, shown by light full lines, to that shown by dotted lines, the edge *ab* will act upon the upper side of the slot, and impart to *B* a motion similar to that obtained in Fig. 156, being that of a crank with an infinite connecting-rod; from 3 to 4 the point *b* will drive the upper side of the slot, *ca* sliding over the lower side, the motion here being also that of a connecting-rod with an infinite link, but decreasing instead of increasing as from 2 to 3. When *b* moves from 4 to 5 there is no motion in *B*; from 5 to 6, *c* acts upon the upper side of the slot, and *B* moves downward; from 6 to 1, *ac* acts on the upper side of the slot, and *B* moves downward to its starting position. The motion of *B* is accelerated from 5 to 6 and retarded from 6 to 1.

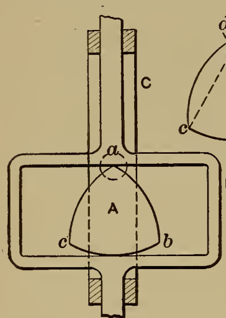


Fig. 214.

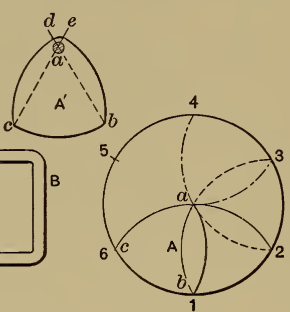


Fig. 215.

At *A′* a form of cam is shown where the shaft *a* is wholly contained in the cam. In this case draw the arcs *de* and *cb* from the axis of the shaft as a centre, making *ce* equal to the width of the slot in *B*; from *c* as a centre with a radius *ce* draw the arc *eb*, and note the point *b* where it cuts the arc *cb*; with the same radius and *b* as a centre draw the arc *dc*, which will complete the cam. In this case the angle *cab* will not be equal to 60°, and the motions in their durations and extent will vary a little from those described above.

Locking Devices.—The principle of the slotted sliding bar combined with that of the Geneva stop is applied in the shipper mechanism shown in Fig. 216, often used on machines where the motion is automatically reversed. The shipper bar *B* slides in the piece *CC*, which also provides a pivot *a* for the weighted lever *wab*. The end of the lever *b* opposite the weight *w* carries a pin which works in the grooved lug *s* on the shipper bar. In the present position of the pieces, the pin *b* is in the upper part of the slot, and the weight w, tending to fall under the action of gravity, holds it there, the shipper being thus effectually locked in its present position. If now the lever be turned left-handed about its axis a until the weight w is just a little to the left of a, gravity will carry the weight and lever into the dotted position shown, where it will be locked until the lever is turned right-handed. The principle of using a weight to complete the motion is very convenient, as the part of the machine actuating the shipper often stops before the belt is carried to the wheel which produces the reverse motion, and the machine is thus stopped. The motion can always be made sufficient to raise a weighted lever, as shown above, and the weight will, in falling, complete the motion of the shipper.

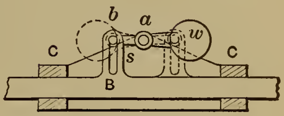


Fig. 216.

The device shown in Fig. 217, of which we may have many forms, serves to retain a wheel A in definite adjusted positions, its use being the same as that of the retaining-pawl shown in Fig. 191. The wheels B and A turn on the shafts c and a, respectively, carried by the link C, which is shown dotted, as it has been cut away in taking the section. Two positions of the wheel B will allow the teeth b of A to pass freely through its slotted opening, while any other position effectually locks the wheel A. The shape of the slot in B and the teeth of A are clearly shown in the figure.

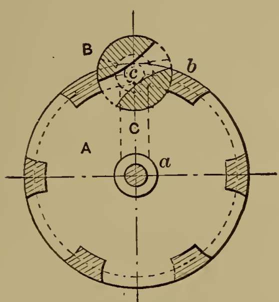


Fig. 217.

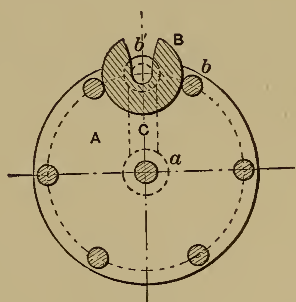


Fig. 218.

Fig. 218 shows another device for locking the wheel A, the teeth of which are round pins; but in this case it is necessary to turn B once to pass a tooth of A. If we suppose the wheel A under the influence of a spring which tends to turn it right-handed, and then turn B uniformly either right- or left-handed, the wheel A will advance one tooth for each complete turn of B, a pin first slipping into the groove on the left and leaving it when the groove opens toward the right, the next pin then coming against the circular part of B opposite the groove. It will be noticed that while there are only six pins on the wheel A, yet there are twelve positions in which A can be locked, as a tooth may be in the bottom of the groove or two teeth may be bearing against the circular outside of B. Devices similar in principle to those shown in Figs. 217 and 218 are often used to adjust stops in connection with feed mechanisms.

Clicks and pawls as used in practice may have many different forms and arrangements; their shape depends very much upon their strength and the space in which they are to be placed, and the arrangement depends on the requirements in each case.

Escapements.

143. An escapement is a combination in which a toothed wheel acts upon two distinct pieces or *pallets* attached to a reciprocating frame, it being so arranged that when one tooth escapes or ceases to drive its pallet, another tooth shall begin its action on the other pallet.

A simple form of escapement is shown in Fig. 219. The frame cc' is arranged to slide longitudinally in the bearings CC, which are attached to the bearing for the toothed wheel. The wheel a turns continually in the direction of the arrow, and is provided with three teeth, b, b', b'', the frame having two pallets, c and c'. In the position shown, the tooth b is just ceasing to drive the pallet c to the right, and is *escaping*, while the tooth b' is just coming in contact with the pallet c', when it will drive the frame to the left.

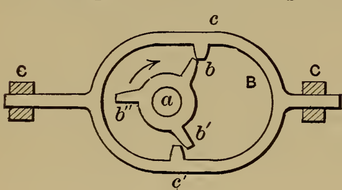


Fig. 219.

While escapements are generally used to convert circular into reciprocating motion, as in the above example, the wheel being the driver, yet, in many cases, the action may be reversed. In Fig. 219, if we consider the frame to have a reciprocating motion and use it as the driver, the wheel will be made to turn in the opposite direction to that in which it would itself turn to produce reciprocating motion in the frame. It will be noticed also that there is a short interval at the beginning of each stroke of the frame in which no motion will be given to the wheel. It is clear that the wheel a must have 1, 3, 5, or some odd number of teeth upon its circumference.

The Crown-wheel Escapement.—The crown-wheel escapement (Fig. 220) is used for causing a vibration in one axis by means of a rotation of another. The latter carries a crown wheel A, consisting of a circular band with an odd number of large teeth, like those of a splitting-saw, cut on its upper edge. The vibrating axis, o, or *verge* as it is often called, is located just above the teeth of the crown wheel, in a plane at right angles to the vertical wheel axis. The verge carries two pallets, b and b_1, located in planes passing through its axis, the distance between them being arranged so that they may engage alternately with teeth on opposite sides of the wheel. If the crown wheel be made to revolve under the action of a spring or weight, the alternate action of the teeth on the pallets will cause a reciprocating motion in the verge. The rapidity of this vibration depends upon the inertia of the verge, which may be adjusted by attaching to it a suitably weighted arm.

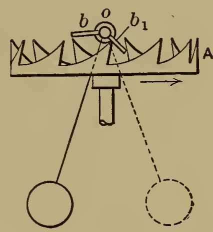


FIG. 220.

This escapement, having the disadvantage of causing a recoil in the wheel as the vibrating arm cannot be suddenly stopped, is not used in timepieces, and but rarely in other places. It is of interest, however, as being the first contrivance used in a clock for measuring time.

144. The Anchor Escapement.—The *anchor escapement* as applied in clocks is shown in the upper portion of Fig. 221. The escape-wheel A_1 turns in the direction of the arrow and is supplied with long pointed teeth. The pallets are connected to the vibrating axis or verge C_1 by means of the arms d_1C_1 and e_1C_1, the axis of the verge and wheel being parallel to each other. The verge is supplied at its back end with an arm C_1p_1, carrying a pin p_1 at its lower end. This pin works in a slot in the pendulum-rod, not shown. The resemblance of the two pallet arms combined with the upright arm to an anchor gave rise to the name "anchor escapement." The left-hand pallet, d_1, is so shaped that all the normals to its surface pass above the verge axis C_1, while all the normals to the right-hand pallet, e_1, pass below the axis C_1. Thus an upward movement of either pallet will allow the wheel to turn in the direction of the arrow, or, the wheel turning in the direction of the arrow, will, when the tooth b_1 is in contact with the pallet d_1, cause a left-handed swing of the anchor; and when b_1 has passed off from d_1 and o_1 reaches the right-hand pallet, as shown, a right-handed swing will be given to the anchor. As the pendulum cannot be suddenly stopped after a tooth has escaped from a pallet, the tooth that strikes the other pallet is subject to a slight recoil before it can move in the proper direction, which motion begins when the pendulum commences its return swing. The action of the escape-wheel on the pendulum is as follows:

Suppose the points l_1 and k_1 to show extreme positions of the point p_1, and suppose the pendulum and point p_1 to be moving to the left; the tooth b_1 has just escaped from the pallet d_1, and o_1 has impinged on e_1, as shown, the point p_1 having reached the position m_1. The recoil now begins, the pallet e_1 moving back the tooth o_1, while p_1 goes from m_1 to l_1. The pendulum then swings to the right and the pallet e_1 is urged upward by the tooth o_1, thus urging the pendulum to the right while p_1 passes from l_1 to n_1, when o_1 escapes. Recoil then occurs on the pallet d_1 from n_1 to k_1, and from k_1 to m_1 an impulse is given to the pendulum to the left, when the above-described cycle will be repeated. As the space through which the pendulum is urged on exceeds that through which it is held back, the action of the escape-wheel keeps the pendulum vibrating. This alternate action *with* and *against* the pendulum prevents it from being, as it should be, the sole regulator of the speed of revolution of the escape-wheel; for its own time of vibration, instead of depending only upon its length, will also depend upon the force urging the escape-wheel round. Therefore any change in the maintaining force will disturb the rate of the clock.

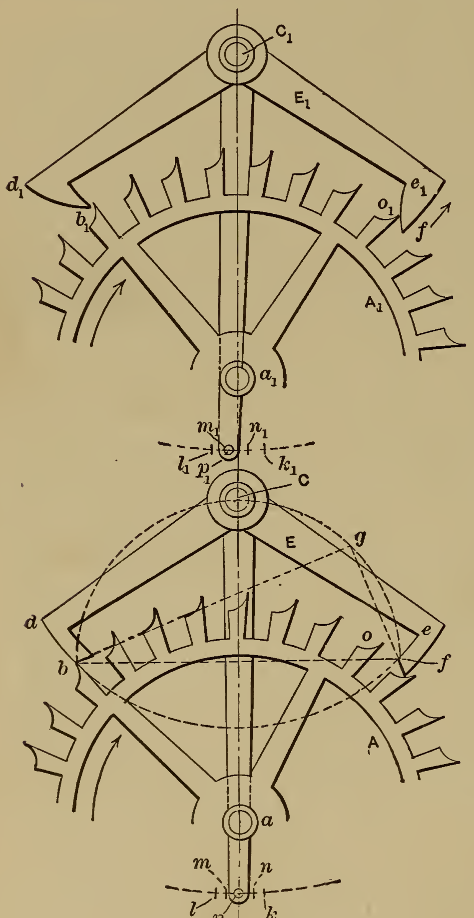


FIG. 221.

145. Dead-beat Escapement.—The objectionable feature of the anchor escapement is removed in Graham's *dead-beat* escapement, shown in the lower portion of Fig. 221. The improvement consists in making the outline of the lower surface, *db*, of the left-hand pallet, and the upper surface of the right-hand pallet, arcs of a circle about *C*, the verge axis; the oblique surfaces *b* and *f* complete the pallets. The construction indicated by dotted lines in the figure insures that the oblique surfaces of the pallets shall make equal angles, in their normal position, with the tangents *bC* and *fC* to the wheel circle not shown. If we suppose the limits of the swing of the point *p* to be *l* and *k*, the action of the escape-wheel on the pendulum is as follows: The pendulum being in its right extreme position, the tooth *b* is bearing against the circular portion of the pallet *d*; as the pendulum swings to the left under the action of gravity, the tooth b will begin to move along the inclined face of the pallet when the centre line has reached n, and will urge the pendulum onward to m, where the tooth leaves the pallet, and another tooth o comes in contact with the circular part of the pallet e, which, with the exception of a slight friction between it and the point of the tooth, will leave the pendulum free to move onward, the wheel being locked in position. On the return swing of the pendulum, the inclined part of the pallet e urges the pendulum from m to n. Hence there is no recoil, and the only action against the pendulum is the very minute friction between the teeth and the pallets. The impulse is here given through an arc mn, very nearly bisected by the middle point of the swing of the pendulum, which is also an advantage. The term "dead-beat" has been applied because the second hand, which is fitted to the escape-wheel, stops so completely when the tooth falls upon the circular portion of a pallet, there being no recoil or subsequent trembling such as occurs in other escapements.

In watches the pendulum is replaced by a balance-wheel swinging backward and forward on an arbor under the action of a very light coiled spring, often called a "hair-spring"; the pivots of the arbor are very nicely made, so that they turn with very slight friction.

146. The Graham Cylinder Escapement.—This form of escapement is used in the Geneva watches. Here the balance verge o (Fig. 222) has attached to it a very thin cylindrical shell rs centred at o, the axis of the verge, and the point of the tooth b can rest either on the outside or inside of the cylinder during a part of the swing of the balance. As the cylinder turns in the direction of the arrow (Fig. 222, A), the wheel also being urged in the direction of its arrow, the inclined surface of the tooth bc comes under the edge s of the cylinder, and thus urges the balance onward; this gives one impulse, as shown in Fig. 222, B. The tooth then passes s, flies into the cylinder, and is stopped by the concave surface near r. In the opposite swing of the balance the tooth escapes from the cylinder, the inclined surface pushing r upward, which gives the other impulse in the opposite direction to the first; the action is then repeated by the next tooth of the wheel.

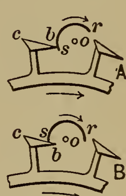


Fig. 222.

This escapement is, in its action, nearly identical to the *dead-beat;* but the impulse is here given through small equal arcs, situated at equal distances from the middle point of the swing.

147. The Chronometer Escapement is shown in Fig. 223. Here the verge o carries two circular plates, one of which carries a projection p, which serves to operate the detent d; the other carries a projection n, which swings freely by the teeth of the escape-wheel when a tooth is resting upon the pallet d, but encounters a tooth when the wheel is in any other position.

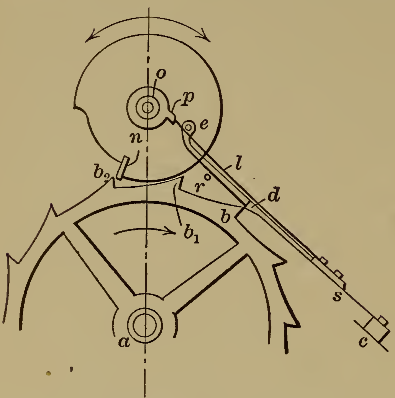


FIG. 223.

The detent d has a compound construction and consists of four parts:

1° The locking-stone d, a piece of ruby on which the tooth of the escape-wheel rests.

2° The discharging-spring l, a very fine strip of hammered gold.

3° A spring s on which the detent swings, and which attaches the whole to the frame of the chronometer.

4° A support e, attached to the body of the detent, to prevent the strip l from bending upward.

A pin r prevents the detent from approaching too near the wheel.

The action of the escapement is as follows: On a right-hand swing of the balance the projection p meets the light strip l, which, bending from its point of attachment to the detent, offers but very little resistance to the balance. On the return swing of the balance, the projection p meets the strip l, which can now only bend from e, and raises the detent d from its support r, thus allowing the tooth b to escape, the escape-wheel being urged in the direction of the arrow. While this is occurring, the tooth b_2 encounters the projection n, and gives an impulse to the balance; the detent meanwhile has dropped back under the influence of the spring s, and catches the next tooth of the wheel b_1.

It will be noticed that the impulse is given to the balance immediately after it has been subject to the resistance of unlocking the detent d, thus immediately compensating this resistance; also that the impulse is given at every alternate swing of the balance.

The motion of the balance is so adjusted that the impulse is given through equal distances on each side of the middle of its swing.

CHAPTER X.

WHEELS IN TRAINS.

148. Since rolling cylinders cannot be relied upon to transmit much force, and at the same time maintain a constant velocity ratio, they are provided with teeth upon their rims, as shown in Fig. 224, which represents a short section of the rim of a toothed wheel. When rolling cylinders, either external or internal, are thus supplied with teeth, they are called *gear-wheels*, or, better, *spur gears*; rolling cones similarly supplied with teeth are called *bevel-wheels* or *bevel gears*. In the latter case, when the axes are at right angles with each other, the bevel gears are commonly known as *mitre gears*. A small spur-wheel is called a *pinion*.

Toothed wheels are said to be *in gear* when they are capable of moving each other, and *out of gear* when they are so shifted in position that the teeth cease to act.

The teeth of gear-wheels are so shaped that they give, by their sliding contact, the same velocity ratio as the rolling cylinders or cones, from which they are derived, give by their rolling contact. The rolling cylinders or cones, as the case may be, that give the same velocity ratio by their rolling contact as the teeth give by their sliding contact, are called *pitch cylinders* and *pitch cones* respectively. The intersection of the pitch cylinder with a plane perpendicular to its axis is called the *pitch circle*. In bevel gearing the largest intersection of the perpendicular plane, or the base of the rolling cone, is commonly called the *pitch circle*. Fig. 224 shows a pair of spur gears, the circles in contact at *c* being the pitch circles. The *pitch* of a gear-wheel is the distance measured on the *pitch circle* from a point on one tooth to a similar point on the next tooth, as *ab*, Fig. 224. In all cases the pitch must be an aliquot part of the pitch circle, and only wheels having the same pitch will gear with each other. In what follows, gear-wheels will be represented by their pitch circles, their pitch cylinders, or their pitch cones, as may be convenient in each case.

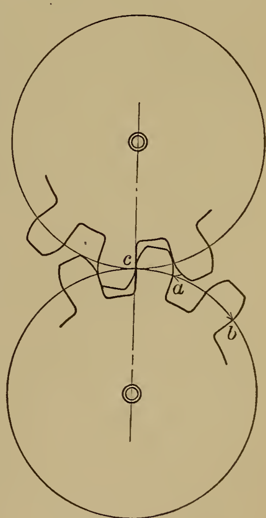


FIG. 224.

149. In the preceding chapters, the elementary combinations discussed have, in general, consisted of two principal parts only, a driver and a follower; and it has been shown how to connect them to produce the required velocity ratio.

There may occur cases, however, in which, although *theoretically* possible, it may be *practically* inconvenient to effect the required communication of motion by a single combination. In such a case a series or *train* of combinations is employed, in which the follower of the first combination of the train is carried by the same axis or sliding piece to which the driver of the second is attached; the follower of the second is similarly connected to the driver of the third, and so on. In this section revolving pieces only, or *trains of wheels*, will be discussed.

Fig. 225 shows an example of a train of wheels where the follower B and driver C are placed on the same axis, A being the first driver. The second follower, D, and the third driver, E, are also on the same axis. The numbers preceding the letters t indicate the numbers of teeth on the wheels upon which they are placed.

150. Value of a Train of Wheels.—By the *value of a train* we mean the ratio of the angular velocities of the first and last axes, or, what is equivalent, the ratio of their rotations in a given time. Thus in the train shown in Fig. 225, if we let n_1 represent the turns of A and n_4 the turns of F, the value of the train will be

$$\frac{\text{turns of } F}{\text{turns of } A} = \frac{n_4}{n_1}.$$

If we let n_2 and n_3 represent the turns of the second and third axes respectively, the value of the train could be written

$$\frac{\text{turns of } F}{\text{turns of } A} = \frac{n_2}{n_1} \times \frac{n_3}{n_2} \times \frac{n_4}{n_3} = \frac{n_4}{n_1}. \qquad (52)$$

That is, the value of the train may be found by multiplying together the separate ratios of the synchronal rotations of the successive pairs of axes.

In § 148 it was stated that two wheels that will work together must have the same pitch. Therefore the numbers of teeth on any two wheels which will work together are proportional to the diameters of the respective pitch circles. It has already been shown that the diameters of two rolling cylinders are inversely proportional to the rotations. From this it follows that the rotations of a pair of wheels

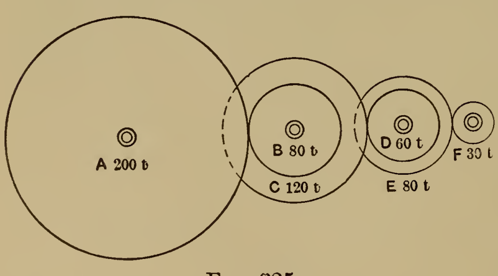


FIG. 225.

are inversely proportional to the numbers of teeth on the wheels. Thus in Fig. 225 we should have

$$\frac{\text{turns of } B}{\text{turns of } A}=\frac{200}{80}, \quad \frac{\text{turns of } D}{\text{turns of } C}=\frac{120}{60}, \quad \frac{\text{turns of } F}{\text{turns of } E}=\frac{80}{30},$$

or

$$\frac{\text{turns of } F}{\text{turns of } A}=\frac{200}{80}\times\frac{120}{60}\times\frac{80}{30}=\frac{40}{3}.$$

When the first and last axes of a train turn in the same direction the value of the train is considered *positive*, and when in opposite directions *negative*. The value of the train in Fig. 225 should then be written

$$\frac{\text{turns of } F}{\text{turns of } A}=-13\tfrac{1}{3}.$$

[When discussing the more complex epicyclic trains in Chapter XI it becomes necessary to use the plus or minus signs in the solutions, as the algebraic sum of two or more component motions will be required.]

Similarly, the value of the belt train shown in Fig. 226, where the diameters of the pulleys are given in inches, is

$$\frac{\text{turns of } F}{\text{turns of } A}=\frac{40}{16}\times\frac{24}{12}\times\frac{16}{6}=\frac{40}{3}=+13\tfrac{1}{3}.$$

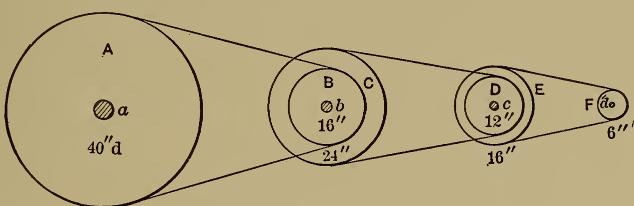


Fig. 226.

151. General Example.—It is not necessary that all the separate velocity ratios be expressed in the same terms as previously explained. For example, let us take a train of six axes, and let:

1° The first axis turn once per minute, and the second once in fifteen seconds.

2° The second axis turn three times while the third turns five times.

3° The third axis carry a wheel of sixty teeth, driving a wheel of twenty-four teeth on the fourth axis.

4° The fourth axis carry a pulley of twenty-four inches diameter, driving by means of a belt a pulley twelve inches in diameter on the fifth axis.

5° The fifth axis have an angular velocity two-thirds of that of the sixth axis.

Then

$$e=\frac{60}{15}\times\frac{5}{3}\times\frac{60}{24}\times\frac{24}{12}\times\frac{3}{2}=50,$$

or the last axis will make fifty turns while the first turns once.

152. Directional Relation in Trains.—This depends on the number and manner of connection of the different axes. When the train consists solely of spur-wheels or pinions on fixed parallel axes, the direction of rotation of the successive axes will be alternately in opposite directions. In any arrangement the directional relation can be traced by placing arrows on the different wheels, showing their direction of rotation and following through the entire combination. It is frequently the case that two separate wheels of a train, as A and B, Fig. 227, may revolve about the same axis, and if they revolve in the same direction, they may be connected by means of two other spur-wheels revolving on an axis parallel to that of the wheels; while if they revolve in opposite directions, Fig. 228, they might be connected by one bevel-wheel placed on an axis making an angle with that of the wheels.

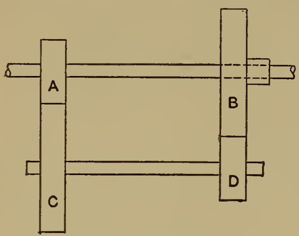


FIG. 227.

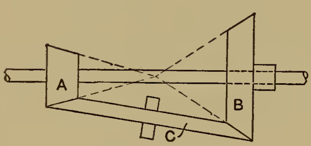


FIG. 228.

When an *annular wheel,* that is, a wheel having teeth on the inside of an annular ring, is used in connection with a pinion, it will be noticed that they both revolve in the same direction.

Idle Wheel.—A wheel called an *idle wheel,* which acts both as a driver and a follower, is often placed between two others. In such a case the velocity ratio of the two extreme wheels is not affected, but the directional relation is changed, the extreme wheels now rotating in the same direction, while if they geared directly, they would rotate in opposite directions.

153. Examples of Wheels in Trains.—Fig. 229 shows the train in a cotton carding-machine. For the train we have the value

$$\frac{\text{turns } B}{\text{turns } A}=\frac{135}{17}\times\frac{37}{20}\times\frac{130}{26}\times\frac{17}{33}=+37.84.$$

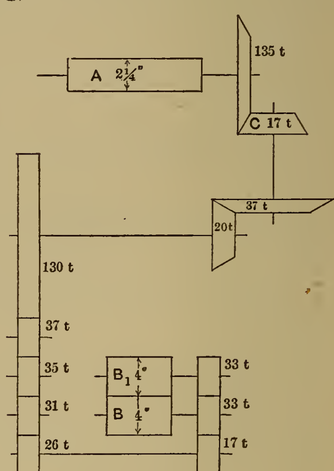


FIG. 229.

In the machine the lap of cotton passing under the roll A is much drawn out in its passage through the machine, and it becomes necessary to solve for the ratio of the surface speeds of the rolls B and A. For this we have, since the surface speed equals $2\pi\times$turns$\times$radius or turns$\times\pi\times$diameter,

$$\frac{\text{surface speed } B}{\text{surface speed } A}=\frac{\text{turns } B\times\text{diam. } B}{\text{turns } A\times\text{diam. } A}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

$$\therefore \frac{\text{surface speed } B}{\text{surface speed } A}=37.84\frac{4}{2.25}=67.27.$$

A train of spur gears is often used in machines for hoisting where the problem would be to find the ratio of the weight lifted to the force applied. In Fig. 230 the value of the train is

$$\frac{\text{turns } B}{\text{turns } A}=\frac{21}{100}\times\frac{25}{84}=\frac{1}{16};$$

then, if $D=15''$ and $R=2\frac{1}{2}$ ft.,

$$\frac{\text{l.v. } W}{\text{l.v. } F}=\frac{1}{16}\times\frac{15}{60}=\frac{1}{64};$$

$$\therefore \frac{F}{W}=\frac{\text{l.v. } W}{\text{l.v. } F}=\frac{1}{64},$$

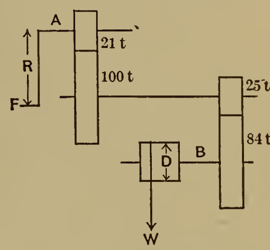


Fig. 230.

or if F were 50 lbs., W would be 3200 lbs. *if loss due to friction were neglected.*

Fig. 231 shows a train of wheels used to connect a set of three lower drawing rolls, such as used in cotton machinery for drawing cotton.

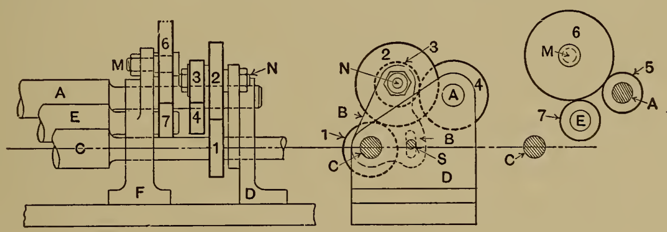


Fig. 231.

The figure at the left is an elevation, and the other figures are end views of the trains connecting the rolls C and A, and A and E, respectively. The top rolls placed over A, E, and C, and turning with them, are not shown in the figures. The cotton to be drawn passes between the top and bottom rolls, pressure being applied to the top rolls to keep them in contact with the lower ones.

The train connecting the rolls C and A consists of four wheels, 1, 2, 3, and 4; that connecting the rolls A and E of three wheels, 5, 6, and 7, and it may easily be seen that the rolls all turn in the same direction. Suppose the roll C to be $1\frac{1}{8}''$ in diameter, and A and E both $1''$ diameter, the wheel 1 to have 36 teeth; 2, 80 teeth; 3, 35 teeth; 4, 56 teeth; 5, 22 teeth; 6, 56 teeth; and 7, 20 teeth. The value of the train will be

$$\frac{\text{turns } C}{\text{turns } A}=\frac{56}{35}\times\frac{80}{36}=+\frac{32}{9},$$

and

$$\frac{\text{surface speed } C}{\text{surface speed } A}=\frac{32}{9}\times\frac{9}{8}=\frac{4}{1},$$

or the surface of C moves four times as fast as that of A. Thus a cotton sliver passing in between the rolls A is drawn to four times its length on emerging from the rolls C. This is termed the *draft* of the rolls.

The value of the train connecting the rolls A and E is

$$\frac{\text{turns } E}{\text{turns } A}=\frac{22\times 56}{56\times 20}=\frac{22}{20}=1.1,$$

or E turns 1.1 times while A turns once, the draft here being 1.1, since the rolls are of the same diameter.

In order to arrange the rolls for different staples of cotton and different drafts we must be able to change the distance between the axes of the rolls to suit the former, and the value of the train to suit the latter. The front roll C revolves in fixed bearings; the bearings of the remaining rolls are made adjustable, which necessitates making the axes of the wheels 2 and 3, and 6 adjustable.

The value of the train connecting A and E is never changed, and it is only necessary here to provide a slot for the adjustable stud M, which furnishes the bearing for the wheel 6. The value of the train connecting C and A, however, is changed to give the required draft, and this is usually accomplished by arranging so that one of the wheels, as 3, called a *change gear*, can be removed easily, and replaced by another. The arrangement is as follows:

An arm B, centred on the shaft C, carries a stud N on which turn the two wheels 2 and 3. The *change gear* 3 fits over the extended hub of the wheel 2, and is made to turn with it by means of a pin in the hub, a nut and washer on the end of the stud serving to keep the wheels in place. The arm B is provided with a circular slot whose centre is in the axis of C, this slot being supplied with a set-screw S in the standard supplying the bearing of C. After once adjusting the stud N in the arm B so that the wheels 2 and 1 are in proper relative position, and placing the necessary change wheel upon the hub of 2, the set-screw S being loose, the arm and wheels can be turned on the shaft C until 3 and 4 come into gear; then tightening the screw S will secure the whole in position. The slot is made long enough to allow for the largest and smallest change gear 3. This principle of first adjusting a stud, carrying one or two gears, on an arm turning about one of the shafts which it is desired to connect, and then swinging this arm until the second contact is made, is often made use of in placing change gears on lathes and other machinery.

The standard D furnishes the bearings for the rolls C and A, and the support for the arm B, and the standard F supplies bearings for all three rolls and a support for the adjustable stud M of the wheel 6.

In a machine of this kind a so called *draft factor* is often determined, assuming the change gear to have one tooth only. The draft factor when divided by the draft desired will give the number of teeth in the change gear; and conversely, the draft factor divided by the number of teeth in the change gear will give the draft. In this case (Fig. 231) the draft factor is

$$\frac{56}{1}\times\frac{80}{36}\times\frac{9}{8}=140.$$

With a draft gear of 35 teeth the draft is

$$\frac{140}{35}=4,$$

that is,

$$\frac{\text{surface speed } C}{\text{surface speed } A}=4.$$

Engine-lathe Train.—Fig. 232 gives an elevation and end view of the headstock of an engine lathe, showing the method of connecting

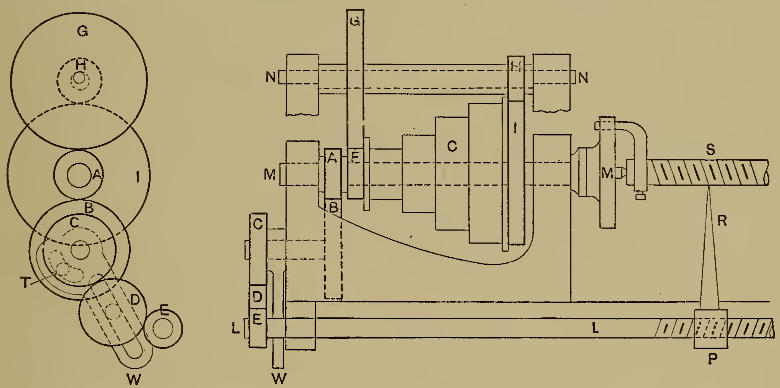


FIG. 232.

the *spindle* or *mandrel* M with the lead screw L. The "back gears" G, H on the shaft NN have been drawn in position above the mandrel, instead of back of it where they are usually placed, so that one figure will show the whole arrangement, a convention often adopted in drawings of headstocks.

The step pulley C turns loose on the mandrel M, and carries the gear F at its left-hand end; the gear wheels I and A are attached to the mandrel, and turn with it. The gear-wheels G and H, connected by a hollow shaft, turn upon an arbor whose axis is parallel to the axis of the mandrel. This arbor is provided with two eccentric bearings NN, so that by turning it slightly the gears G and H with their hollow shaft, commonly known as *back gears*, can be thrown in or out of gear with F and I respectively. An adjustable catch is arranged between the step pulley C and the wheel I, so that the wheel can be connected directly to the pulley C by throwing in the catch, the back gears being out of gear, or indirectly through the back gears, which are now thrown in, the catch being adjusted so that the pulley turns freely on the mandrel. This catch consists of an adjustable pin, moving in a radial slot in the gear wheel I, it being held in position by a spring or thumb-nut. When the catch is at the inner end of the slot, it works in a circular groove in the pulley C; but when it is at the outer end of the slot, it is located in a radial notch cut from the groove, thus compelling C and I to move together.

Thus, by combining a four-step pulley with back gears, a series of eight speeds can be obtained for the mandrel—four with the back gears in, and four with the cone only. The cone and back-gear train should be so proportioned that the speeds are in geometric progression.

If it is desired to have a certain value for the train from F to I, it becomes necessary to find suitable numbers of teeth for the four gears F, G, H, and I. It is to be noticed that, since the axes M and N are parallel, the diameter of $F+$ diam. of G will equal diam. of $H+$ diam. of I; therefore, if the same pitch were to be on all four gears, we should have teeth on $F+$ teeth on $G=$ teeth on $H+$ teeth on I. In a lathe the pitch on H and I would usually be a little greater than on F and G, since the teeth on H and I bear a greater stress.

For example, let the value of the train be

$$\frac{\text{turns } I}{\text{turns } F}=\frac{1}{10};$$

this ratio may be separated into two ratios nearly alike; thus,

$$\frac{\text{turns } I}{\text{turns } F}=\frac{1}{10}=\frac{3}{10}\times\frac{1}{3}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (54)$$

and if we are to use not less than 24 teeth on any wheel, the train could be as follows:

$$\frac{\text{turns } I}{\text{turns } F}=\frac{\text{teeth on } F}{\text{teeth on } G}\times\frac{\text{teeth on } H}{\text{teeth on } I}=\frac{3}{10}\times\frac{1}{3}=\frac{24}{80}\times\frac{24}{72},$$

which would be a train suitable for back gears. If it were required that the pitch should be the same on all the gears, the train could be

$$\frac{3}{10}\times\frac{1}{3}=\frac{24}{80}\times\frac{26}{78}, \quad \text{where} \quad 24+80=26+78.$$

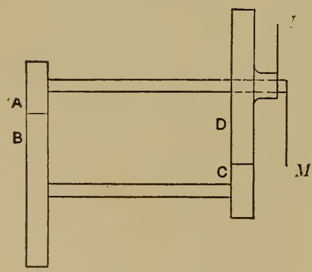


FIG. 233.

This same type of train occurs in clockwork between the minute and hour hands. The value of the train (Fig. 233) would be

$$\frac{\text{turns } M}{\text{turns } H}=\frac{12}{1}=\frac{3}{1}\times\frac{4}{1}=\frac{30}{10}\times\frac{32}{8},$$

or, letting A, B, C, and D represent the numbers of teeth,

$$\frac{A}{B}\times\frac{C}{D}=\frac{10}{30}\times\frac{8}{32}=\frac{1}{12}=\frac{\text{turns } H}{\text{turns } M}.$$

In this arrangement the wheels are all of the same pitch.

154. Screw-cutting.—One of the most frequent uses of an engine lathe is that of cutting screws. To this end, the lathe is provided with an accurately cut screw L (Fig. 232), called a *lead screw,* which serves to move the carriage of the lathe by turning in a nut P made fast to the carriage. This carriage travels upon the bed of the lathe, on ways parallel to the axis of the mandrel, and carries the tool-holder; the whole is here represented by the pointer R, which illustrates the action just as well.

A train of gear-wheels connects the mandrel with the lead screw, an intermediate shaft, as that carrying the wheels B and C, commonly known as the "stud," being generally used. This stud is then connected to the lead screw by means of the train CDE, D being an idle wheel, and C and E change gears. Let us suppose that the pitch of the lead screw L is $\frac{1}{6}''$ L.H. and that we wish to cut a screw S having ten threads per inch R.H. While the lead screw turns six times the cutting-tool represented by R will travel one inch; therefore the screw S must make ten turns in the same time.

$$\therefore \frac{\text{turns } S}{\text{turns } L} = \frac{10}{6} = \frac{\text{teeth on } E}{\text{teeth on } C} \times \frac{\text{teeth on } B}{\text{teeth on } A}; \quad . \quad . \quad . \quad (55)$$

or, we may always write

$$\text{turns } S \times \text{pitch of } S = \text{turns } L \times \text{pitch of } L, \quad . \quad . \quad . \quad (56)$$

from which

$$\frac{\text{turns } S}{\text{turns } L} = \frac{\frac{1}{6}}{\frac{1}{10}} = \frac{10}{6}.$$

The numbers of teeth on A and B between the mandrel and the stud would be known, and it would be necessary to find suitable numbers of teeth for the gears C and E, and also to determine whether an odd or an even number of idlers D would be needed to give the desired thread. If A and B have respectively 24 and 36 teeth,

$$\frac{\text{turns } S}{\text{turns } L} = \frac{\text{teeth on } E}{\text{teeth on } C} \times \frac{36}{24} = \frac{10}{6};$$

$$\therefore \frac{\text{teeth on } E}{\text{teeth on } C} = \frac{10}{9} = \frac{40}{36};$$

or the wheel on the stud could have 36 teeth and that on the lead screw 40 teeth. In this way a table may be calculated for any lathe and the gears for each pitch of screw to be cut, arranged in tabular form, as given below for the above case.

Threads per Inch to be cut.	Gear on Stud, C.	Gear on Lead Screw, E.
3	72	24
4	72	32
5	72	40
6	36	24
7	36	28
8	36	32
9	36	36
10	36	40
etc.	etc.	etc.

In arranging such a table the same gears are used as often as possible, and so planned that *both* gears need not be changed any oftener than is necessary.

It will be noticed that when the screws S and L revolve in the same direction, the threads will both be either right- or left-handed; while if they revolve in opposite directions, as in the figure, one screw must be right- and the other left-handed. In the figure, a right-handed screw is being cut, the lead screw in this case being left-handed. To cut a left-handed screw, another idle wheel should be inserted in the connecting train. In many lathes it is arranged that either one or two idle wheels can be thrown between A and B at pleasure, by simply moving an arm placed near B in the headstock casting.

In gearing the stud with the lead screw, a vibrating slotted arm, similar to that described in connection with the roll train, is made use of. The wheel C is first placed on the stud, and then E is placed on the lead screw; a wheel D is then selected from among the change gears, and placed on the movable stud of the arm W, the stud being adjusted so that C and D gear with each other; the arm is then swung over until D and E gear with each other, and clamped in position by means of the screw T.

155. Clockwork.—A familiar example of the employment of wheels in trains is seen in clockwork. Fig. 234 represents the trains of a common clock; the numbers near the different wheels denote the number of teeth on the wheels near which they are placed.

The verge or anchor O vibrates with the pendulum P, and if we suppose the pendulum to vibrate once per second, it will let one tooth of the escape-wheel pass for every double vibration, or every two seconds (§ 144). Thus the shaft A will revolve once per minute, and is suited to carry the second hand S.

The value of the train between the axes A and C is $\frac{\text{turns } C}{\text{turns } A}=$ $\frac{8\times 8}{60\times 64}=\frac{1}{60}$, or the shaft C revolves once for sixty revolutions of A; it is therefore suited to carry the minute hand M. The hour hand H is also placed on this shaft C, but is attached to the loose wheel F by means of a hollow hub. This wheel is connected to the shaft C by means of a train and intermediate shaft E. The value of this train is

$$\frac{\text{turns } H}{\text{turns } M} = \frac{28 \times 8}{42 \times 64} = \frac{1}{12}.$$

The drum D, on which the weight-cord is wound, makes one revolution for every twelve of the minute hand M, and thus revolves twice each day. Then, if the clock is to run eight days, the drum must be large enough for sixteen coils of the cord. The drum is connected to the wheel G by means of a ratchet and click, so that the cord can be wound upon the drum without turning the wheel.

Clock trains are usually arranged as shown in the figure, the wheels being placed on shafts, often called "arbors," whose bearings are arranged in two parallel plates which are kept the proper distance apart by shouldered pillars (not shown) placed at the corners of the plates. When the arbor E is placed outside, as shown, a separate bearing is provided for its outer end.

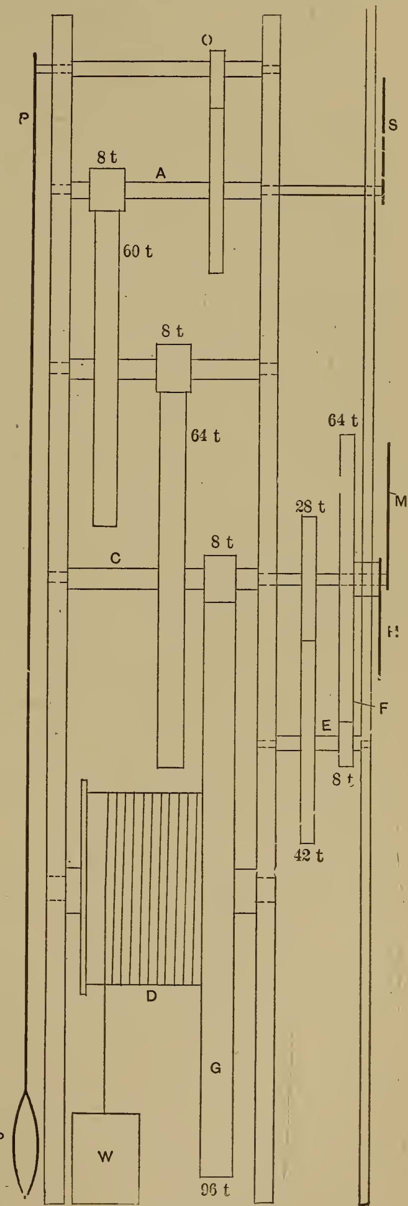


FIG. 234.

156. Frequency of Contact between Teeth. Hunting Cog.—Let T and t be the numbers of the teeth on two wheels in gear, and let $\frac{T}{t} = \frac{a}{b}$ when reduced to its lowest terms. It is evident that the same teeth will be in contact after a turns of t, and b turns of T. Therefore the smaller the numbers a and b, which express the velocity ratio of the two axes, the more frequently will the same pair of teeth be in contact.

Assume the velocity ratio of two axes to be nearly as 5 to 2. Now if we make $T=80$ and $t=32$, we shall have

$$\frac{T}{t}=\frac{80}{32}=\frac{5}{2}, \text{ exactly,}$$

and the same pair of teeth will be in contact after five turns of t or two turns of T.

If we now change T to 81, then $\frac{T}{t}=\frac{81}{32}=\frac{5}{2}$, very nearly, the angular velocity ratio being scarcely distinguishable from what it was originally. But now the same teeth will be in contact only after 81 turns of t or 32 turns of T.

The insertion of a tooth in this manner prevents contact between the same pair of teeth too often, and insures greater regularity in the wear of the wheels. The tooth inserted was called a *hunting cog,* because a pair of teeth, after being once in contact, would gradually separate and then approach by one tooth in each turn, and thus appear to hunt each other as they went round. In cast gears, which will be more or less imperfect, it would be much better if an imperfection on any tooth could distribute its effect over many teeth rather than that all the wear due to such imperfection should come always upon the same tooth. This result is most completely obtained when the numbers of teeth on the two gears are prime to each other, as above when T and t were 81 and 32 respectively.

This same principle is used in a form of stop motion. Suppose the wheel A, Fig. 235, to have 61 teeth and B to have 30 teeth. The same teeth will be in contact after 30 turns of A or 61 of B. If now one of the wheels A is arranged on a movable axis and two false teeth are supplied on the wheels so as to meet at a point, one wheel can be made to push the other one side and operate a stop motion for a machine after a certain desired motion, as 61 turns in the above example, of the fixed wheel B has taken place.

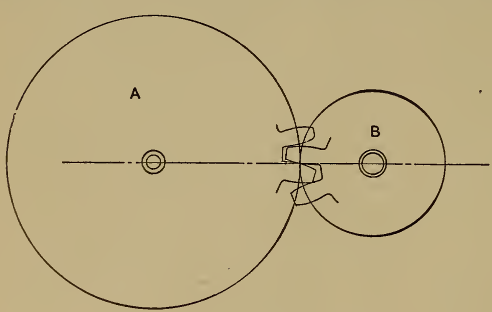


Fig. 235.

Where it is necessary to have some exact value to the train, as in clockwork, the above principle can only be employed to a limited extent.

157. Methods of Designing Trains.—Let it be required to connect two axes, A and B, by a train of spur gears so that the value of the train shall be

$$\frac{\text{turns } B}{\text{turns } A}=C,$$

and further let the largest number of teeth allowed on any wheel be T and the smallest number be t. Then one pair of wheels using teeth T and t respectively would give an a.v. ratio $\frac{T}{t}$ to the axes thus connected.

It is first necessary to determine the least number of pairs of gears required to obtain the value of the train. This is easily done by finding the power to which the ratio $\frac{T}{t}$ must be raised to give a result not less than the value of the train. If $m=$the number of pairs of gears,

$$C=\left(\frac{T}{t}\right)^m; \quad m=\frac{\log C}{\log \frac{T}{t}}. \qquad (57)$$

For example, suppose $\frac{\text{turns } B}{\text{turns } A}=400$, and $\frac{T}{t}=\frac{150}{25}$; from equation (57),

$$m=\frac{\log 400}{\log 6}=3.344;$$

or, three pairs is not sufficient, therefore four pairs of gears are required.

Where the ratio $\frac{T}{t}$ is some number easily raised to successive powers the least number of pairs can be obtained without using equation (57). Thus, in the above example, $\frac{T}{t}=6$; $6\times6\times6=6^3=216$, the result of using three pairs, which is not enough; $6^4=1296$ is much more than enough, but four pairs are required, and the fact that four pairs would give so much more than is desired means simply that we can use less than 150 teeth on the large wheels.

After determining the least number of pairs of gears needed, one of two methods may be used in solving for the desired train, the choice depending largely on whether it is required that the desired value of the train shall be exactly obtained, or whether some deviation is allowable.

1° Let the value of the train be 360, and $\frac{T}{t}=\frac{120}{20}=6$. The least number of pairs of gears required will be four (since $6^3=216<360$).

Separate 360 into its prime factors, and place these factors in the numerator, letting the denominator be written as shown, the four *units* representing the four small gears in the final train; thus,

$$\frac{360}{1}=\frac{2\cdot2\cdot2\cdot3\cdot3\cdot5}{1\cdot1\cdot1\cdot1}.$$

Since 20 is the least number of teeth to be used, we may next substitute for the four *units* four *twenties,* multiplying the numerator also by the same amount, giving

$$\frac{360}{1}=\frac{2\cdot2\cdot2\cdot3\cdot3\cdot5\cdot20\cdot20\cdot20\cdot20}{20\cdot20\cdot20\cdot20}.$$

The factors of the numerator are now to be combined into *four* numbers which are to be the teeth for the large gears, and to obtain the best result the *twenties* in the numera or should be factored as well as the 360. Two results are given below:

$$\frac{360}{1}=\frac{100}{20}\times\frac{100}{20}\times\frac{90}{20}\times\frac{64}{20},$$

or, better,

$$\frac{360}{1}=\frac{100}{20}\times\frac{96}{20}\times\frac{80}{20}\times\frac{75}{20}.$$

If, however, it be required that the numbers of teeth on the wheels which are in gear should be prime to each other, it will be necessary to choose such numbers of teeth for the small gears as will give when combined with the factors of the value of the train numbers which can fulfil the requirements.

For example, let the value of the train be 400 and $\frac{T}{t}=\frac{150}{25}=6$. Four pairs of wheels will be found necessary. Then we may write

$$\frac{400}{1}=\frac{2\cdot2\cdot2\cdot2\cdot5\cdot5\cdot27\cdot27\cdot26\cdot26}{27\cdot27\cdot26\cdot26}$$
$$=\frac{104}{27}\times\frac{104}{27}\times\frac{135}{26}\times\frac{135}{26}.$$

For the other train, when the value was 360, with *prime* gears, and to have the large gears as small as possible, the small gears will probably need to be all different; thus,

$$\frac{360}{1}=\frac{2\cdot2\cdot2\cdot3\cdot3\cdot5\cdot20\cdot21\cdot(2\cdot11)\cdot23}{20\cdot21\cdot22\cdot23}$$
$$=\frac{99}{20}\times\frac{92}{21}\times\frac{105}{22}\times\frac{80}{23}.$$

Should the numerator or denominator of the fraction expressing the value of a train be a large prime number, or contain inconveniently large prime factors, then an approximate ratio can be sought having terms that can be factored.

In an orrery the earth must rotate on its axis once in 24 hours = 86400 seconds, and revolve round the sun in 365 days 5 hours 48 minutes 48 seconds = 31556928 seconds. Then the value of the train will be

$$\frac{31556928}{86400}=\frac{164359}{450}=\frac{269\times47\times13}{10\times9\times5}=365.2421.$$

Here 269 is a large prime number, and to avoid its use let us introduce in its place

$$269.001=\frac{269001}{1000}=\frac{81\times81\times41}{10\times10\times10}=269, \text{ very nearly.}$$

Then arranging the factors, we obtain the following train:

$$\frac{81}{10}\times\frac{81}{10}\times\frac{41}{10}\times\frac{47}{45}\times\frac{13}{10}=365.2436,$$

which gives a very close approximation.

This principle of adding a very small amount to one of the prime factors of the value of a train, in order to obtain a number which can be factored, may be applied in many cases; the conditions in each case can alone determine whether the variation is permissible or not.

2° Where an error of a certain amount is allowable, as would very often be the case, the following method may be used to advantage.

For example, let the value of the train be 60 and $\frac{T}{t}=\frac{100}{2}=5$. It will be found that *three* pairs of gears are needed. Therefore take the *cube* root of 60, which is 3.91+, and write

$$\frac{3.91}{1}\times\frac{3.91}{1}\times\frac{3.91}{1}=60, \text{ nearly.}$$

Since the small gears are not to have less than 20 teeth, and since $20\times3.91=78+$, we may write as a first approximation

$$\frac{79}{20}\times\frac{79}{20}\times\frac{79}{20},$$

which will be found to equal 61.63; if this result is too greatly in error, a reduction of one or two teeth in the numerator or an increase in the denominator may give a closer result, as

$$\frac{77}{20}\times\frac{79}{20}\times\frac{79}{20}=60.07.$$

158. Mangle-wheels.—Mangle-wheels are used to produce reciprocating motion from the uniform rotation of a pinion, and derive their name from the first machine in which they were applied.

Figs. 236, 237, and 238 show three forms of mangle-wheels. In Fig. 236 the teeth are drawn in on only a part of the pitch curve *PP*, and in Fig. 237 the pitch curves only are shown.

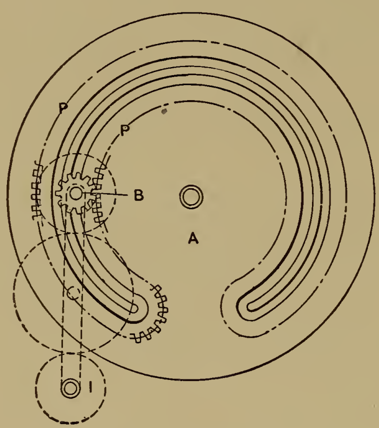


Fig. 236.

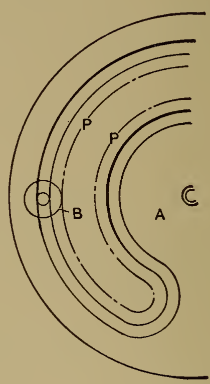


Fig. 237.

In Figs. 236 and 237 the cycle of motion of the wheel is divided into two parts, each part having its own definite velocity ratio, which is here constant except for a small space at each end of the motion, when the pinion is being guided from one pitch circle *P* to the other, they being joined at their ends by short circular arcs.

Fig. 236 has the teeth cut upon the edge of an annular groove in the disc *A*, these teeth being properly formed to gear with the pinion *P*, the shaft of which is so supported as to allow the pinion to gear with both the inner and outer sides of the groove. The pinion's shaft projects below the pinion, and works in a groove, the width of the groove being a little greater than the diameter of the pinion's shaft. This groove serves to keep the pinion always in gear. If we suppose the pinion to rotate right-handed, the wheel *A* will first make about $\frac{3}{4}$ of a rotation left-handed, and then about one rotation right-handed, and so on. It will be noticed that the change of motion is gradual at each end when the pinion is passing from one position to the other.

In Fig. 237 the wheel *A* (only one-half of which is shown) has teeth cut upon the outside of an annular ring projecting from the face of *A*, the pinion now travelling outside of the pitch circle *PP*, a groove being supplied for the shaft as before. Here the difference between the velocity ratios is less than in the previous case, and by making the two pitch lines *PP* to coincide, as has been done in Fig. 238, the velocity ratios are made the same.

In Fig. 238 the wheel *A* is supplied with a series of pins *P*, the pinion *B* working alternately on the inside and then on the outside of the pins. One method of connecting the pinion to a fixed shaft *I* is shown (see also Fig. 236). The arm *D*, turning on the shaft *I*, carries the pinion shaft *H* and a train of gear-wheels *EFG*, connecting the shafts *H* and *I*; these gear-wheels, being above the arm, are shown dotted. The shaft *H* projects below the pinion, and can only move between the positions *H* and H_1; the guards *JJ* against which it comes when the last pin is reached serve to guide it from one side of the teeth to the other. As the teeth of the pinion cannot be shaped to give an exact velocity ratio, and at the same time work both on the inside and outside of the pins, this arrangement cannot be used where great uniformity of motion is desired.

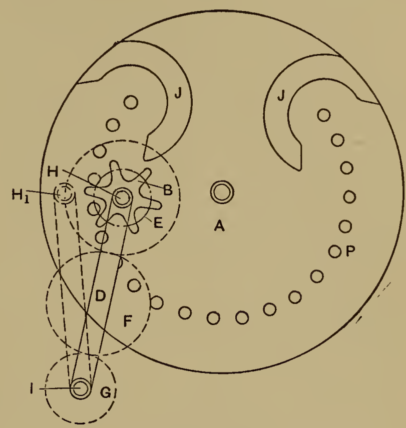


FIG. 238.

The wheels illustrated have their teeth arranged in circular rows, but this is not necessary, as by properly curving the rows of teeth the velocity ratio may be varied at pleasure; a pause can also be introduced by making the row radial for the corresponding distance.

As examples of the use of mangle-wheels, we have the "flat stripping device" on flat carding-engines, and the mechanism for moving the

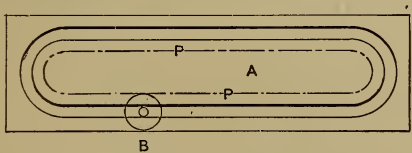


FIG. 239.

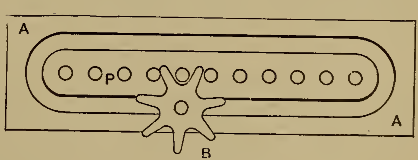


FIG. 240.

rails up and down in "spoolers," used for winding warp yarn from the bobbins on to the spools.

Figs. 239 and 240 show two forms of mangle-racks, which are lettered similarly to the wheels, the shafts of the pinions being constrained to move in vertical lines.

In Fig. 240 the pinion *B* will gear correctly with the circular pins *P*, and in both cases the velocity ratios will be the same.

Sometimes the pinion is fixed and the rack shifts laterally, it being arranged to move in suitable guides or to be governed by linkages properly arranged.

Mangle-racks are used in some forms of cylinder printing-presses to actuate the table.

CHAPTER XI.

AGGREGATE COMBINATIONS.

159. **Aggregate Combinations** is a term applied to such assemblages of pieces in mechanism in which the motion of the follower is the resultant of the motions given to it by more than one driver. The number of independently-acting drivers which give motion to the follower is generally two, and cannot be greater than three, as each driver determines the motion of at least one point of the follower, and the motion of three points in a body fixes its motion.

By means of aggregate combinations we may produce very rapid or slow movements and complex paths, which could not well be obtained from a single driver.

160. **Aggregate Motion by Linkwork.**—Figs. 241 and 242 represent the usual arrangement of such a combination. A rigid bar ab has two points, as a and b, each connected with one driver, while c may be connected with a follower. Let aa_1 represent the l.v. of a, and bb_1 the l.v. of b: to find the l.v. of c. Consider the motions to take place separately; then if b were fixed, the l.v. aa_1 given to a would cause c to have a velocity represented by cc_1. Considering a as fixed, the l.v. bb_1 at b would give to c a velocity cc_2. The aggregate of these two would be the algebraic sum of cc_1 and cc_2. In Fig. 241 we have cc_1 acting to the left, while cc_2 acts to the right; therefore the resulting l.v. of c will be $cc_3 = cc_1 - cc_2$ acting to the left, since $cc_1 > cc_2$. In Fig. 242, where both cc_1 and cc_2 act to the left, the result is $cc_3 = cc_1 + cc_2$ acting to the left. It will be seen that the same results could have been obtained by finding the instantaneous centre o of ab in each case, when we should have l.v. c:l.v. $a = co:ao$.

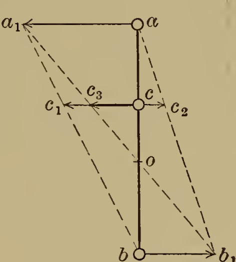


FIG. 241.

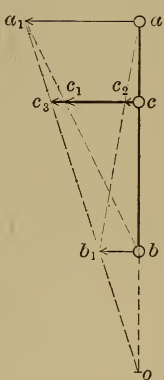


FIG. 242.

In many cases the lines of motion are not exactly perpendicular to the link, nor parallel to each other, neither do the points a, b, and c necessarily lie in the same straight line, but often the conditions are approximately as assumed in Figs. 241 and 242, so that the error introduced by so considering them may be sufficiently small to be practically disregarded.

As examples of aggregate motion by linkwork we have the different forms of link motions as used in the valve gears of reversing steam engines. Here the ends of the links are driven by eccentrics, and the motion for the valve is taken from some intermediate point on the link whose distance from the ends may be varied at will, the nearer end having proportionally the greater influence on the resulting motion.

A wheel rolling upon a plane is a case of aggregate motion, the centre of the wheel moving parallel to the plane, and the wheel itself rotating upon its centre. The resultant of these two motions gives the aggregate result of rolling.

161. **Pulley-blocks for Hoisting.**—The simple forms of hoisting-tackle, as in Fig. 243, are examples of aggregate combinations. The

FIG. 243.

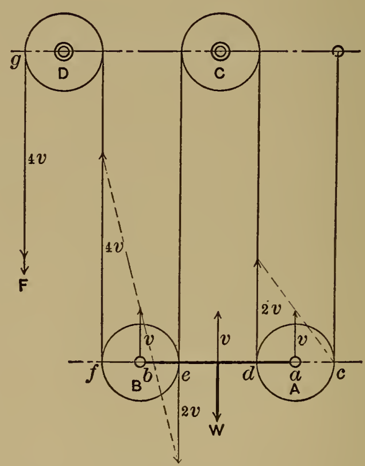


FIG. 244.

sheaves C and D turn on a fixed axis, while A and B turn on a bearing from which the weight W is suspended. Fig. 244 is in effect the same as Fig. 243, but gives a clearer diagram for studying the l.v. ratio. Assume that the bar ab with the sheaves A and B and the weight W has an upward velocity represented by v. The effect of this at the sheave A, since the point c at any instant is fixed, is equivalent to a wheel rolling on a plane, and there would be an upward l.v. at $d=2v$. At the sheave B there is the aggregate motion due to the *downward* l.v. at $e=2v$ and the upward l.v. of the axis $b=v$, giving for the l.v. of f, $4v$ upwards.

$$\therefore \frac{\text{l.v.}\,F}{\text{l.v.}\,W}=\frac{4}{1}=\frac{W}{F}.$$

Many elevator-hoisting mechanisms are arranged in a similar manner, the force being applied at W, and the resulting force being given at F. This means a large force acting through a relatively small distance, producing a relatively small force acting through a much greater distance.

162. Differential Pulley-block.—Fig. 245 shows a diagram of the Weston differential pulley-block, which consists of a double chain sheave A turning on a fixed axis c with a single sheave B below it. The chain is endless, passing around the larger circumference of A with the radius ac, then down and around B, whose radius is a mean between the two upper radii, so that the chain hangs parallel; from B the chain passes up and around the smaller circumference on A, the radius of which is bc. To find the l.v. of W if the l.v. of F is represented by v, lay off the velocity v downward at the point a as shown. The chain leaving the smaller sheave at b will have a downward l.v. represented by $v_1=v\dfrac{bc}{ac}$. The effect of these velocities on the lower sheave will be to give an upward velocity v at d, and a downward velocity v_1 at e. The aggregate of these two will give for a resultant

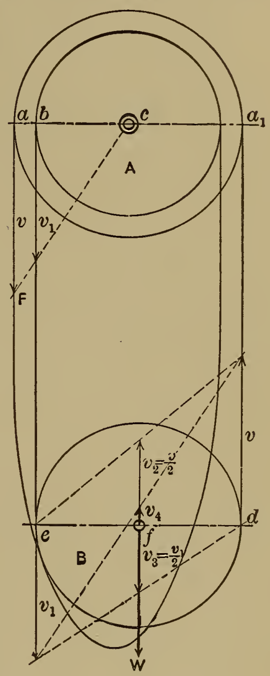


FIG. 245.

at the axis f a velocity $v_4=v_2-v_3=\dfrac{v}{2}-\dfrac{v_1}{2}$; and since $v_1=v\dfrac{bc}{ac}$, we have l.v. $W=\dfrac{v}{2}\left(1-\dfrac{bc}{ac}\right)$.

$$\therefore \frac{\text{l.v. } W}{\text{l.v. } F}=\frac{\frac{v}{2}\left(1-\frac{bc}{ac}\right)}{v}=\frac{ac-bc}{2ac}=\frac{F}{W}. \quad . \quad . \quad . \quad . \quad (58)$$

For example, if $ac=7\frac{1}{2}''$ and $bc=7''$, we should have $\dfrac{\text{l.v. } W}{\text{l.v. } F}=\dfrac{7.5-7}{15}=\dfrac{1}{30}$, and a force of 100 lbs. at F would raise a weight of 3000 lbs. at W *if friction were neglected.*

The motion of W might have been derived as follows: Suppose the wheel A to turn once L.H.; then an amount of chain $2\pi ac$ will be taken up at d, and an amount $2\pi bc$ will be delivered at e, thus giving a motion to W equal to $\pi(ac-bc)$; thus the l.v. ratio is

$$\frac{\text{l.v. } W}{\text{l.v. } F}=\frac{\pi(ac-bc)}{2\pi ac}=\frac{ac-bc}{2ac}.$$

One great advantage of this block is that it will retain the load. The pull at d, having but little more leverage than the equal pull at e, in the block as practically constructed, is not sufficient to more than overcome the friction of the chain and bearing c.

163. **Epicyclic Trains** may be defined as trains of wheels in which some or all of the wheels have a motion compounded of a rotation about an axis and a revolution, or translation, of that axis.

The wheels are usually connected by a rigid link, such as D (Fig. 246). This link is usually called the *train arm,* and it often rotates upon the axis of the first wheel A of the train: the last wheel of the train may or may not be placed upon this axis.

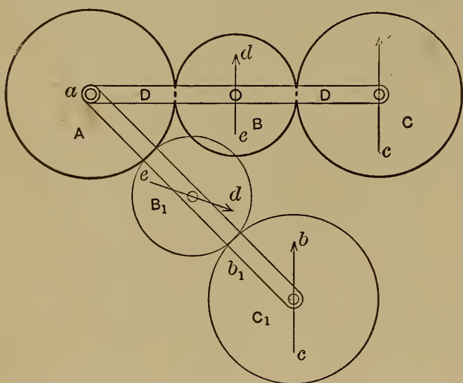


FIG. 246.

In what follows we will consider a wheel to have made one turn or rotation when an arrow placed upon it, as in Fig. 246, comes again into a position parallel to its first position with the head upward, after a continuous angular motion, either right- or left-handed. [The word *turn* will be used in place of rotation, as it is much shorter; right-handed turns will be considered *positive,* and left-handed turns *negative.*] In Fig. 246 the wheel A is assumed to be fixed, and for $+\frac{1}{8}$ of a turn of the arm D, as shown, the wheel B has turned a little more than $+\frac{1}{4}$, as indicated by the position of the arrow de, while the wheel C has not turned, the arrow bc having moved parallel to itself.

164. There are two methods by which these trains may be solved:

First. By resolving the resultant motion into its components and then allowing these to take place in succession, the motion of the arm being considered first.

In Fig. 247 let A have 100 teeth and B 50; also let A make $+5$ turns about the fixed axis a while the arm D makes -6 turns; to find the number of turns of B.

1° Suppose the mechanism locked so that the wheels cannot turn in relation to the arm, and then turn the whole -6 turns about a, the number of turns which the arm is to make. This is expressed by the first line of the following table.

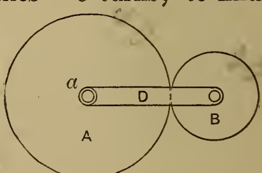


FIG. 247.

2° Unlock the train, and put the wheel for which the motion is given, in this case A, where it should be. A is to make $+5$ turns, but it has been turned -6 turns with the arm when the train was locked; it must therefore be now turned $+11$ in order that its resultant turns shall be $+5$.

This will cause B to make -22 turns, since the value of the train $\dfrac{\text{turns } B}{\text{turns } A} = -2$. These motions are expressed by the second line of the table.

3° Taking the algebraic sum of the above component motions will give the resultant motions, as expressed by the third line of the table.

	A	B	Arm
1° Train locked.	-6	-6	-6
2° Train unlocked, arm fixed.	$+11$	22	0
3° Resultant turns.	$+5$	-28	-6

If in the same train it were required to find the number of turns which the arm must make in order that B shall make -28 turns, while A makes $+5$, we may let x be the number of turns of the arm and proceed as above.

1° With the train locked, turn the whole x turns.

2° With the train unlocked, and arm fixed, turn A $(-x+5)$, which will cause B to turn $(-2)(5-x)$.

3° Adding these component motions and equating the turns of B thus found with the given number of turns will give an equation from which x may be found. These steps are expressed in the following table:

	A	B	Arm
1° Train locked	x	x	x
2° Train unlocked, arm fixed. .	$-x+5$	$(-2)(5-x)$	0
3° Resultant turns	$+5$	$(-2)(5-x)+x$	x

But turns of $B=-28=(-2)(5-x)+x$;

$$\therefore\ x=-6=\text{turns of arm.}$$

Second Method.—By the use of a general equation which may be written for any of these trains, which is usually the more convenient method.

In the problem just solved we could write the following general statement, in the form of an equation, and a similar statement could be made for any epicyclic train:

$$\frac{\text{turns of } B \text{ relative to arm}}{\text{turns of } A \text{ relative to arm}} = \frac{\text{absolute turns of } B - \text{turns of arm}}{\text{absolute turns of } A - \text{turns of arm}}. \tag{59}$$

In this equation it will be seen that the first term can always be expressed in terms of the numbers of teeth on the gears, as it is simply the value of the train assuming the arm fixed. *It is absolutely essential that this value of the train be expressed as + or −, depending on whether the gears considered turn in the same or in opposite directions relative to the arm.*

Substituting the data of the first problem, given under the tabular method, would give

$$\frac{\text{relative turns } B}{\text{relative turns } A} = -2 = \frac{\text{absolute turns } B - \text{turns arm}}{\text{absolute turns } A - \text{turns arm}} = \frac{B-(-6)}{5-(-6)},$$

where B in the last term represents the absolute turns of B; therefore

$$-2=\frac{B+6}{5+6};$$

$$B=-28 \text{ turns.}$$

With the data as given in the second case under the tabular method, where x represents the turns of the arm, and B and A the absolute turns of B and A respectively,

$$-\frac{2}{1}=\frac{B-\text{arm}}{A-\text{arm}}=\frac{-28-x}{5-x};$$

$$-10+2x=-28-x;$$

$$x=-6 \text{ turns.}$$

165. Problems of this kind may have the data so given that the numbers of teeth on the gears or on some of them may be required. Thus in Fig. 248 suppose that A is to make $+5$ turns, and B -28, while the arm turns -6; to find the number of teeth needed on the wheel B if A has 100 teeth, and to determine whether or not an idle wheel is needed between them. By the general equation,

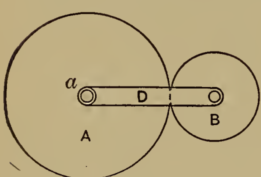


FIG. 248.

$$\frac{\text{relative turns } B}{\text{relative turns } A}=\frac{\text{absolute turns } B-\text{turns arm}}{\text{absolute turns } A-\text{turns arm}}=\frac{-28+6}{5+6}=-2,$$

or the value of the train relative to the arm must be -2; so B must have 50 teeth, and no idle wheel is required, since the value of the train is negative.

166. Examples of Epicyclic Trains.—Fig. 249 shows an application of the two-wheel train commonly known as the *Sun and Planet Wheels*, first devised by Watt to avoid the use of a crank, which was patented; but in his device the pin b worked in a circular groove around the centre a, which took the place of the link ab, and kept the two wheels in gear. The rod B is attached to the beam of the engine, and a represents the engine shaft. While in this case we cannot say that the wheel C does not turn, yet its action on the wheel D, for an interval of one rotation of the arm, is the same as though it did not turn, as the position at the start and stop is the same. Then to find the turns of D, for one turn of the link ab R.H., we will first disconnect the rod and lock the train, thus obtaining the first line of the following table. Then, as C has been turned $+1$, we unlock the train, fix the arm, and turn C -1, giving the second line.

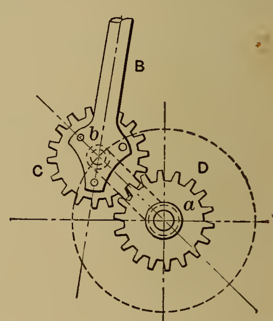


FIG. 249.

	C	D	ab
1° Train locked.	$+1$	$+1$	$+1$
2° Train unlocked, arm fixed.	-1	$+1$	0
3° Resultant motions.	0	$+2$	$+1$

Adding, we find that the wheel D makes 2 turns R.H.

In the three-wheeled train, Fig. 250, let A have 55 teeth, and C

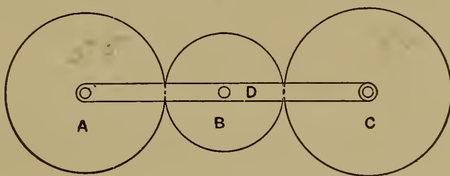


Fig. 250.

have 50. A does not turn; find the turns of C while the arm D makes $+10$ turns. By the tabular method we have:

	A	C	Arm.
1° Train locked.	$+10$	$+10$	$+10$
2° Train unlocked, arm fixed.	-10	$-10\ (\frac{55}{50})$	0
3° Resultant motions.	0	-1	$+10$

Or the wheel C turns -1 while the arm D turns $+10$.

This type of train is made use of in rope-making machinery, to give the bobbins which carry the strands such a motion that the strands are not untwisted in laying the rope, the twist in laying being opposite to that of the strands; also in wire-rope machinery, where the individual wires cannot be twisted in the laying. The arrangement usually adopted is shown in Fig. 251. The bobbins for the strands, or wires (shown dotted), are attached to the wheels B by means of the spindles b, turning in bearings in the large disc which turns with the rope-laying block on the shaft a. The idle wheels C, turning on pins placed in the disc, connect the wheels B with a stationary wheel A, on the axis a, having the *same number* of teeth as the wheel B for wire rope. Then, on rotating the disc, the bobbins will be carried round with the laying block, but the wires will not twist. This arrangement, it will be noticed, gives the same result as that shown in Fig. 115, and both are used for the same purpose.

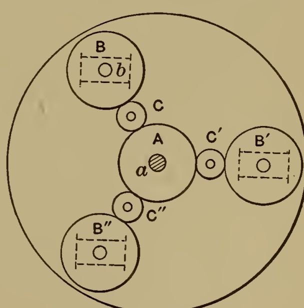


Fig. 251.

Fig. 252 shows an epicyclic train which gives the same result as above when the pulleys A and B, which are here connected by a belt, are of the same size. This train has also been used in fibre rope-making machinery, the pulleys B being often made slightly smaller in diameter

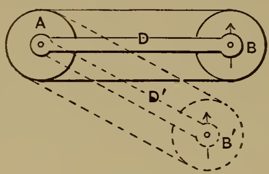


FIG. 252.

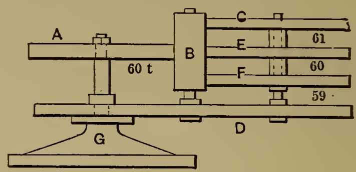


FIG. 253.

than A, when it will be found that they will turn slowly in the opposite direction to that of the arm D, giving a slight additional twist to the strands of the rope as they are being laid, and making a rope less liable to untwist.

If the wheel C in Fig. 250 has more teeth than A, it will turn in the *same* direction as the arm. Fig. 253, called *Ferguson's paradox*, shows an arrangement giving the three cases, the wheel E having the same number of teeth as A, C one more than A, and F one less than A, B being an idle wheel connecting A with the other three. The arm D turns freely on the axis of the stand G, while A is fast to the stand. If D makes $+1$ turn, we have for the other wheels, by the tabular method:

	A	D	C	E	F
1° Train locked................	$+1$	$+1$	$+1$	$+1$	$+1$
2° Train unlocked, arm fixed....	-1	0	$-\frac{60}{61}$	-1	$-\frac{60}{59}$
3° Resultant motions...........	0	$+1$	$+\frac{1}{61}$	0	$-\frac{1}{59}$

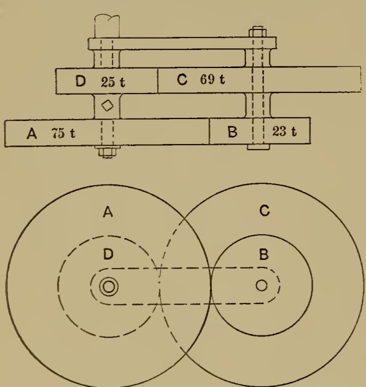


FIG. 254.

Thus, C turns slowly R.H., and F slowly L.H., in respect to the wheel E, all gearing with the pinion B.

167. The train may be a compound train, and also the last wheel of the train may turn about the same axis as the first, as shown in Fig. 254. Let the numbers of teeth be as indicated on the wheels; to find the number of turns of A while D makes $+15$ turns, and the arm makes -3 turns; also to find the turns of BC. Using the tabular method we have:

	A	BC	D	Arm.
1° Train locked.................	-3	-3	-3	-3
2° Train unlocked, arm fixed.....	$+2$	$(-\frac{25}{69})(18)$	$+18$	0
3° Resultant motions...........	-1	$-\frac{219}{23}$	$+15$	-3

Or A will make -1 turn, and the sleeve on which the wheels B and C are fast will make $-9\frac{12}{23}$ turns.

168. Annular wheels frequently enter into the construction of these trains, and the first and last wheels of the train would usually turn about the same axis. In Fig. 255 let A have 20 teeth and C have 200 teeth; C is to make -16 turns while A makes $+50$ turns. To find the number of turns which the arm D must make. Using the general equation for this,

$$\frac{\text{relative turns } A}{\text{relative turns } B}=-10$$

$$=\frac{\text{absolute turns } A-\text{turns } D}{\text{absolute turns } B-\text{turns } D}=\frac{50-x}{-16-x};$$

$$-10=\frac{50-x}{-16-x};$$

$$x=-10.$$

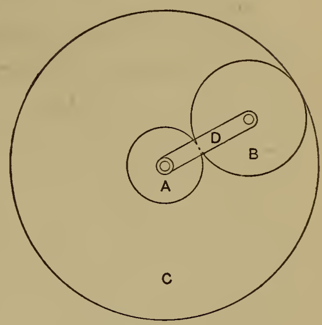


Fig. 255.

Therefore the arm D must make -10 turns.

The *Triplex Pulley-block*, Fig. 256, is an example of the use of an annular epicyclic train. Here the annular D is made fast to the casing E so that it does not turn. The shaft H turns in bearings in the block frame or casing E, its axis coinciding with that of the annular D. The gears B and C (of which there are three equidistant sets to distribute the

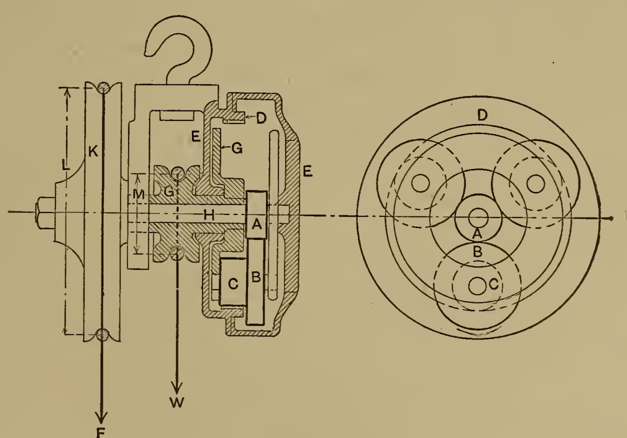


Fig. 256.

stress evenly about the axis) connect A with D and turn loosely on studs fast to the disc G, which in turn is made fast to the hollow shaft of the chain wheel G turning in bearings as shown in the casing E. A hand chain wheel K is attached to the end of the shaft H, turning loosely in the hollow shaft of the chain wheel G, and serves to actuate the train.

The problem is to find the l.v. ratio of W to F. This involves finding first the a.v. ratio of G to K.

It will be seen that G is the *arm; D*, the fixed annular, is the last wheel and A the first wheel of the train; the arm turns the lifting chain wheel, and the hand chain wheel K turns A. It is thus necessary to find the number of turns of G for one turn of A. Knowing the a.v. ratio of G to A, and the diameters L and M of the chain wheels, we may obtain the l.v. ratio of the chains.

169. Epicyclic Bevel Trains.—Fig. 257 represents a common form of epicyclic bevel train, consisting of the two bevel-wheels D and E attached to sleeves free to turn about the shaft extending through them. This shaft carries the cross at F which makes the bearings for the idlers GG connecting the bevels D and E (only one of these idlers is necessary, although the two are used to form a balanced pair, thus reducing friction and wear). The shaft F may be given any number of turns by means of the wheel A, at the same time the bevel D may be turned as desired, and the problem will be to determine the resulting motion of the bevel E. The shaft and cross F here correspond with the arm of the epicylic spur-gear trains, for we may assume the bevels locked so that they do not turn relative to F, and then give the desired motion to F; the bevels may then be unlocked and, while F remains fixed, such motion may be given to either of them as the data require. For example, in Fig. 257 let A make $+5$ turns while B makes -4; to find the resulting motion of C. When the bevels are arranged as in Fig. 257 the wheels D and E must have the same number of teeth. [It will be found clearer in these problems to assume that the motion is positive when the nearer side of the wheel moves upward, in which case a downward motion would be negative; or if a downward motion is assumed as positive, then upward motion would be negative.] Solving the problem by the tabular method, we have:

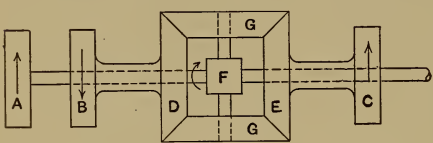


Fig. 257.

	D	E	F
1° Train locked.	$+5$	$+5$	$+5$
2° Train unlocked, arm fixed.	-9	$+9$	0
3° Resultant motion.	-4	$+14$	$+5$

Or the wheel C will make 14 turns in the same direction as A.

Solving this problem by the general equation, and letting D, E, and F, in the latter part of the equation, represent the absolute turns of D,

E, and F respectively, we have

$$\frac{\text{relative turns } E}{\text{relative turns } F}=-1=\frac{E-F}{D-F}=\frac{E-5}{-4-5};$$
$$\therefore\ E=+14.$$

The bevel train in any machine may be driven through some external train of gears, in which case the first step must be to determine from the given train or trains the absolute turns of the respective parts of the epicyclic train. For example, in Fig. 258 let it be required to find the turns of B for $+1$ of A. From the trains at the left we find that $+1$ turn of A will give -2 turns to D and $+\frac{1}{2}$ turn to F; from these we may determine the turns of E,

$$-1=\frac{E-\frac{1}{2}}{-2-\frac{1}{2}};\quad \therefore\ E=+3.$$

$+3$ turns of E will give -6 turns to B, or B makes 6 turns in the opposite direction to A.

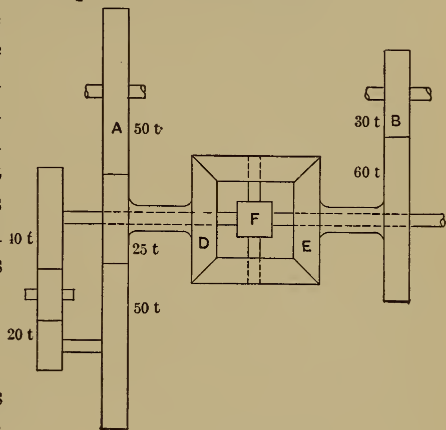


Fig. 258.

The bevel train may be a compound train, as shown in Fig. 259, the essential difference in this case being that the value of the bevel train relative to the arm is no longer -1. In Fig. 259 if B makes -10 turns, and A $+40$, find the turns of C. With the numbers of teeth as given in the figure, we should have

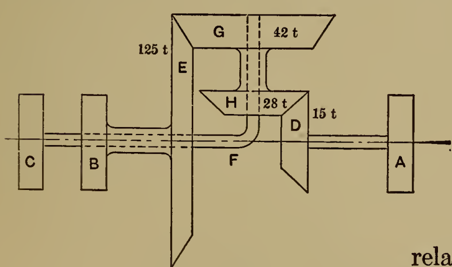


Fig. 259.

$$\frac{\text{relative turns } D}{\text{relative turns } E}=-\frac{125}{42}\times\frac{28}{15}=-\frac{50}{9};$$

$$-\frac{50}{9}=\frac{D-F}{E-F}=\frac{+40-F}{-10-F};$$

$$\therefore\ F=-\frac{140}{59}.$$

Or, C will turn $\frac{140}{59}$ turns in the same direction as B turns. To check this we may use the tabular method:

	D	E	F
1° Train locked.	$-\frac{140}{59}$	$-\frac{140}{59}$	$-\frac{140}{59}$
2° Train unlocked, arm fixed. . . .	$+\frac{140}{59}+40$	$\left(-\frac{9}{50}\right)\left(\frac{140}{59}+40\right)$	0
3° Resultant motion.	$+40$	-10	$-\frac{140}{59}$

170. Other Examples of Epicyclic Trains.—An epicyclic bevel train is used in connection with a train containing a pair of cone pulleys, in a form of water-wheel governor for regulating the supply of water to the wheel. Fig. 260 is a diagram for this train, the position of the belt connecting the cone pulleys being regulated by a ball governor connecting by levers with the guiding forks of the belt. The governor is so regulated that when running at the mean speed the belt will be in its mid-position, at which place the turns of E and D should be equal, and opposite in direction, in which case the arm F will not be turning. If the belt moves up from its mid-position, and if A turns as shown, the arm F will turn in the same direction as the wheel E. As an example, find the diameters x and y if C is tc turn downward 1 turn for 25 turns of A in the direction shown; and must the belt be crossed or open? Solving by the general equation, we have

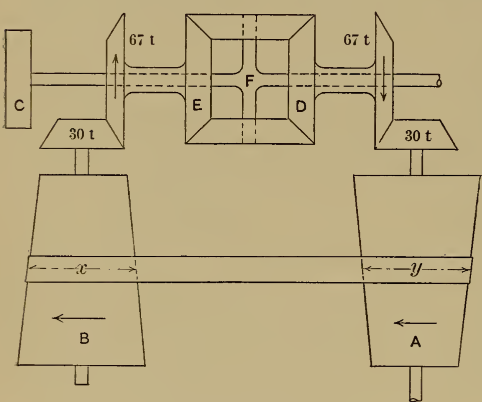


FIG. 260.

$$-1=\frac{D-F}{E-F}=\frac{(-25)\frac{30}{67}+1}{E+1};$$

$$E=+\frac{616}{67}.$$

The plus sign indicates that E must turn upward or opposite to D, and therefore an *open* belt is required. To find the ratio $\frac{y}{x}$ equate the turns of E, found above, to the turns of E for 25 turns of A through the belt; thus,

$$\frac{616}{67}=\frac{y}{x}\cdot\frac{30}{67}\cdot 25;$$

$$\therefore \frac{y}{x}=\frac{308}{375}.$$

Or if the diameter $x=7\frac{1}{2}$ inches, then the diameter y would be 6.16 inches.

Epicyclic trains are used, in connection with a screw, in cylinder boring-bars to feed the cutter collar along the bar. Let *F*, Fig. 261, represent such a bar, supported in the centres of a lathe. *E* represents

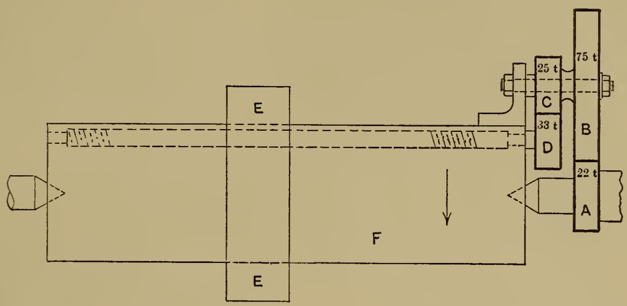


FIG. 261.

the cutter-holding collar. The cylinder to be bored is made fast to the bed of the lathe, and the problem is to feed the collar *E* through the cylinder while *E* turns with *F*. To this end *E* is paired with a screw running in a groove in the bar, and driven by a gear *D*. The motion of *D* is due to the epicyclic train *ABCD*, the stud on which the wheels *B* and *C* are mounted being fast to the bar *F* and so causing *BC* to revolve about the gear *A* made fast to the tailstock of the lathe.

In Fig. 261 if *F* turns as shown, and if the screw has four threads per inch R.H., find the resulting motion of the collar *E* for one turn of *F*. Calling the motion of *F* positive and solving by the tabular method:

	A	D	F
1° Train locked............	$+1$	$+1$	$+1$
2° Train unlocked, arm fixed	-1	$-\frac{2}{9}$	0
3° Resultant motion.......	0	$+\frac{7}{9}$	$+1$

But the turns of *D* which we need to determine the motion of *E* will be the turns relative to *F*, or $+\frac{7}{9}-1=-\frac{2}{9}$, since the motion of *E* along *F* is due only to the turns of the screw in *F*. If the screw makes $\frac{2}{9}$ of a turn in a direction opposite to the motion of *F*, the collar *E* will travel $\frac{1}{18}''$ to the right, which is, therefore, its travel for one turn of the bar.

It will be evident that, since *A* is fixed and *F* turns downward, the same result would be obtained by assuming *F* fixed and turning *A* upward once. The train from *A* to *D* is arranged so that one gear, *C*, will be a change gear, which makes it possible to vary the rate of motion of *E*.

Fig. 262 shows an application of the *sun and planet wheels,* which will give the same feed on a boring-bar as the arrangement just described. In this case the screw S is attached to the collar in the same way as before, and the end of the screw projects beyond the end of the boring-bar and carries a spur-wheel A. A pinion B, supported by an adjustable stud in the end of the bar, gears with the wheel A. This pinion is prevented from turning by the slotted link working on the stationary pin P, which is usually placed much further below the boring-bar than here shown. The action of the wheel B is the same as that of C, Fig. 249.

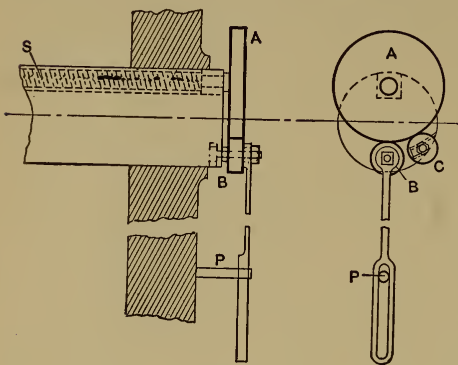


Fig. 262.

Let the pitch of the screw S be $\frac{1}{4}''$ R.H., and let A have 90 teeth and B 20 teeth, the bar turning right-handed as seen from the right. Calling m the turns of B, n the turns of A, a the turns of the bar, and $e=\frac{\text{relative turns } A}{\text{relative turns } B}$, we have $m=0$, $a=+1$, and $e=-\frac{2}{9}$; then $n=+1\frac{2}{9}$, and the nut turns $+1$; therefore the feed is $\frac{2}{9}\times\frac{1}{4}=\frac{1}{18}''$, and the collar is drawn toward the right.

To reverse the feed in this machine, the stud of the wheel B is dropped so that A and B are no longer in gear; then the idle wheel C is adjusted so that it gears with A and B, the stud of the wheel C moving in a T slot concentric with the screw S. Now $m=0$, $a=+1$, and $e=+\frac{2}{9}$; then $n=+\frac{7}{9}$; and the nut turns $+1$; therefore the feed is $\frac{2}{9}\times\frac{1}{4}=\frac{1}{18}''$, as before, but now the nut moves toward the left.

171. Roberts's Winding-on Motion.—On a mule the spun yarn is wound upon a slightly tapering spindle in conical layers, as shown in Fig. 263, forming what is called a *cop.* The formation of the cop may be divided in two parts: 1° the forming of the *copbottom* upon a bare spindle by superposing a series of conical layers of yarn with a continually increasing vertical angle; and 2° the building of the body of the cop by winding the yarn in a series of nearly conical layers. If we conceive the winding on to begin at the base of the cone, forming the copbottom, it is evident that the speed of the spindles must increase in order to wind on the same amount of yarn for the same travel of the spindle.

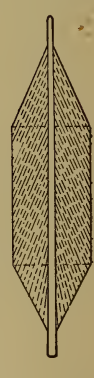


Fig. 263.

Fig. 264 shows the principal parts of a mule that are concerned in the winding on of the yarn. The carriage GG travels in and out from the rolls L and carries the spindles T, arranged in a row and driven by a long drum and cords, as shown. The cop is shown at K, and the position of winding upon the cop is governed by a wire, called the *faller wire,* attached to the ends of the arms V', carried by a shaft just inside of and parallel to the shaft V, which itself carries the *counter-faller wire* V by a series of arms. The yarn, in being wound upon the cop, passes over the counter-faller wire and under the faller wire, the shafts of both wires being supported by the carriage.

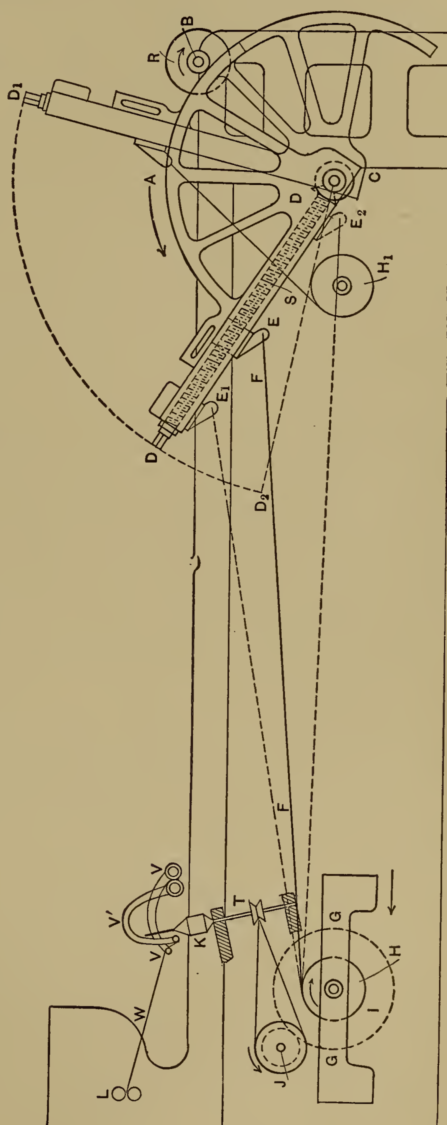


Fig. 264.

While spinning, the carriage moves from a position where the spindles are about six inches from the rolls L to the right, until the drum H of the carriage reaches the position H_1; the spindles meanwhile are driven by a rope and grooved pulley (not shown) attached to the long drum shaft. The counter-faller wire is then below the tops of the spindles and the faller wire above them, the yarn spinning from the tops of the spindles.

The winding-on mechanism consists of three parts:

1° The vibrating arm DD (centred upon the stud C, fixed to the mule frame), governed in its motion by the pinion B, whose shaft turns in bearings attached to the frame. The pinion B is connected to the carriage in such a way that they have a constant velocity ratio, and the proper relative movement, a rope and drum R being usually employed.

2° The train HIJ (attached to the carriage), which is made up of a chain drum H fixed to a spur gear I, gearing with the pinion J, which pinion is attached to the drum shaft by a ratchet similar to that shown in Fig. 199. This allows the drum to turn freely while spinning is going on, and secures the connection of J and the shaft whenever J turns in the direction of the arrow.

3° A chain FF, which connects the drum H with an adjustable block E in the quadrant arm. A screw S serves to regulate the position of the block in the quadrant arm. This screw may be turned by a wrench applied to the squared end D or by means of the bevel gears at C.

When winding on begins, the carriage is at its extreme right position, the drum H being at H_1, and the quadrant in the position D_1. Now if we suppose the quadrant to be fixed in the position D_1 and allow the carriage to move in, the drum H will rotate in the direction of the arrow with a very nearly uniform velocity; in this case it will gradually accelerate, as the chain is attached to a point not in line with the motion of the top of the drum. If, on the other hand, we allow the carriage to stand still and swing the quadrant arm from D_1 to D_2, the drum will rotate in a direction opposite to that of the arrow with a constantly decreasing angular velocity proportional to the perpendicular let fall from C to the line of the chain, which is the line of connection. Allowing both of these motions to take place simultaneously, by connecting the carriage as described with the pinion B, it will be found that the motion of the arm in passing from D_1 to D_2 will subtract from the first nearly uniform motion of unwinding of the chain a continually decreasing amount, and thus the spindles, which are driven by the drum H through the train and ratchet, will have a constantly accelerating angular velocity.

The faller wire V' regulates the position of winding upon the cop, and the counter-faller wire V takes up any slack by moving upward, the winding on being so planned that only an amount of yarn equal to the travel of the carriage is wound upon the cop for each run-in of the carriage.

This motion would be suitable for winding a conical layer of a certain size, but a different motion is necessary to form the copbottom.

As the first layer of the copbottom is placed upon the bare spindle (or upon a thin coptube), a nearly uniform motion is called for in the winding. This is obtained by placing the block E in the position E_2, the proper one to start the copbottom: the motion of H will now be a very gradually accelerating one. As the cop gradually builds up, the travel of the faller wire being higher and higher, as shown by the successive layers in Fig. 263, the screw S is turned by means of the bevel gears at C, and the position of the nut E is so regulated that the proper amount of yarn is wound upon the spindle for each run-in of the carriage. The movement of the nut E, outward, is regulated by the counter-faller wire V, which throws into gear a train of mechanism operating the screw S whenever the threads W become so taut as to draw the wire below its normal position, movement of the nut E away from C causing less and less motion to the spindle.

The motion of the faller wire is governed by an inclined rail, the incline being varied so that in building the copbottom the wire will move so as to make the winding closer at the bottom, thus increasing the diameter more rapidly there than at the top.

The nut E is moved outward until it reaches a stop which is adjusted in position to build the cop upon the copbottom, the winding motion being then the same for each run-in of the carriage, the path of the faller wire being, however, successively higher and higher.

The figure shows the carriage running in, the directions of motion of the different paths being shown by arrows. The nut E is now in the proper position to build the cop upon the copbottom; for a larger-sized cop it would be necessary to allow E to move farther out toward E_1, which is the extreme position.

172. **A Fusee** is a contrivance adopted in some of the older forms of watches in order to maintain a uniform force upon the train of wheels and compensate for the decreasing power of the main-spring. In this case the fusee consists of a groove of a helical nature, traced upon a conoid, formed by revolving a hyperbola on one of its axes. The spring is placed in a cylindrical drum, and this drum is connected to the fusee by a cord or chain. As the spring uncoils and its force diminishes, the cord being drawn to the cylinder acts on a continually increasing arm, the groove being so made that this arm increases in the proper ratio.

In mechanism the fusee is frequently employed to transmit motion, and then it enables us to derive a continually increasing or decreasing motion from the uniform motion of the fusee shaft.

The groove of the fusee may be traced upon a cone or other tapering surface, or it may be compressed into a flat, spiral curve; in all cases the effect produced will be that due to a succession of arms which radiate in perpendicular directions from the fixed axis, and continually increase or decrease in length.

Two fusees may be combined (Fig. 265) so that the motion produced may be increasing at first and then decreasing at the last. Such a device, known as a *scroll*, is used to operate the carriage of a spinning-mule, which should have an accelerated motion up to the middle of its path, and then a retarded motion to the end of its path, the start and stop being slow.

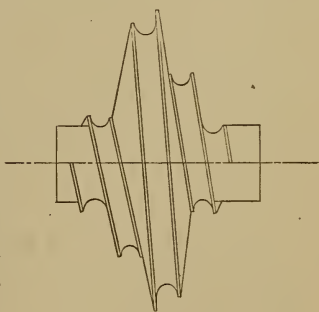


Fig. 265

rength of gear teeth; see tooth gearing
Kimball & Barr Mach. Design.

CHAPTER XII.

GEARING.—CONSTRUCTION OF GEAR-TEETH.

173. In Chapter IV, § 44, it was stated that teeth could be formed from rolling cylinders of such shape that by their sliding action the same a.v. ratio could be obtained for the axes of the rolling bodies as would be obtained if the rolling bodies were assumed to drive each other without slipping.

In order to discuss the action of such gears, and to design them, it will be necessary to understand the following definitions of the various terms constantly used:

1° **Pitch Surface.**—The pitch surface of a toothed wheel, or rack, is the elementary surface from which the tooth curves are formed, as either one of a pair of rolling cylinders.

2° **Pitch Line.**—The pitch line of a gear-wheel is the trace of the pitch surface on a plane perpendicular to the axis of the wheel; in a cylinder this would be a circle; in a rack where the teeth are formed from a plane it would be a straight line. Where the teeth are formed from rolling cones the pitch lines are commonly taken as the largest intersections of the perpendicular planes, thus giving the bases of the rolling cones as their pitch circles.

3° **Pitch Point.**—The pitch point of a pair of gear-wheels is the point of contact of their pitch lines, as the point c, Fig. 266. The *pitch point of a tooth* is the point where the tooth curve crosses the pitch line, as a_1, Fig. 266.

4° **Pitch.—Diametral Pitch.**—The pitch is the distance measured on the pitch line from a point on one tooth to the corresponding point on the next tooth, as ca_1, Fig. 266, and is equal to the thickness of the tooth plus the space between the teeth, $a_1b_1 + b_1c$. In all cases the pitch must be an aliquot part of the pitch line, and in order that two wheels may gear with each other they must have the same pitch. Thus in Fig. 266 ca_1 must equal ca_2.

Diametral Pitch.—To lay out the teeth on a pair of wheels, it is necessary to use the pitch; but if the pitch is arbitrarily assumed, it would usually give awkward decimals in the dimensions of the diameters. Practically, it is much more important that the diameters should be even numbers or convenient fractions, and that the pitch should be deduced from the diameter and the number of teeth. The *diametral pitch* is the diameter divided by the number of teeth. Thus, if the gear is 10 inches in diameter and has 20 teeth, the diametral pitch is $\frac{1}{2}$ inch. This is commonly expressed by saying that the gear is 2-pitch, or 2-P., meaning that it has two teeth corresponding to each inch of diameter; thus a 5-P. gear having 50 teeth would be 10 inches in diameter.

Since the circular pitch is the circumference divided by the number of teeth, and the diametral pitch is the diameter divided by the number of teeth, it follows that

$$\frac{\text{circular pitch}}{\text{diametral pitch}}=\frac{\text{circumference}}{\text{diameter}}=\pi;$$

or, circular pitch $=$ diametral pitch $\times\pi$. Thus, a 3-P. gear has a diametral pitch of $\frac{1}{3}$ inch, and a circular pitch of $\frac{\pi}{3}$ inches $=1.047$ inches.

5° **Backlash.**—The backlash is the difference between the space on one wheel and the thickness of the tooth on the other: in Fig. 266

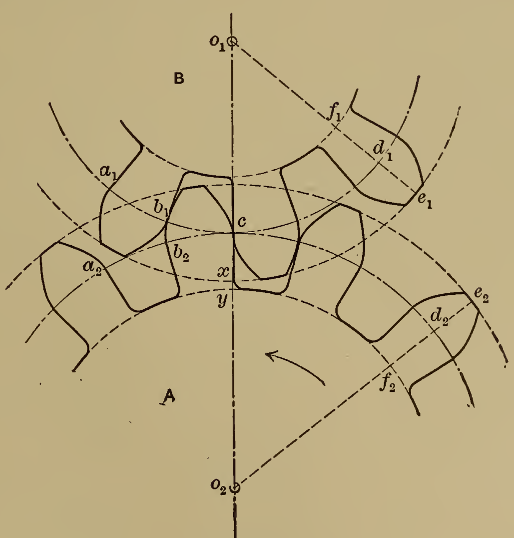


Fig. 266.

$a_2b_2-a_1b_1$ will be the backlash. In most wheels the thickness of the tooth is the same on both the wheels which are in gear, so that the backlash would be the difference between the space and the thickness of the tooth, but for constructive reasons the thicknesses may not be equal, so that the former definition is better. Backlash prevents the non-acting sides of the teeth from touching, and some should always be provided, but it may be very small in accurately cut gears.

6° **Addendum.**—The addendum is the term applied to the length of the tooth outside the pitch circle, as d_2e_2, Fig. 266, and a circle drawn through the point e_2 with o_2 as a centre is called the *addendum line* or *circle*, and limits the tops of the teeth.

7° **Root Circle.**—The root line or circle is a line drawn through the bottoms of the spaces, as through f_2, Fig. 266.

8° **Clearance.**—The clearance is the difference between the radial distance from the pitch line of one wheel to its root circle, and the addendum of the other wheel, and is the amount by which the tops of the teeth of one wheel clear the bottoms of the spaces of the other, as they pass the line of centres; thus, in Fig. 266, xy is the clearance on the wheel A and is equal to $d_2f_2-d_1e_1$. In most wheels the addendum is the same on each of two wheels in gear, so that the clearance would be the difference between the radial distances from the pitch circle to the root and addendum circles, as $d_2f_2-d_2e_2$.

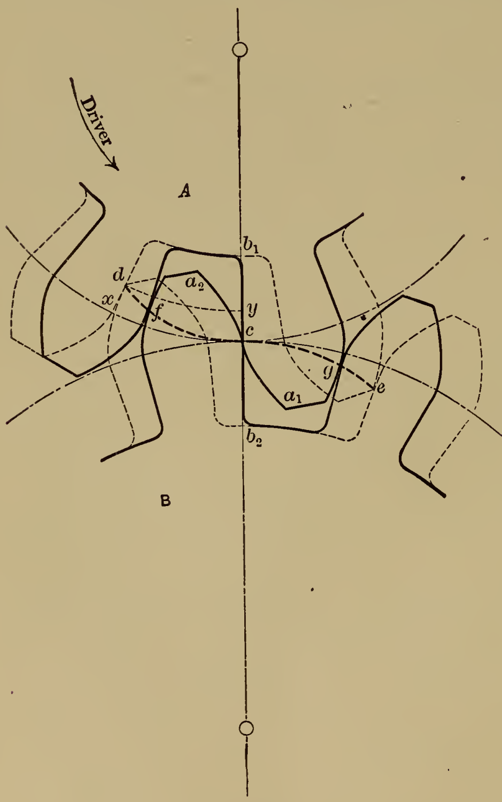


Fig. 267.

9° **Length.**—The length of a tooth is the distance between the addendum and root circles measured on a radial line, as e_2f_2, Fig. 266.

10° **Breadth.**—The breadth of a tooth is the distance measured on an element of the pitch surface, between the two bounding surfaces of the tooth.

11° **Parts of the Teeth.—Face and Flank.**—The face of a tooth is that part of the tooth curve extending beyond the pitch circle, as ca_1, ca_2, Fig. 267, and the flank is that part of the curve within the pitch circle, as cb_1, cb_2. If the wheel A is the driver and turning as shown, it will be seen that the flank of the driver acts upon the face of the follower during the approaching action, that is, while the teeth are sliding toward each other, and that during the receding action the face of the driver will drive the flank of the follower. The *acting flank* is the part of the flank which comes into contact with the face of the tooth of the other wheel, for it will be evident from Fig. 267 that the entire flank cannot come into contact. Thus the acting flank of the tooth on A is $dx = cy$, d being the first point on the flank of A to come into contact with the face on B.

12° **Point of Contact.—Path of Contact.**—Points of contact are points where the teeth touch each other, as f and g, Fig. 267, and the path of contact is a smooth curve drawn through the successive points of contact of a pair of teeth while in action, as the curve dce, Fig. 267. The path of contact is always limited by the addendum circles, as at d and e, and will be found to always pass through the pitch point of the wheels.

13° **Arcs and Angles of Action.—Arcs and Angles of Approach and Recess.**—In Fig. 268 let C_1 and D_1 be a pair of teeth in contact at

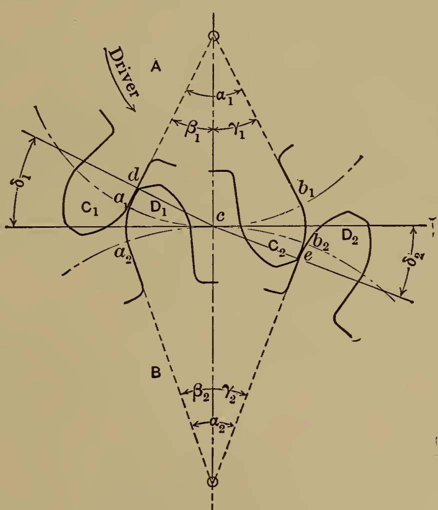


Fig. 268.

the point d where the contact begins, and let C_2 and D_2 be the same pair of teeth in the position where contact just ends at e; then the *arc of action* is the arc through which the pitch line of either wheel moves while a pair of teeth are in contact, as $a_1b_1 = a_2b_2$. The *angles of action* are the angles through which the wheels move while a pair of teeth are in action, and if the diameters are not alike, these angles will be inversely as the radii of the pitch circles, since the arcs which subtend them are equal. For A (Fig. 268) the angle of action is α_1, and for B, α_2.

The *arcs of approach* are $a_1c=a_2c$, the arcs through which the pitch lines move while the teeth are moving toward each other, which action ends when the points a_1 and a_2, the pitch points of the teeth, meet each other at c. Similarly the *arcs of recess* are $cb_1=cb_2$. The angles of approach and of recess for the wheel A are the angles subtended by the arcs of approach and of recess respectively, β_1 and γ_1. For the wheel B the angles of approach and of recess are β_2 and γ_2 respectively.

In order that one pair of teeth shall not cease their action until the next pair are in contact, the arc of action must be at least equal to the pitch, and in practice it should be considerably more if much force is to be transmitted, so that usually two pair of teeth, or more, are always in contact.

The approaching action is more injurious than the receding action, for in approach the friction between the teeth *adds* to the pressure on the bearings of the wheels, while in recess the reverse is the case.

14° **Line of Action or Line of Connection.—Obliquity of Action.**— The line of action, or line of connection, is a line normal to the tooth curves at their point of contact, as is always the case in pieces in sliding contact (see § 111). The component motions along this line will be the same for the two points in contact, as will also be the case for the force components transmitted. The *obliquity of action*, or *angle of obliquity*, is the angle which any line of action makes with the common tangent to the two pitch lines. Thus in Fig. 268 the teeth C_1 and D_1 in contact at d will have for the line of action the normal dc, and the angle of obliquity in that position will be δ_1. The maximum angle of *obliquity in approach* would be the angle of obliquity at the beginning of approach, which is δ_1 in the figure, and δ_2 would be the maximum angle of *obliquity in recess*, since ce is the line of action at the end of the path of contact.

The angle of obliquity should not be large, as the force required to overcome a given resistance would increase if the angle of obliquity increased, since the moment arm of the force along the line of connection would decrease in the driven wheel if the angle of obliquity increased, necessitating a greater pressure by the driving tooth.

174. **Gearing Classified.**—The following are the varieties of tooth wheels commonly met with in practice.

1° **Spur Gearing** (Fig. 269).—Here the axes of the wheels are parallel. If we suppose the number of teeth to be increased indefinitely, their size being at the same time correspondingly diminished, they will finally become mere lines, or elements of surfaces in contact, thus giving the figure at the right, which figure shows the pitch surfaces.

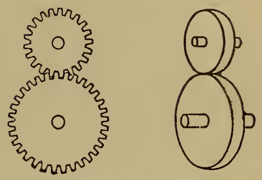


FIG. 269.

If the teeth of one of the wheels are replaced by pins, and the teeth of the other are made to work properly with the pins, we have what is called *pin gearing.*

Should the radius of one of the wheels be made infinite, the pitch line becomes a straight line, and we have a *rack.*

2° **Bevel Gearing** (Fig. 270).—Here the axes intersect, and the pitch surfaces are cones, having a common apex at the point of intersection of the axes. When the axes intersect at right angles and the wheels are equal, the gears are called *mitre-wheels.*

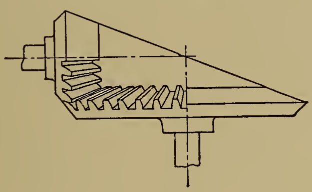


FIG. 270.

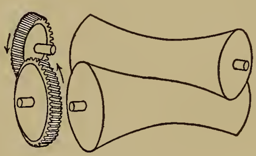


FIG. 271.

3° **Skew Gearing** (Fig. 271).—Here the axes lie in different planes, and the pitch surfaces are hyperboloids of revolution. This class of gearing is not very generally used, owing to the difficulty of forming the teeth, it being possible, in most cases, to make the connection by means of two sets of bevel gears.

In cases 1°, 2°, and 3° the teeth touch each other along straight lines, parallel to the axes in 1°, passing through the apexes of the pitch cones in 2°, and approximating in their general direction to the common element of the two hyperboloids in 3°.

4° **Twisted Gearing** (Fig. 272).—If we suppose one of a pair of engaging circular wheels, belonging to either of the first three classes, to be uniformly twisted on its axis, each successive transverse plane being rotated through a greater angle, the other wheel receiving a corresponding twist as shown, we shall have a case of twisted gearing.

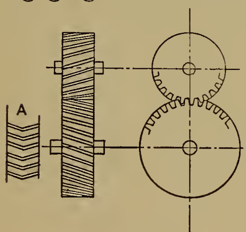


FIG. 272.

The wheels thus formed will gear together as well as before. The teeth have faces of a helicoidal nature, and by increasing the number of teeth indefinitely, helical lines or elements would result, giving the same pitch surface as before.

In the case shown, pressure will result along the axes of the gears. To neutralize this axial pressure, the twisting can be made to start at the central plane of the wheel, and proceed the same on each side, as shown at A.

5° **Screw Gearing** (Fig. 273).—Here the teeth also have a helicoidal form, as in twisted gearing, and reduce to helical lines; but these helices lie upon cylinders whose axes are in different planes, the pitch surfaces touching in a single point only. As illustrated by the "worm and wheel," it is the screw-like action alone of one wheel on the other which transmits the motion.

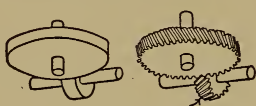


FIG. 273.

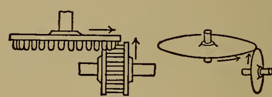


FIG. 274.

6° **Face Gearing** (Fig. 274) is not much used in modern machinery. The teeth generally consist of turned pins projecting from circular discs, but may be arranged on other surfaces than planes; the axes also may be inclined to each other.

In this case the teeth are circular in section, and touch only in one point; and when the number is increased indefinitely, two circles touching each other at their circumference will result. Face-wheels, then, have no pitch surfaces properly so called, but surfaces of some kind are required to support the teeth. This class of gearing was best adapted to wooden mill machinery, and at one time was used almost exclusively for that purpose.

175. Fundamental Law Governing the Shapes of Curves Suitable for Tooth Curves.—In Fig. 275 let A and B be two pieces in sliding contact at the point d, with nn as the common normal to the acting surfaces at d, and assume the piece B, turning R.H., to be driving the piece A. Draw o_1a and o_2b perpendicular to nn. The direction of motion of the point a in A, and also the direction of motion of b in B, are, in this position, along nn; therefore the l.v. of a must be equal to the l.v. of b, for if l.v. of a were greater than l.v. of b, the curves in contact at d would separate. This fact is also evident by noticing that nn is the *line of connection* between the sliding pieces in contact at d, and the components along nn of the l.v's of any points in either A or B situated on nn must be equal; a and b, which are, in the given position, moving along nn, must therefore have their l.v's equal.

$$\text{l.v. } a = \text{a.v. } A \times o_1a \quad \text{and} \quad \text{l.v. } b = \text{a.v. } B \times o_2b.$$

But
$$\text{l.v. } a = \text{l.v. } b;$$

$$\therefore \text{ a.v. } A : \text{a.v. } B = o_2b : o_1a.$$

But by construction the triangles o_1ac and o_2bc are similar, from which

$$o_2b : o_1a = o_2c : o_1c;$$
$$\therefore \text{ a.v. } A : \text{a.v. } B = o_2c : o_1c.$$

If now we draw the two circles shown dotted through the point c with the centres o_1 and o_2, and assume the circles to move in rolling contact, we should have

$$\text{a.v. } o_1 : \text{a.v. } o_2 = o_2c : o_1c.$$

Therefore the two pieces A and B, with their axes at o_1 and o_2 respectively, have an a.v. ratio, due to their sliding contact, exactly the same

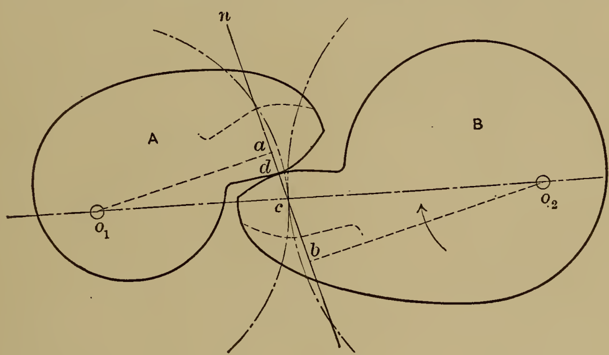


FIG. 275.

at this instant as that of two rolling cylinders on the same axes and in contact at c. Thus if a constant a.v. ratio is to be maintained, which is the special function of gearing, it is only necessary that *the tooth curves shall be so shaped that at any point of contact the common normal to the curves shall pass through the pitch point of the wheels.* For if, on moving the pieces A and B into some other position, and drawing the normal to the acting curves at the new point of contact, it were found that this normal passed through some other point than c on the line of centres, it could be proved that the a.v. ratio was the same as that of some other pair of rolling cylinders in contact at the point where the new normal crosses the line of centres.

This **Law**, that *the normal to the tooth curves at any point of contact must pass through the pitch point of the gears*, is fundamental to all types of gearing if constant a.v. ratio is to be obtained.

Fig. 276 shows a pinion and annular wheel with one pair of teeth in contact at d, nn being the common normal to the curves at d. o_1a and o_2b are drawn perpendicular to nn. Then, by the same reasoning as in Fig. 275,

$$\text{l.v. } a = \text{l.v. } b;$$
$$\text{l.v. } a = \text{a.v. } o_1 \times o_1a \quad \text{and} \quad \text{l.v. } b = \text{a.v. } o_2 \times o_2b;$$
$$\therefore \text{ a.v. } o_1 : \text{a.v. } o_2 = o_2b : o_1a = o_2c : o_1c;$$

or, the teeth in contact at d would give to the axes o_1 and o_2 the same a.v. ratio as that of the two cylinders in internal contact at c.

FIG. 276.

176. Rate of Sliding.—To determine the rate of sliding of one tooth upon another at any position it will be necessary to find the l.v. of the point of contact d in each of the teeth, and resolve these l.v's into their components along the common normal and the common tangent. In Fig. 276 let de represent the l.v. of d around o_1; the components of de along the common normal and tangent are df and dg respectively. The direction of the motion of d around o_2 is along the line dh. To find the magnitude of its l.v. we have df as its component along the common normal, since this normal is the line of connection between the two sliding surfaces, and components along the line of connection must be equal. This will give dh as the l.v. of d around o_2, and dk as its component along the common tangent. The rate of sliding will be found to be gk, equal to $dg+dk$, since the components along the tangent act in opposite directions.

177. General Problem.—Conjugate Curves.—Given the face or flank of a tooth of one of a pair of wheels, to find the flank or face of a tooth of the other. The solution of this problem depends on the fundamental law, § 175. In Fig. 277 let the flank and face of a tooth on A be given. If A is considered as the driver, points on the flank, as a and b, will be points of contact in the approaching action, and by the law they can properly be points of contact only when the normals to the flank at these points pass through the pitch point: therefore drawing

ac and bd normals to the flank from the points a and b respectively, and then turning A backward until the points c and d are at the pitch point, we find positions a_1 and b_1 which a and b respectively must occupy when they can be points of contact with the face of a tooth of the other wheel. The point a_1 must be a point on the desired face of a tooth on the wheel B when the pitch circles have been moved backward an arc equal to c_1c, that is, so that c is at the pitch point. To find this point when the teeth are in the original position, it is necessary to move the wheels forward, the wheel B carrying with it the point a_1 and the normal a_1c_1 until the point c_1 has moved through an arc c_1c_2 equal to c_1c; this will carry the point a_1 to a_2, and the normal a_1c_1 to a_2c_2. During this same forward motion the normal a_1c_1 moving with the wheel A will return to its original position ac.

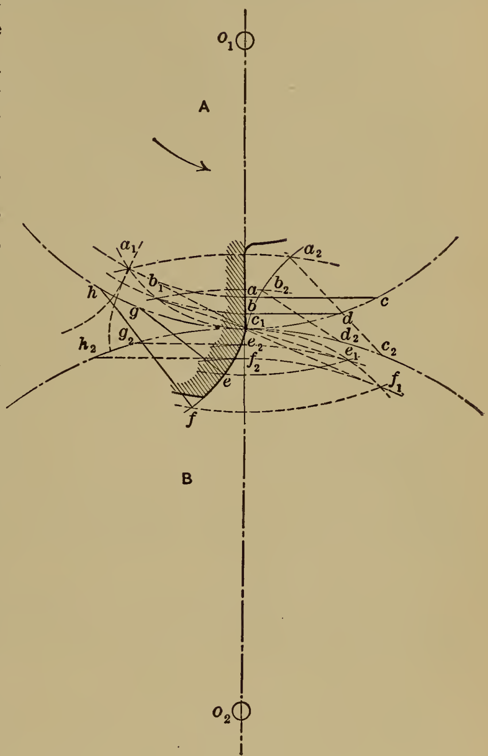


FIG. 277.

In a similar manner the point b_1, which can be a point of contact of the given flank with the desired face, is a point on this face when the pitch circles of the wheels are moved backward an arc equal to c_1d. Moving them forward the same distance, the point b_1 and normal b_1c_1, moving with the wheel B, will be found at b_2 and b_2d_2. This process may be continued for as many points as may be needed to give a smooth curve. The curve drawn through the points $a_2b_2c_1$ will be the required face.

A similar process gives the flank of the tooth on the wheel B which will work properly with the given face. The normals taken in the figure are eg and fh, the positions of e and f when they can be points of contact being e_1 and f_1; and the points on the required flank when in the original position are e_2 and f_2.

A smooth curve passed through the points of contact $a_1b_1c_1e_1f_1$ will be the path of contact, the beginning and end of which will be determined by the addendum circles of B and of A respectively.

Another method of solving the above problem is shown in Fig. 278, where o_1 and o_2 are a pair of plates whose edges are shaped to arcs of the given pitch circles AA_1 and BB_1, due allowance being made for a thin strip of metal, gh, connecting the plates, to insure no slipping of their edges on each other.

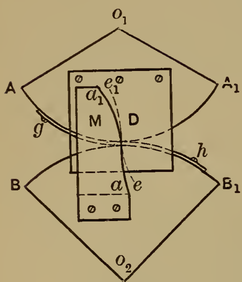


FIG. 278.

Attach to o_2 a thin piece of sheet metal, M, the edge of which is shaped to the given curve aa_1; and to o_1 a piece of paper, D, the piece M being elevated above o_1 to allow space for the free movement of D. Now roll the plates together, keeping the metallic strip gh in tension, and, with a fine marking-point, trace upon the paper D, for a sufficient number of positions, the outline of the curve aa_1. A curve just touching all the successive outlines on D, as ee_1, is the corresponding tooth curve for o_1.

Conjugate Curves.—Any two curves so related that, by their sliding contact, motion will be transmitted with a constant a.v. ratio, as in rolling cylinders, are called conjugate curves. The curves, in this case, are often called *odontoids*.

178. After finding the proper shapes for the tooth curves and knowing the pitch, backlash, addendum, and clearance, the teeth may be drawn as in Fig. 279. A convenient method of laying off the pitch ab_1

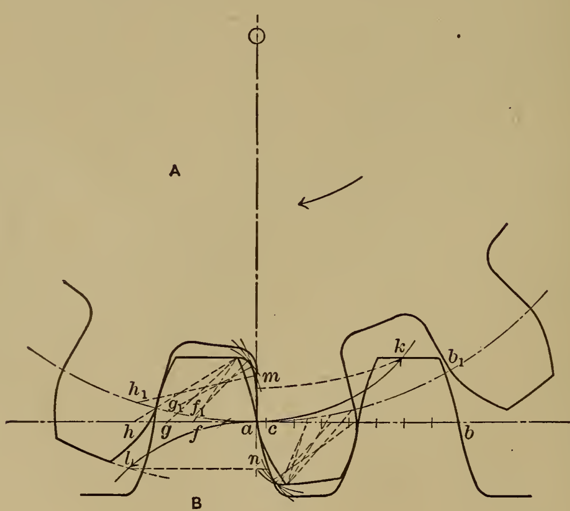


FIG. 279.

on the pitch circle is indicated in the figure; let ab be equal to the pitch, laid off from the pitch point, on the tangent; starting from b space off toward a equal divisions sufficiently small so that when spaced back on the pitch circle the difference between chord and arc may be neglected; one of these divisions, as c, will come sufficiently near to a that it may be considered to be on the pitch circle as well as on the tangent. From this point c space back on the pitch circle the same number of divisions giving the point b_1, and ab_1 will be very nearly equal to ab. This method of spacing may be used to construct the tooth curves in the two systems of gearing to be discussed in this chapter.

If the flanks are extended until they join the root line, a very weak tooth would often result; to avoid this, a *fillet* is used which is limited by the arc of a circle connecting the root line with the flank, and lying outside the actual path of the end of the face of the other wheel. This actual path of the end of the face is called the *true clearing curve*, the construction of which is taken up later in § 179, 5°.

Fig. 279 shows a rack and pinion in gear. If the pinion drives, the path of contact will be *kal*. The ends of the acting flanks are also indicated at m and n.

179. The Drawing of Rolled Curves.—Any curve described by a point carried by one line which rolls upon another may be called a *rolled curve.*

The line which carries the tracing-point is called the *generatrix, describing line*, or *describing circle*, while the one in contact with which it rolls is called the *directrix* or *base line;* either line may be straight, or both may be curved.

1° The Cycloid.—This curve is traced by a point in the circumference of a circle which rolls upon a straight line as a directrix, as the curve aa_2a_3 (Fig. 280) traced by the point a in the circumference of the circle which rolls on the straight tangent line ae.

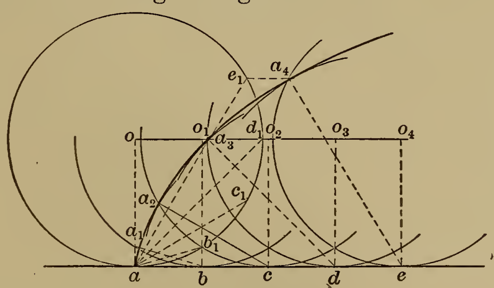


FIG. 280.

To construct the curve, lay off on the circumference of the rolling circle a series of equal divisions as shown, and on the tangent line a similar series of divisions such that the divisions on the arc are of the same length as those on the straight line, and that the corresponding points will roll into each other. This division may be made by carefully using spacing-dividers, setting them so fine that the difference between a chord and its arc is inappreciable, or by rectifying by calculation a portion of the arc of the rolling circle and dividing both into the same number of equal parts, as has been done in the figure. The centre of the rolling circle moves on a line oo_4, parallel to the tangent line, and its position can be found for each division by erecting perpendiculars through the divisions on the directrix. To find a point, as a_4, corresponding to the fourth division: Here the centre of the rolling circle is at o_4, and the circle is tangent to the directrix at e (perpendicularly under o_4), the point e_1 on the circle having rolled to the point e on the directrix. Hence striking an arc from o_4 with a radius o_4e, equal that of the describing circle, and intersecting this with an arc struck from the tangent point e with a radius equal to the cord ae_1, we obtain the point a_4. In the same way other points may be found. The instantaneous centre of the rolling circle is at the tangent point e, and a_4e is the normal, its length representing the radius of curvature of the cycloid at the point a_4. In fact the cycloid might be drawn by making it tangent to a series of arcs struck as above from the divisions of the directrix.

Another method of locating the point a_4 is to draw through the point e_1 on the circle a line e_1a_4 parallel to the directrix and noting where it intersects the corresponding arc of the rolling circle.

2° **The Epicycloid.**—This curve is traced by a point in the circum-

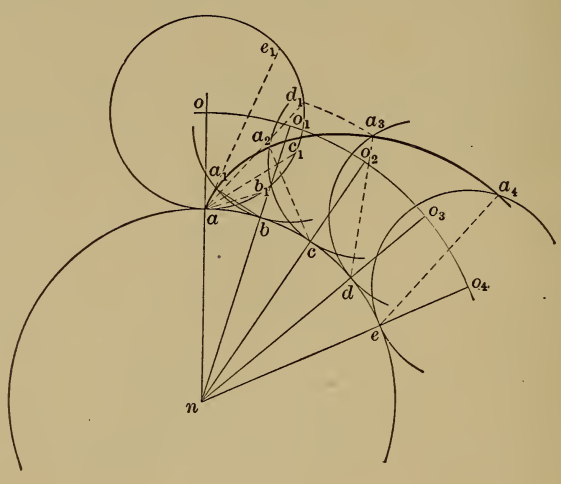


FIG. 281.

ference of a circle which rolls on the *outside* of another circle as a directrix.

The construction, Fig. 281, is similar to that of the cycloid, and the figure is lettered to correspond; the centre of the describing circle moves on the arc oo_1o_4; the successive points of tangency are b, c, d, and the points b_1, c_1, d_1, roll to these points, the point a moving to the points a_1, a_2, a_3, successively.

If the describing circle is larger and rolls *internally* upon the directing circle, an epicycloid will still be rolled. Fig. 282 shows the epicycloid aa_1a_3 so rolled, the large circle ae_2f_2 rolling internally on the circle aef. If, as in Fig. 282, the diameter of the describing circle ae_2f_2 rolling internally is equal to the sum of the diameters of the directing

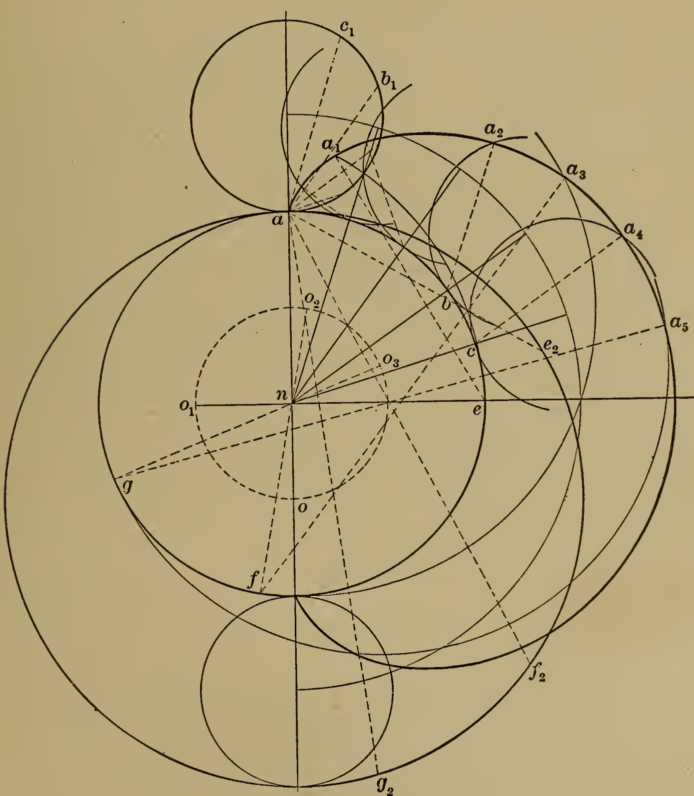


FIG. 282.

circle and the small describing circle ab_1c_1, the two describing circles will trace exactly the same epicycloid, as shown by the points a_2, a_4, found by the small circle, and a_1, a_3, a_5, found by the large circle. This *double generation of the epicycloid* will be referred to in the discussion of the teeth of annular wheels using these curves.

3° **The Hypocycloid.**—This curve is traced by a point in the circumference of a circle rolling *inside* of another circle.

Fig. 283 shows the construction of a hypocycloid, the letters corresponding to those in the previous curves; the small circle ab_1c_1 rolling to the right inside of the large circle, $abcd$, traces the curve $aa_2a_3a_5$. If another circle, ae_2f_2, whose diameter is equal to the difference between the diameters of the directing circle and the describing circle ab_1c_1, as in Fig. 283, is rolled inside the directing circle, exactly the same hypocycloid will be traced as that traced by the circle ab_1c_1, and these hypocycloids

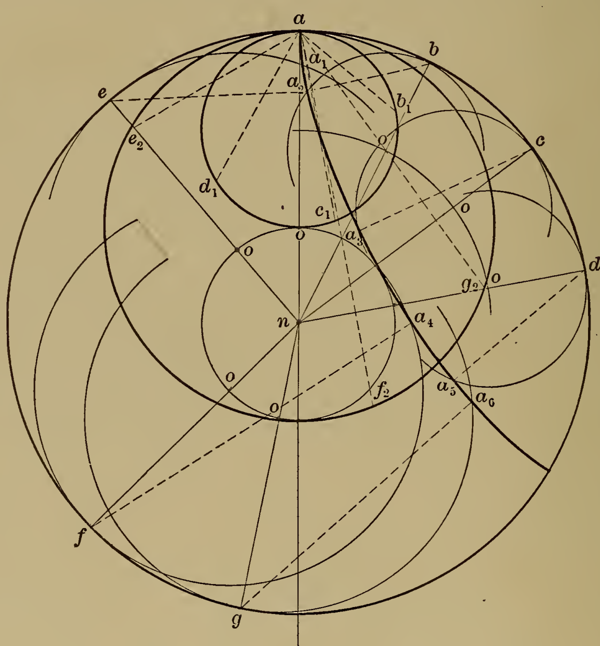


FIG. 283.

will coincide as shown in the figure, provided the describing circles roll in *opposite* directions; thus in the figure the circle ae_2f_2 rolls to the left and gives the points a_1, a_4, a_6, while the circle ab_1c_1 rolls to the right and gives the points a_2, a_3, a_5. This *double generation of the hypocycloid* will be referred to in discussing annular wheels using these curves.

When the diameter of the rolling circle is one-half that of the circle in which it rolls, the hypocycloid becomes a diameter of the directing circle.

4° **The Involute of the Circle.**—This curve may be considered as generated by a point in a straight line which rolls upon a circle. It may also be regarded as generated by unwinding an inextensible fine thread from a cylinder; the thread being always taut and always tangent to the cylinder, its length is thus equal to the arc from which it was unwound.

To find one point, as a_2, in the involute (Fig. 284): Draw the radius oc, and perpendicular to it draw the tangent ca_2. Make the tangent ca_2 equal in length to the arc ac, and a_2 is a point on the involute. The radius of curvature of the involute at a_2 is a_2c. It is generally most convenient to lay off equal divisions on the directing circle, and on some straight line a similar series of divisions equal to those of the circle. Then draw tangents at the divisions of the directrix, and on these lay off distances corresponding to the different arcs, the distances being taken from the divided straight line. From the construction it can be seen that the normal at any part of the involute is always tangent to the directing circle. In gearing where involute curves are used for the teeth, the directing circle is called the *base circle*.

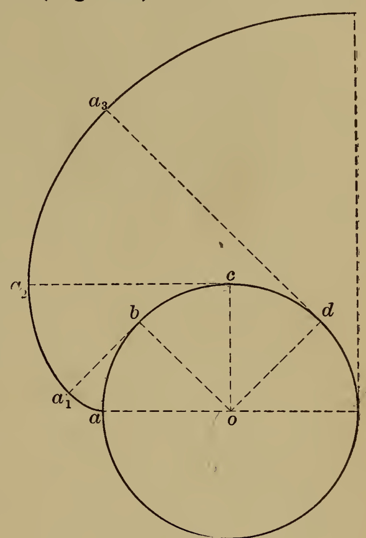


FIG. 284.

5° **Epitrochoid.**—The term epitrochoid is used in a general way to name those curves traced by rolling one circle upon another when the marking-point is not situated upon the circumference of the rolling circle. When the marking-point is situated outside of the circumference of the rolling circle, the looped curve traced is known as a *curtate* epitrochoid; when it is situated inside, the curve is known as a *prolate* epitrochoid.

The above method of drawing the cycloidal curves may here be used; but the tracing-point, now not being on the circumference of the rolling circle, must be located in its different positions by a method of triangulation from points whose positions are known.

In Fig. 285 the circle A with its centre at o rolls on the line B. The point a, carried by A, will trace an epitrochoid. To draw the curve first lay off equal spaces on A and B, $pb_1 = pb$, $pc_1 = pc$, such that b rolls to b_1, c to c_1, etc., and find the corresponding positions of the centre of A, o_1o_2, etc. To locate one point on the curve, as a_3, when d rolls to d_1, we know that a is always situated a distance oa from the centre of the rolling circle, and that when d rolls to d_1 the centre is at o_3; therefore a must be somewhere on an arc struck from o_3 with a radius $o_3a_3 = oa$; also a is distant da from d, and when d rolls to d_1 the point a must be found at a_3 on the arc about o_3 distant $d_1a_3 = da$ from d_1. As d_1 is the instantaneous centre, a is moving about it for the instant. In the same manner other points, as a_1, a_2, may be found.

If the arcs drawn from b_1, c_1, d_1, etc., are extended, it will be found that the epitrochoid is tangent internally to them, and the curve may be drawn without finding the points a_1, a_2, etc., as follows: After laying out the points which roll into each other, as b, b_1, etc., with b_1 as a centre and ab as a radius draw an arc a_1; with c_1 as a centre and ac as radius draw arc a_2, etc.; then draw the curve E internally tangent to these arcs.

The *clearing curve* defined in § 178 is an epitrochoid, and is constructed

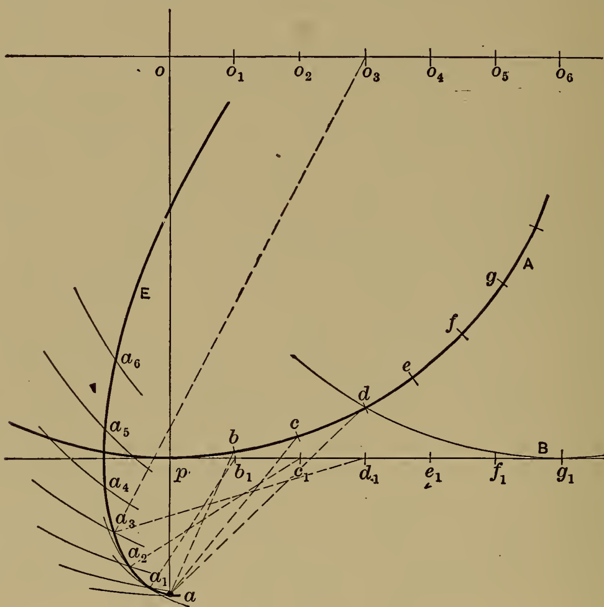


FIG. 285.

by the method just described, as will be evident on referring to Fig. 279, where the construction is clearly shown.

180. Spur Gearing, Cycloidal System.—Generation of the Tooth Outline.—In Fig. 286 let o_1 and o_2 be the centres of the two wheels A and B, their pitch circles being in contact at the point a. Let the smaller circles C and D, with centres at p_1 and p_2, be placed so that they are tangent to the pitch circles at a. Assume the centres of these four circles to be fixed and that they turn in rolling contact; then if the point a on the circle A moves to a_1, a_2, a_3, the same point on B will move to b_1, b_2, b_3, and on C to c_1, c_2, c_3. Now if the point a on the circle C carries a marking-point, in its motion to c_1 it will have traced from the circle A the hypocycloid a_1c_1, and at the same time from the circle B the epicycloid b_1c_1. This can be seen to be true if the circles A and B are now fixed; and if C rolls in A, the point c_1 will roll to a_1, tracing the hypocycloid c_1a_1; while if C rolls on B, c_1 will trace the epicycloid c_1b_1. These two curves in contact at c_1 fulfil the fundamental law for tooth curves, that the normal to the two curves at the point c_1 must pass through a; as C starts to roll on either A or B, a is the instantaneous centre of C and therefore the direction of motion of c_1, and so the tangent to the curves at c_1 are perpendicular to the radius ac_1 to the instantaneous centre. Similarly, if the original motion of the circles had been to a_2, b_2, c_2, the same curves would be generated, only they would be longer and in contact at c_2. If the hypocycloid c_2a_2 is taken for the flank of a

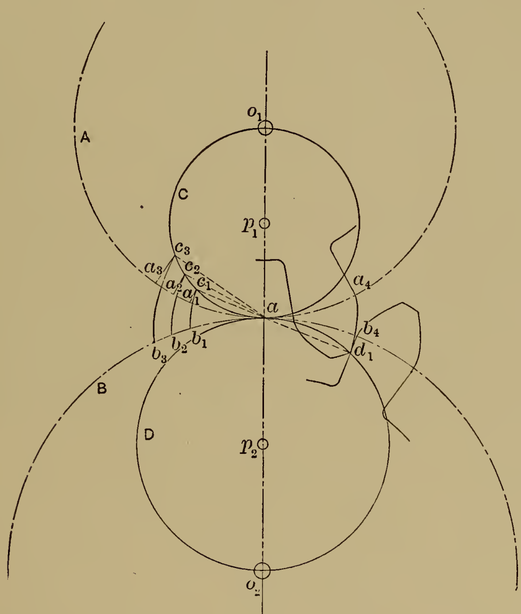


FIG. 286.

tooth on A, and the epicycloid c_2b_2 for the face of a tooth on B, and if c_2a_2 drives c_2b_2 toward a, it is evident that these two curves by their sliding action, as they approach the line of centres, will give the same type of motion to the circles as the circles had in generating the curves, which was pure rolling contact. Therefore the two cycloidal curves rolled simultaneously by the describing circle C will cause by their sliding contact the same a.v. ratio of A and B as would be obtained by A and B moving with pure rolling contact.

If now the circles A, B, and D are rolled in the *opposite* direction to that taken for A, B, and C, and if the point a moves to a_4, b_4, and d_1 on the respective circles, the point a on D while moving to d_1 will trace from A the epicycloid a_4d_1, and from B the hypocycloid b_4d_1. The curve

a_4d_1 may be the face of a tooth on A, and b_4d_1 the flank of a tooth on B, the normal d_1a to the two curves in contact at d_1 passing through a. The flank and face for the teeth on A and B, respectively, which were previously found have been added to the face and flank just found, giving the complete outlines, in contact at d_1.

If now the wheel B is turned L.H., the tooth shown on it will drive the tooth on A, giving a constant a.v. ratio between A and B until the face of the tooth on B has come to the end of its action with the flank which it is driving, at about the point c_2.

The following facts will be evident from the foregoing discussion: in the cycloidal system of gearing, *the flank and face which are to act upon each other must be generated by the same describing circle*, but the describing circles for the face and flank of the teeth of one wheel need not be alike. *The path of contact is always on the describing circles;* in Fig. 286 it is along the line d_1ac_1. See also § 189.

181. Interchangeable Wheels.—A set of wheels any two of which will gear together are called interchangeable wheels. For these the same describing circle must be used in generating all the faces and flanks. The size of the describing circle depends on the properties of the hypocycloid, which curve forms the flanks of the teeth (excepting in an annular wheel). If the diameter of the describing circle is half that of the pitch circle, the flanks will be radial (Fig. 287, A), which

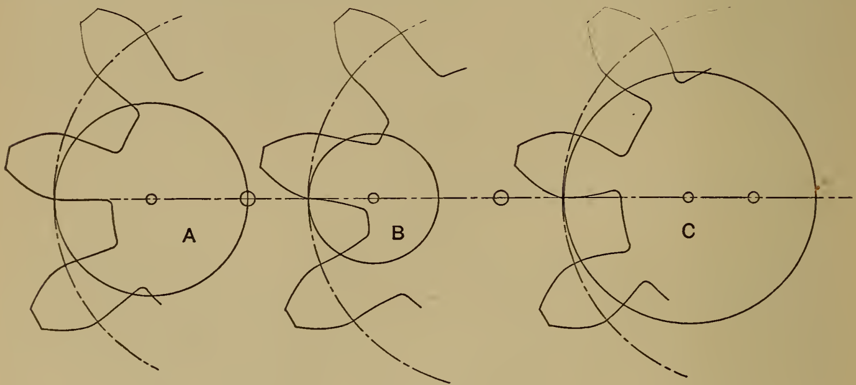


Fig. 287.

gives a comparatively weak tooth at the root. If the describing circle is made smaller, the hypocycloid curves away from the radius (Fig. 287, B) and will give a strong form of tooth; but if the describing circle is larger, the hypocycloid will curve the other way, passing inside the radial lines (Fig. 287, C) and giving a still weaker form of tooth, and a form of tooth which may be impossible to shape with a milling-cutter.

From the above the practical conclusion would appear to be that the diameter of the describing circle should not be more than one-half that of the pitch circle of the smallest wheel of the set. It will be found, however, that when the diameter of the describing circle is taken five-eighths the diameter of the pitch circle, the curvature of the flanks will not be so great, with the ordinary proportions of height to thickness of teeth, that the spaces are any wider at the bottom than at the pitch circle: this being the case, the teeth can be shaped by a milling-cutter.

In one set of wheels in common use the diameter of the describing circle is taken such that it will give radial flanks on a 15-tooth pinion, or five-eighths that of a 12-tooth pinion, the smallest wheel of the set. This describing circle has been used with excellent results.

As an example, given an interchangeable set of cycloidal gears, 2-P., radial flanks on a 15-tooth pinion; a pinion having 24 teeth is to drive one having 30 teeth. The diameter of a 2-P., 15-tooth pinion would be $7\frac{1}{2}$ inches; to give radial flanks on this pinion the diameter of the describ-

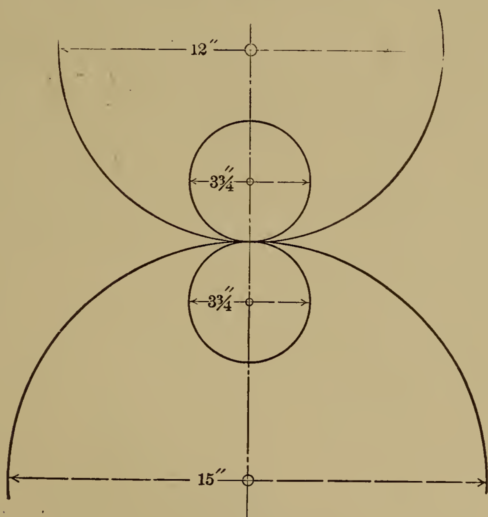


Fig. 288.

ing circle would be $3\frac{3}{4}$ inches. This is the diameter of the describing circle for all the faces and flanks for any gear of the set. The 24-tooth pinion will have a diameter of 12″, and the 30-tooth will have 15″ diameter. This will give the diagram in Fig. 288, ready for the rolling of the tooth outlines.

182. To draw the teeth for a pair of cycloidal wheels, and to determine the path of contact.—In Fig. 289, given the pitch circles A and B and the describing circles C and D, C to roll the faces for B and the flanks for A, while D is to roll the faces for A and the flanks for B. These curves may be rolled at any convenient place. In the figure, if the wheel A is to be the driver and is to turn as shown, any point, as b, on A may be chosen, and a point a on B at a distance from the pitch point

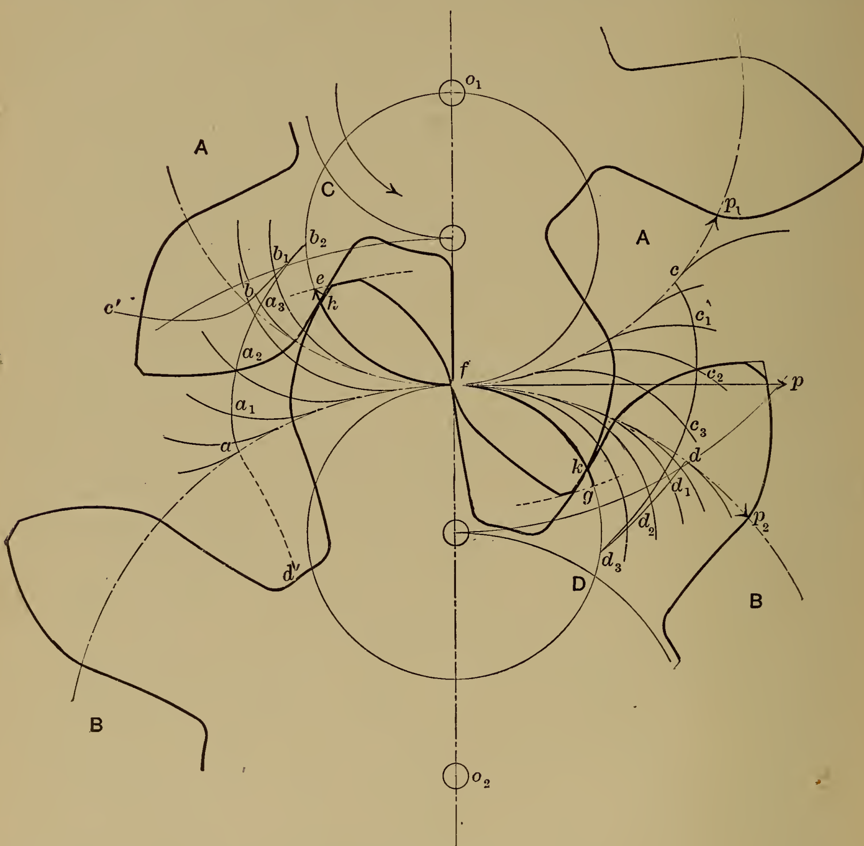


Fig. 289.

$af = bf$. The epicycloid and hypocycloid rolled from a and b respectively, and shown in contact at b_2, would be suitable for the faces of the teeth on B and the flanks of the teeth on A respectively, and could be in action during approach. The curves may be rolled as indicated by the light lines. The method used to roll these curves is shown in Fig. 290, where the circle C is tracing a hypocycloid on A from the point o. Assume the circle C to start tangent to A at o and to roll as shown, drawing it in as many positions as may be desired to obtain a smooth curve, and these positions do not need to be equidistant; thus in the figure the centre of C is at b, c, and d for the three positions used. Since the circle C rolls on A, the distance measured on A from o to a tangent point of C and A is equal to the distance measured on C from that tangent point to the hypocycloid. The method of spacing off these equal arcs for the successive positions is the same as described in § 178, Fig. 279, for laying off the pitch on the pitch circle, and is clearly indicated in Fig. 290.

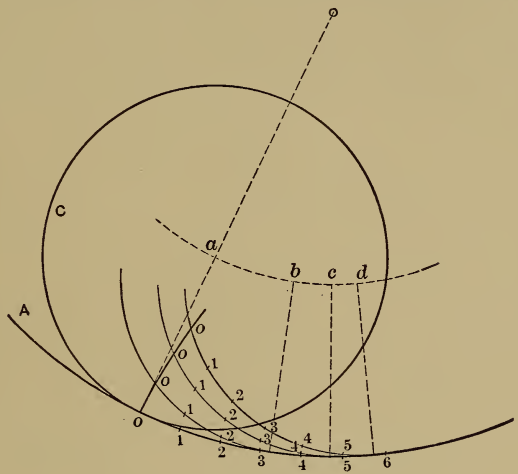


FIG. 290.

Returning to Fig. 289, the circle D is to roll the faces for the teeth on A and the flanks for the teeth on B. These curves may also be rolled from any convenient points, as c and d *equidistant from f.* The face thus found from A may be traced and then transferred to the flank already found for the teeth on A at the point b, giving the curve b_2bc', the entire acting side of a tooth on A. Similarly by transferring the flank dd_3 to the point a we have b_2ad', the shape of the teeth for the wheel B. It will be seen that the face on A could have been rolled from b as well as from c, so that the entire tooth curve could be rolled from b, and similarly the other tooth curve could have been rolled from the point a. After finding the tooth curves, and knowing the addendum, clearance, and backlash, the teeth may be drawn. In Fig. 289 the teeth are drawn without backlash, and in contact on their acting surfaces at h and k. The *path of contact* is efg on the describing circles and is limited by the addendum circles.

183. Limits of the Path of Contact.—Possibility of any desired Action.—If, in Fig. 289, the teeth of either wheel are made longer, the path of contact and arc of action are increased; the extreme limit of the path of contact would therefore be when the teeth become pointed.

It is often desirable to find whether a desired arc of action in approach or in recess may be obtained before rolling the tooth curves. Given the pinion A driving the rack B as shown in Fig. 291; to determine if an arc of approach equal to ab is possible. The path of contact must then begin at c, where the arc ac is equal to ab. The *face* of the *rack's tooth* must be long enough to reach from b to c, and this depends on the thickness of the tooth measured on the pitch line, since the non-acting side of the tooth must not cause the tooth to become pointed before the point c is reached. To see if this is possible without the tooth curves, draw a line from c parallel to the line of centres (in general this line is drawn to the centre of the wheel; the rack's centre being at infinity gives

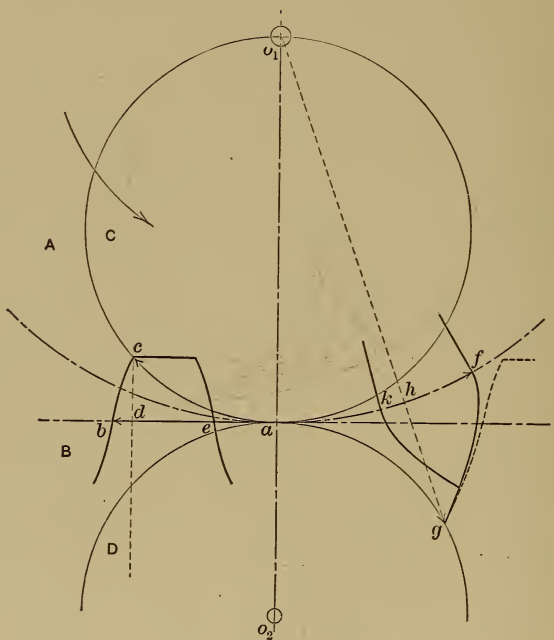


Fig. 291.

the line parallel to the line of centres), and note the point d where this line crosses the pitch line of the rack. If bd were just equal to one-half the thickness of the tooth, the tooth would be pointed at c, and the desired arc of approach would be just possible; if bd were *less* than one-half the thickness of the tooth, the tooth would not become pointed until some point beyond c was reached, so that the action would be possible and the teeth not pointed, as shown by the figure.

If it is desired to have the arc of recess equal to the arc af, then the path of contact must go to g, and the face of the pinion must remain in contact with the flank of the rack until that point is reached, or the face must be long enough to reach from f to g. Drawing a line from g to the centre of the pinion A, we find that the distance fh is greater than one-half of fk, which is taken as the thickness of the tooth; therefore the desired arc of recess is not possible even with pointed teeth.

184. Annular Wheels.—Fig. 292 shows a pinion A driving an annular wheel B, the describing circle C generating the flanks of A and the faces of B, which in an annular wheel lie inside the pitch circle, while D generates the faces of A and the flanks of B. The describing circle C is called the *interior describing circle*, and D is called the *exterior describing circle*. The method of rolling the tooth curves, and the action of the teeth, are the same as in the case of two external wheels, the path of contact being

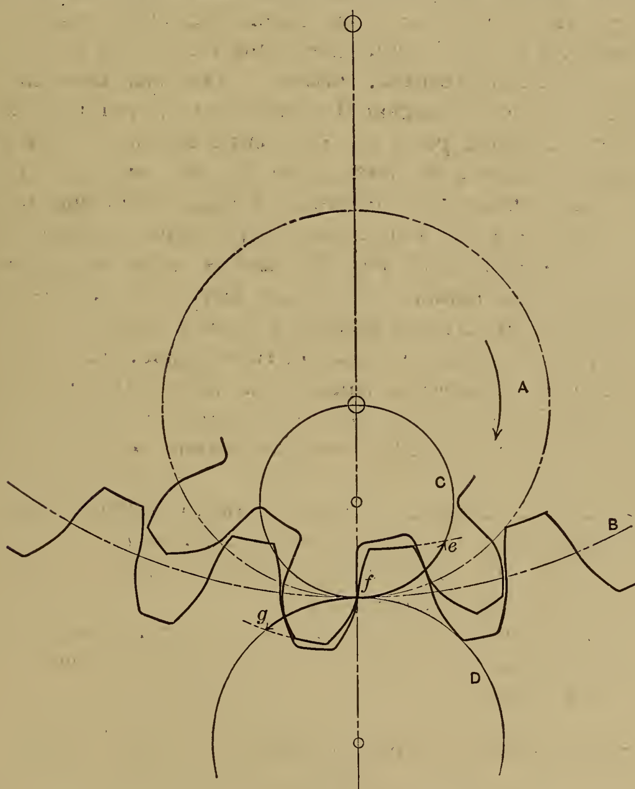


FIG. 292.

in this case efg when the pinion turns R.H. If these wheels were of an interchangeable set, the describing circles would be alike and found as explained in § 181, and the annular would then gear with any wheel of the set excepting for a limitation which is discussed in the following paragraph.

185. Limitation in the Use of an Annular Wheel of the Cycloidal System.—Referring to Fig. 292, it will be evident that, if the pinion drives, the *faces* of the pinion and annular will tend to be rather near each other during recess (during approach also on the non-acting side of the teeth). The usual conditions are such that the faces do not touch; but the conditions may be such that the faces will touch each other without interference, for a certain arc of recess; or, finally, the conditions may be such that the faces would interfere, which would make the wheels impossible.

To determine whether a given case is possible it is necessary to refer to the double generation of the epicycloid and of the hypocycloid, § 179, 2° and 3°. The acting face of the pinion, Fig. 292, is rolled by the exterior describing circle D, while the acting face of the annular is generated by the interior describing circle C. Two such faces are shown in Fig. 293 as they would appear if rolled from the points g and h, equidistant from the pitch point k. The acting face of A is an epicycloid, and is made by rolling the circle D to the *right* on A; in § 179, 2°, it was seen that a circle whose diameter is equal to the sum of the diameters of A and B would roll the same epicycloid if rolled in the same direction. This circle is E, Fig. 293, and is called the *intermediate describing circle of the pinion.* The acting face of the annular is a hypocycloid rolled by the interior describing circle C rolling to the *left* inside of B; in § 179, 3°, it was seen that the same hypocycloid would be rolled by a circle whose diameter is equal to the difference between the diameters of B and C, *provided it is rolled in the opposite direction.* This circle is F, Fig. 293, and is called the *intermediate describing circle of the annular.*

If now the four circles A, B, E, and F turn in rolling contact, through arcs each equal to kg, the point k will be found at g, h, m, and n on the respective circles, the point k on E having rolled the epicycloid gm, while k on F rolls the hypocycloid hn.

To determine whether these faces do or do not touch or conflict, assume that the given conditions gave the circles E and F coincident as in Fig. 294, where

$$\text{diam. } A+\text{diam. } D=\text{diam. } E=\text{diam. } B-\text{diam. } C=\text{diam. } F.$$

Here if the three circles A, B, and (EF) turn in rolling contact, the point k moving to g on A will move to h on B and to (mn) on the common intermediate circle. This means that the common intermediate circle could simultaneously generate the two faces; therefore the two faces are in perfect contact on the intermediate circle. This contact would continue until the addendum circle of one of the wheels crosses the intermediate circle, the addendum circle crossing first necessarily limiting the path of contact.

The above may be stated as follows: *If the intermediate describing* *circles of the pinion and annular coincide, the faces will be in contact in recess, if the pinion drives, in addition to the regular path of contact.*

If in Fig. 294 the exterior describing circle, for example, should be made smaller, as in Fig. 295, then the intermediate of the pinion would

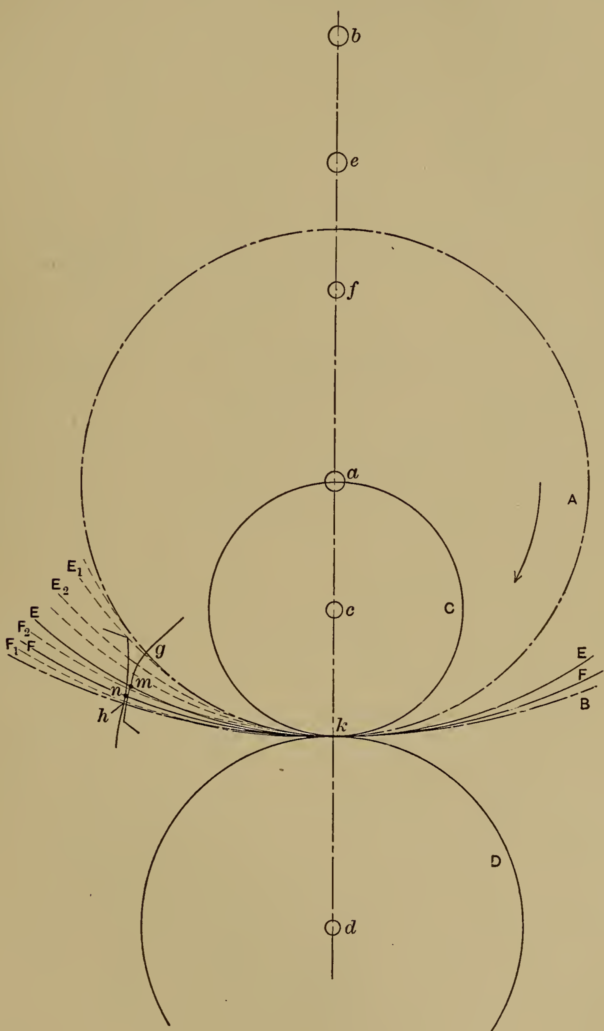


FIG. 293.

be smaller than that of the annular; but if the exterior describing circle is smaller, the face *gm* will have a greater curvature and will evidently curve away from the face *hn*, so that no contact between the faces can occur, as is shown in Fig. 295. Here no additional path of contact occurs, and it is evident, if the arcs *kg, km, kn,* and *kh* are equal as they must be, if the circles move in rolling contact, that the smaller *D* be-

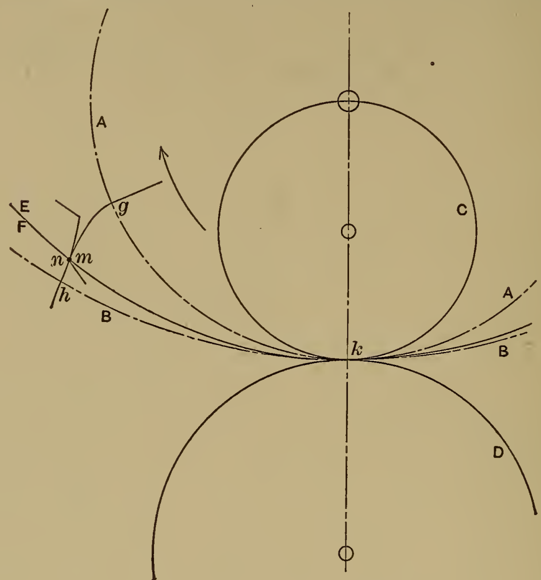


Fig. 294.

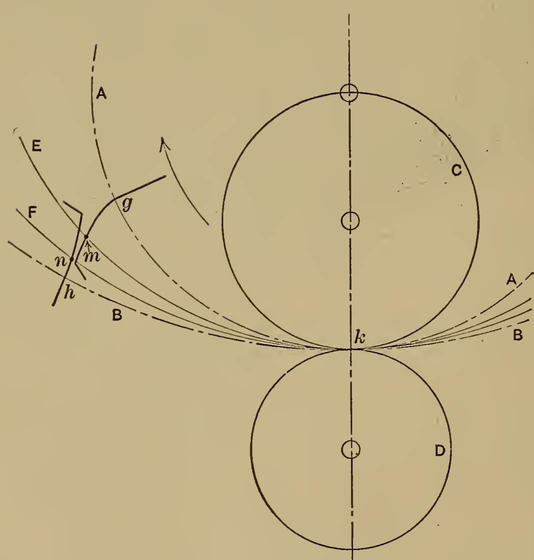


Fig. 295.

comes (and consequently *E*) the greater will be the space between the faces.

This may be stated as follows: *If the intermediate describing circle of the pinion is smaller than that of the annular, the faces do not touch, and the action is in all respects similar to the cases of external wheels.*

In Fig. 296 the exterior describing circle D is made larger than it is

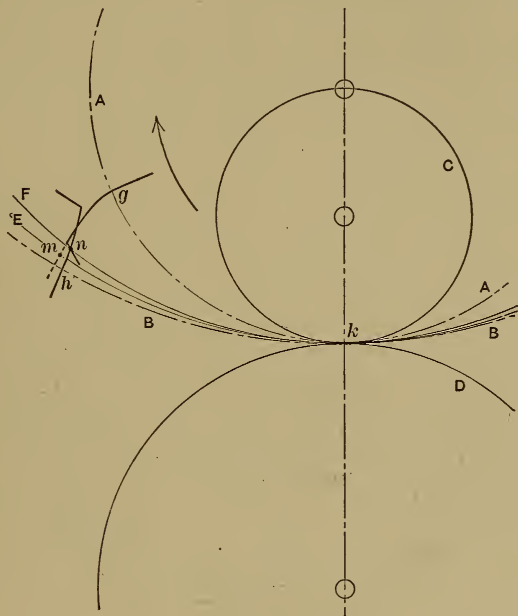


Fig. 296.

in Fig. 294, so that the intermediate E of the pinion is larger than F, that of the annular. Making the circle D larger would give the face of the pinion less curvature, which would cause the curve gm to cross the curve hn, giving an impossible case. Therefore, *if the intermediate of the pinion is greater than that of the annular, the action is impossible.*

186. To find the smallest annular to gear with a given pinion.—This will be most clearly shown by solving a problem. Let the data be as follows: Cycloidal, interchangeable gears, 8-P., radial flanks on a 12-tooth pinion; find the smallest annular which a 20-tooth pinion can drive, and show the path of contact if the pinion turns R.H. The successive steps are shown in Fig. 297. The exterior and interior describing circles are $\frac{3}{4}''$ diameter, the gears being 8-P., with radial flanks on a 12-tooth pinion. The diameter of the 20-tooth pinion is $\frac{20}{8}=2\frac{1}{2}''$; and the diameter of the intermediate of the pinion will be $2\frac{1}{2}+\frac{3}{4}=3\frac{1}{4}''$. The smallest annular will be of such size that its intermediate circle will coincide with that of the pinion; but the diameter of the intermediate of the annular is equal to the difference between the diameters of the annular and of the interior describing circle, or

$$\text{diam. of annular} - \tfrac{3}{4}'' = 3\tfrac{1}{4}'';$$
$$\therefore \text{ diam. of annular} = 3\tfrac{1}{4} + \tfrac{3}{4} = 4'',$$

and the smallest annular will have 32 teeth.

The path of contact, limited by the addendum circles drawn in Fig.

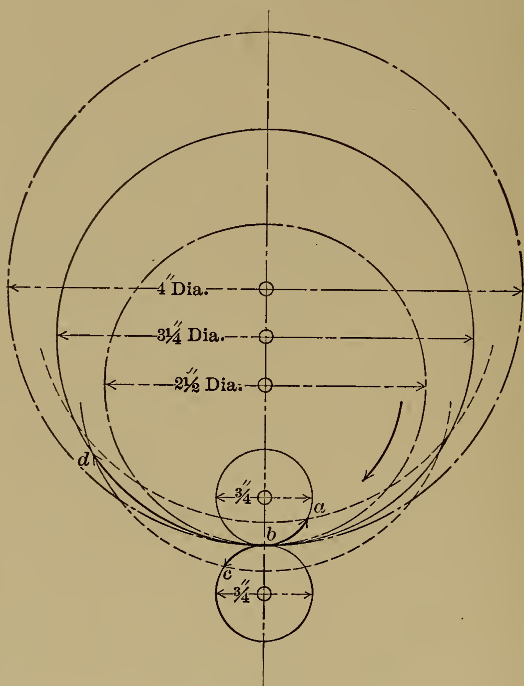


Fig. 297.

297, is *abc*, the usual path, and in addition the path *bd* on the common intermediate.

187. **Low-numbered Pinions, Cycloidal System.**—The obliquity of action in cycloidal gears is constantly varying; it diminishes during the approach, becoming zero at the pitch point, and then increases during the recess. For wheels doing heavy work it has been found by experience that the maximum obliquity should not in general exceed 30°, giving a mean of 15°. When more than one pair of teeth are in contact, a high maximum is less objectionable.

As the number of teeth in a wheel decreases, they necessarily become longer to secure the proper path of contact, and both the obliquity of action and the sliding increase. From the preceding considerations the practical rule is deduced that, for millwork and general machinery, no pinion of less than twelve teeth should be used if it is possible to avoid it.

It often becomes necessary, however, to use wheels having less than twelve teeth, in light-running mechanism, such as clockwork. In such cases a greater obliquity may be admissible, and for light work the flank-describing circle may be made large.

Let it be required to determine the possibility of using two equal pinions, having six teeth, with radial flanks, the arcs of approach and recess each equal to one-half the pitch, and to find the maximum angle of obliquity. Fig. 298 is the diagram for two such gears. The path of

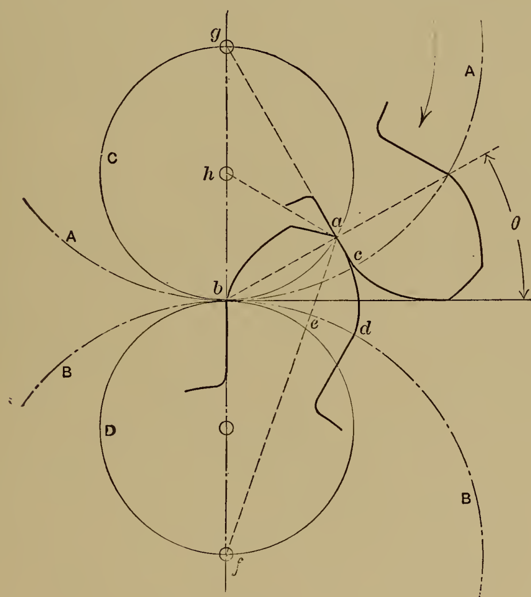


FIG. 298.

contact is to begin at a, the arcs ab, cb, and db each being equal to one-half the pitch; then the face of the pinion B must be long enough to be in contact with the flank of A at a. Drawing the line aef from a to the centre of B, we find that the distance de is less than one-half the thickness of the tooth, and that the approach is possible. Since the pinions are alike, the recess is also possible. The maximum angle of obliquity in approach is the angle θ, and this may be found in degrees as follows. The arc bc on the pitch circle A subtends an angle bgc equal to one-half the pitch angle, the arc of approach being equal to one-half the pitch; in this case the angle bgc is 30°. The arc ab on the describing circle C is equal to bc and therefore subtends an angle bha, which is to the angle bgc inversely as the radii of the respective circles. In this case these radii are as 2 to 1, making the angle *bha* equal to 60°. (It is important to notice that the line *gc* does not pass through the point *a* excepting in the single case of a radial flank gear.) The angle θ between the tangent and the chord *ab* will always be one-half the angle *ahb* subtended by the arc *ab*. This gives the angle of obliquity 30°. Therefore we find that two pinions with six teeth and radial flanks will work with arcs of approach and recess each equal to one-half the pitch and with a maximum angle of obliquity of 30°. By allowing a greater angle of obliquity the teeth may be made a little longer and so give an arc of action greater than the pitch, which should be the case in practice.

Two pinions with five teeth each will work with describing circles having diameters three-fifths the diameter of the pitch circles, and arcs of approach and recess each equal to one-half the pitch, as shown by Fig. 299, the path of contact beginning at *a*, the arcs *ab*, *cb*, and *db* each

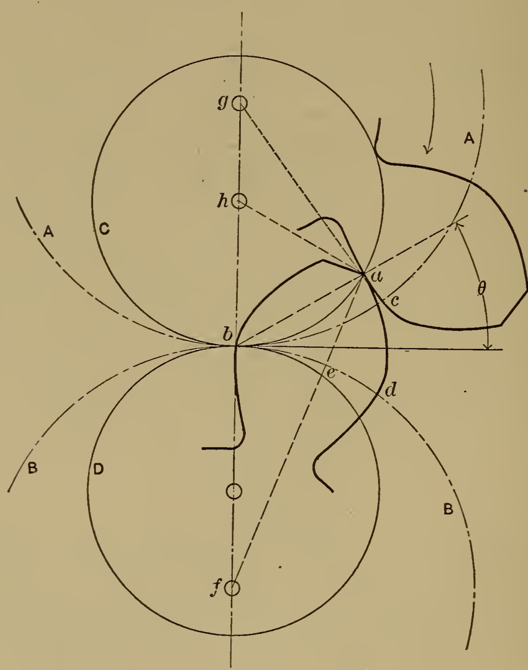


FIG. 299.

being equal to one-half the pitch. The action is possible, since *de* is less than one-half the thickness of the tooth. The maximum angle of obliquity is 30°, the angle *bgc* being 36° and *bha* being $\frac{5}{3}$ of 36°, or 60°.

Two pinions with four teeth each will just barely work with describing circles having diameters five-eighths the diameter of the pitch circle, and with no backlash, the arcs of approach and recess each being one-half the pitch. Fig. 300 shows the diagram for this case, and the teeth are apparently pointed, which would be the case if *de* were just one-half the thickness of the tooth. To determine the possibility of the action the angle *dfe* may be calculated. It should not be greater than $22\frac{1}{2}°$ to allow the desired arc of approach. It will be found to be 22° 27′ 19″,

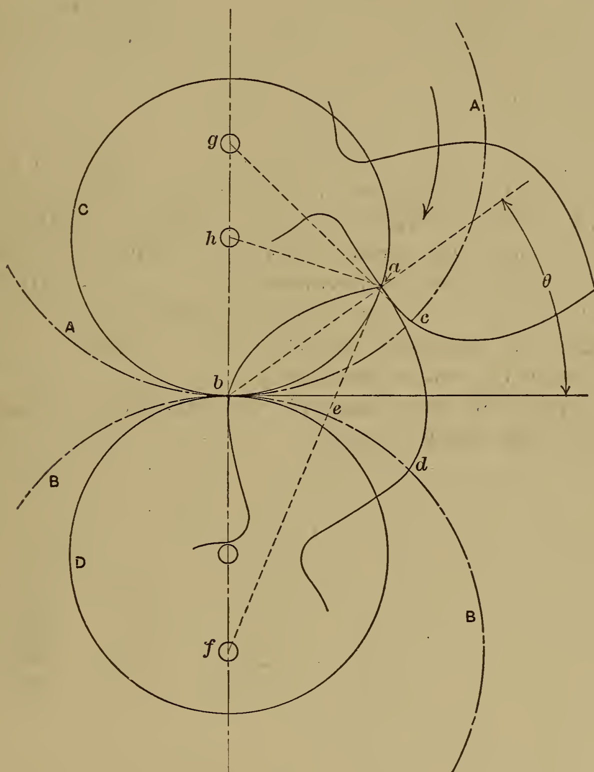


Fig. 300.

so that the action is just possible. The maximum angle of obliquity θ will be found to be 36°.

A pinion with four teeth will work with a pinion having four teeth or any higher number, if the arc of action is not required to be greater than the pitch, the maximum angle of obliquity not exceeding 36°.

The requirements may be very different from the above in every respect; an arc of action greater than the pitch would usually be required; it might be desired to have the arc of recess greater than the arc of approach; it might not be admissible to have so great an angle of obliquity or to have the teeth cut under so far as a describing circle five-eighths the pitch circle would require. The results would of course vary with the conditions imposed.

188. Arbitrary Proportions.—The teeth of gear-wheels may be designed by the preceding methods so as to fulfil any proposed conditions of approaching and receding action. In the majority of cases the exact lengths of the approaching and receding action are not important provided they are long enough. It is a very common practice to make the whole length of a tooth a certain fraction of the pitch; the part which projects outside of the pitch circle being made a little less than that within to allow the proper clearance.

None of the arbitrary proportions can be considered absolute, as the proper amounts of clearance and backlash depend on the precision with which the tooth outlines are laid out to begin with, and then on the accuracy with which the teeth are made to conform to these outlines.

In the best cut gears manufactured to-day the teeth barely clear each other when the fronts are in contact, and in any case the allowance made should depend on the accuracy of workmanship. In cast gears more clearance is necessary to allow for irregular shrinkage and rapping, or for slight derangements of the mould.

The following table gives some of the proportions in common use, P representing the circular pitch.

	1	2	3	4
Length	$^7/_{10}\,P$	$0.67\ P$	$^4/_5\,P$	$0.750\,P$
Clearance	$^1/_{10}\,P$	$0.07\ P$	$^1/_{15}\,P$	$0.060\,P+0.04''$
Working depth	$^6/_{10}\,P$	$0.60\ P$	$^{11}/_{15}\,P$	$0.690\,P-0.04''$
Addendum	$^3/_{10}\,P$	$0.30\ P$	$^{11}/_{20}\,P$	$0.345\,P-0.02''$
Thickness	$^5/_{11}\,P$	$0.475\,P$	$^7/_{15}\,P$	$0.470\,P-0.02''$
Space	$^6/_{11}\,P$	$0.525\,P$	$^8/_{15}\,P$	$0.530\,P+0.02''$
Backlash	$^1/_{11}\,P$	$0.05\ P$	$^1/_{15}\,P$	$0.060\,P+0.04''$

In the first three systems the percentage of backlash is constant, the backlash thus increasing with the pitch. It is better, however, to decrease the percentage of backlash as the pitch grows larger; for the larger the pitch, the smaller will be the proportion borne to it by any unavoidable error in workmanship. The last system (4) of Fairbairn and Rankine is based on this view of the proper proportions of backlash; the amount of backlash given is, however, much larger than that now generally allowed.

Teeth proportioned by any of the four systems will in general be of good shape, and answer their purpose. Should the wheel have less than twelve teeth, or should the exact amount of approaching or receding action be of importance, no arbitrary system is to be used, but the proper dimensions should be determined by the methods as explained.

The backlash and clearance in any case should always be as small as the quality of workmanship will permit.

Since theoretically the teeth can be in contact on both sides at once, the backlash has been disregarded in the diagrams. Should it be introduced, the thickness of the tooth, instead of being one-half the pitch, would be one-half the pitch minus one-half the backlash.

In using the diametral pitch the working depth of the tooth is usually taken as twice the diametral pitch; the clearance is then commonly taken as one-eighth the diametral pitch, thus making the length of the tooth $2\frac{1}{8}$ times the diametral pitch. This system is used by the Pratt & Whitney Co. In an 8-pitch wheel the working depth is $\frac{2}{8}'' = \frac{1}{4}''$, the addendum is $\frac{1}{8}''$, and the clearance is $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}''$. The diameter of the "gear-blank," or the addendum circle, could easily be found by adding two to the number of teeth on the wheel and then multiplying by the diametral pitch. Thus the diameter for a 60-tooth, 8-pitch wheel-blank would be

$$\frac{60+2}{8} = 7\tfrac{3}{4}''.$$

189. Conditions for a Uniform Velcity Ratio.—It is to be observed that the conditions for a uniform velocity ratio would have been satisfied had the acting tooth curves been traced by a marking-point not in the circumference of the describing circle. And it is also to be observed that the tracing-point need not be carried by a circle; any other describing curve that would roll on the pitch circles would fulfil the above conditions just as well, the resulting curves having a common point of tangency and a common normal at that point which passes through the pitch point. Hence we may in general say that any proper tooth outlines must be such as can be simultaneously traced upon the planes of rotation of the two wheels, while in action, by a marking-point which is carried by a describing curve moving in rolling contact with both pitch circles. And conversely, for any set of proper tooth curves there is a corresponding describing curve.

190. Spur Gearing, Involute System.—Generation of the Tooth Outline.—Let o_1 and o_2 be the centres of the two wheels A and B, Fig. 301, whose pitch circles are in contact at a. *The angle of obliquity is constant in a pair of involute wheels,* which means that the path of contact will lie on a straight line, which is called the *line of obliquity.* The tooth curves are not rolled from the pitch circles, but from circles called *base circles,* derived from the pitch circles as follows: Draw the line of obliquity bac making the given angle of obliquity bae with the tangent dae. From the centres of the wheels draw the circles C and D tangent to the line of obliquity at the points b and c respectively; these circles are the *base circles.* Draw the lines o_1b and o_2c from the respective centres to the tangent points of the base circles and the line of obliquity. Then

$$o_1a : o_2a = o_1b : o_2c; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

or, in a pair of involute wheels the radii of the base circles are directly proportional to the radii of the pitch circles. If the teeth can be rolled from the base circles in such a way as to give a constant a.v. ratio inversely proportional to their radii, then the desired result of a constant

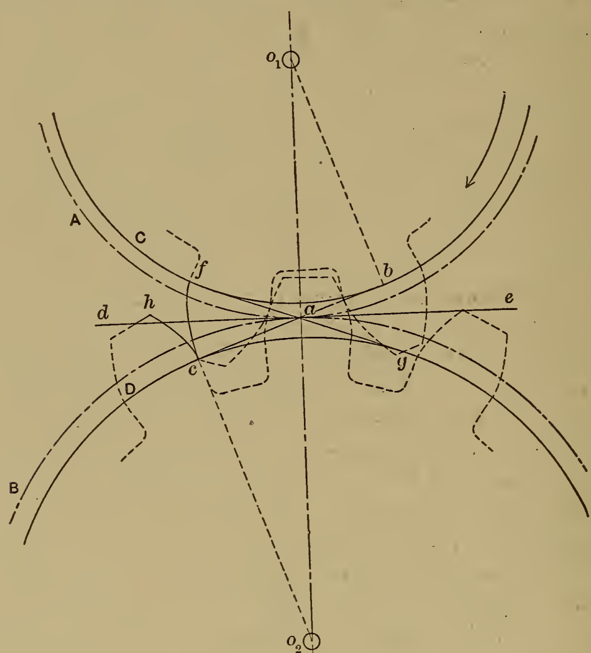


FIG. 301.

a.v. ratio inversely proportional to the radii of the pitch circles will be obtained.

Imagine the base circles to be connected by inextensible bands *bc* and *fg* similar to a crossed belt connecting a pair of pulleys, and assume that no slipping of the imaginary band occurs as the base circles are turned; also let the line *bc* carry a marking-point. The curves which this marking-point would trace on the planes of the respective base circles would be suitable for tooth curves. Thus, if we assume the marking-point to start at *b* and move through a distance *bc*, the point *b* on the base circle *C* would have moved to *f*, where *bf* is equal to *bc*, and the marking-point will have traced the involute *fc* of the base circle *C*. At the same time a point *g* on the base circle *D*, *cg* being equal to *cb*, will have moved to *c*, and the marking-point *b* will have traced the involute hc of the base circle D. These two involutes will be suitable for tooth curves, for they will evidently give by their sliding action the same motion to the base circles as the base circles had while the curves were being generated, which was a linear motion of the circle C equal to the linear motion of D, and hence the a.v. of C : a.v. of D as $o_2c : o_1b$, and from equation (60) this a.v. ratio is the same as that of the pitch circles A and B in rolling contact. Therefore these involutes by their sliding action will give to the wheels an a.v. ratio the same as the pitch circles would give by their rolling contact.

It will also be seen in Fig. 301 that the fundamental law of gearing is fulfilled, that is, the normal to the tooth curves at any point of contact passes through the pitch point, this common normal being always the line of obliquity.

The involutes thus found form both the face and flank of the tooth; and although the face may be longer even until the teeth become pointed, the acting flank (that is, the part of the flank which can be in proper contact with the face of the other wheel) cannot pass inside the base circle, the additional part of the flank necessary to carry the tooth down to the root circle not being part of the involute.

Knowing the pitch and the backlash, both of which are laid off on the *pitch circles*, the teeth may be drawn in as shown in Fig. 301, the addendum being limited, however, as will be seen in the following paragraph.

191. Path of Contact.—Relation between the Path of Contact and the Arc of Action.—Limit of the Addendum.—As was seen in the previous paragraph, the path of contact is on the line of obliquity. It is limited, as in all systems, by the respective addendum circles, and the addendum would be a maximum when the teeth are pointed. In the involute system, however, the addendum circles are limited. Fig. 302 shows the acting side of a pair of teeth as they appear when in contact at the point b, where the base circle C is tangent to the line of obliquity; at the pitch point a; and at the point c, where the base circle D is tangent to the line of obliquity. If the face ce of the wheel A is made longer (which it could be if the tooth were not yet pointed), no apparent conflict would occur; but if the wheels were turned further, so that the point of contact would tend to be beyond c, the additional involute face has no longer an involute with which to be in gear, and the radial flank extension, as drawn in Fig. 302, would not be conjugate to the additional involute face. Conflict would then occur, for the curve conjugate to the involute would lie within the radial flank. This conjugate of the involute could be used, but one of the chief advantages of the involute system, which will be noticed later, would thereby be destroyed.

Therefore, in Fig. 302, the contact cannot begin sooner than at the point b with A driving R.H., and cannot go beyond c; or, in a pair of external involute wheels *the addendum is limited in each wheel by the tangent point of the line of obliquity and the base circle of the other wheel.*

To insure perfect action the arc of action must be at least equal to the pitch. In the involute system the path of contact is not equal to the arc of action, but is equal to the arc through which the base circles move while a pair of teeth are in contact, while the arc of action is the arc through which the pitch circles move in the same time. Since the pitch circles and base circles turn together, the arcs moved through

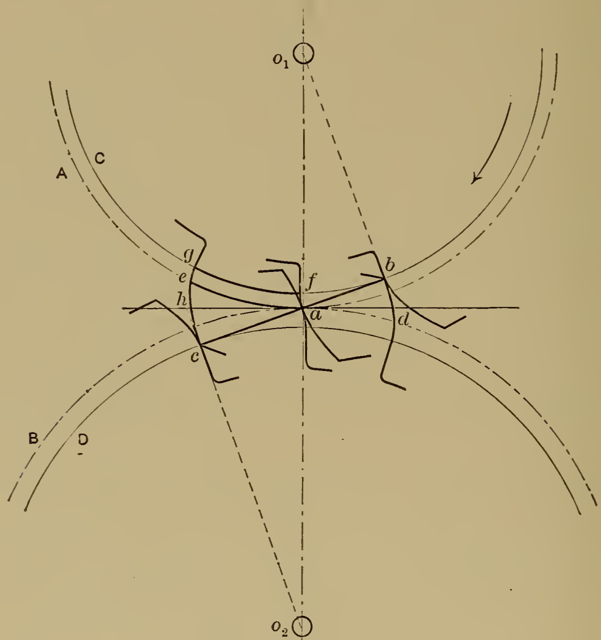


FIG. 302.

would subtend equal angles, and are proportional to the respective radii; therefore

$$\frac{\text{path of contact}}{\text{arc of action}}=\frac{\text{arc moved through by base circle}}{\text{arc moved through by pitch circle}}$$

$$=\frac{\text{radius of base circle}}{\text{radius of pitch circle}}=\text{cosine of angle of obliquity.} \quad . \quad (61)$$

The last term of the above equation is obtained by noticing that the radius of the base circle from b and of the pitch circle from a are respectively perpendicular to the line of obliquity and to the tangent.

The above relation is indicated in Fig. 302 for the arc of recess. The two teeth in contact at a move through the arc of recess ae when they will be in contact at c; the path of contact is ac equal to the arc fg, which the teeth move through on the base circle. Noticing that the arcs ae and fg must subtend equal angles at the centre of the wheel A, we have

$$\frac{ac}{ae}=\frac{fg}{ae}=\frac{\text{radius of base circle}}{\text{radius of pitch circle}}=\cos cah.$$

Thus for involute gearing the following law holds good: *The path of contact is to the arc of action as the cosine of the angle of obliquity.*

By this relation we have a simple method for determining the beginning or end of the path of contact for any desired arc of action. In Fig. 302 let it be required to find the end of the path of contact for a given arc of recess. Lay off from the pitch point a on the tangent a distance ah equal to the arc of recess; draw from h a line hc perpendicular to the line of obliquity; the point c is the end of the path of contact, for, by construction,

$$\frac{ac}{ah}=\cos cah=\frac{\text{path of contact in recess}}{\text{arc of recess}}.$$

192. Normal Pitch.—The normal pitch in an involute wheel is the distance from one tooth to the corresponding side of the next tooth measured on a normal to the curves, and from the method of generating the curves this distance is a constant, and *is equal to the distance between the corresponding sides of two teeth, measured on the base circle.*

In a pair of involute wheels the path of contact cannot be less than the normal pitch, the corresponding arc of action being the circular pitch, for it will be evident, from the discussion in the preceding paragraph, that

$$\frac{\text{normal pitch}}{\text{circular pitch}}=\text{cosine of angle of obliquity.} \quad . \quad . \quad . \quad (62)$$

193. To determine the possibility of any desired action in a pair of involute wheels.—The solution is similar to that in the cycloidal system, the essential difference being due to the fact that the path of contact is not equal to the arc of action. In Fig. 303 let it be required to determine if the arc of recess can be equal to $\frac{3}{4}$ of the pitch. Lay off from a on the tangent the distance $ab=\frac{3}{4}$ of the pitch. Draw bc perpendicular to the line of obliquity; c will be the end of the path of contact for the given arc of recess. If the point c came beyond d, the tangent point of the line of obliquity and the base circle, the action would be impossible since no contact can occur beyond d. But if, as in Fig. 303, the point c comes between a and d, it is necessary to determine if the face of the tooth on A can reach to c. Lay off on the pitch circle A the arc $ae=ab=\frac{3}{4}$ of the pitch; the face of the tooth on A will then pass through c and e. Draw the line co from c to the centre of A, and note the point f where it cuts the pitch circle A. If ef is less than one-half the thickness of the tooth, the action can go as far as c and the teeth will not be pointed. In the figure, assuming no backlash, the thickness of the tooth would be eg, and ef is less than $\frac{1}{2}eg$; therefore the action is possible, as is shown by the two teeth drawn in contact at c.

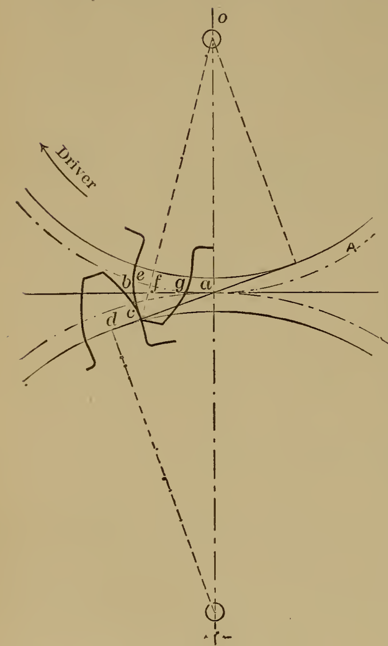


Fig. 303.

194. Involute Pinion and Rack.—Fig. 304 shows a pinion driving a rack. The path of contact cannot begin before the point a, but the recess is not limited excepting by the addendum of the pinion, since the base line of the rack is tangent to the line of obliquity at infinity. For the same reason it will be evident that the sides of the teeth of the rack will be *straight lines* perpendicular to the line of obliquity. In the figure the addendum on the rack is made as much as the pinion will allow, that is, so that the path of

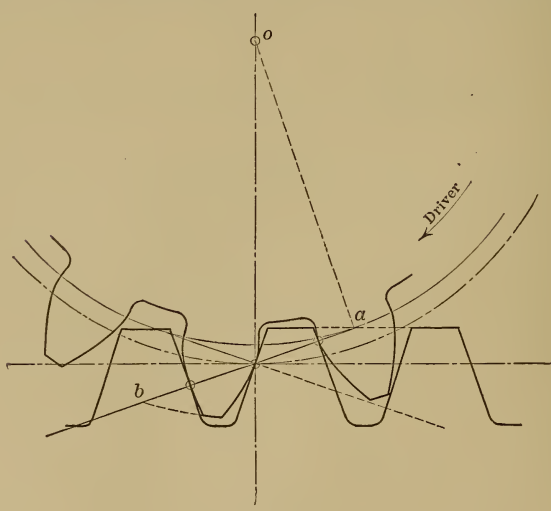


Fig. 304.

contact will begin at a. The addendum of the pinion will give the end of the path of contact at b.

In Fig. 305, the diagram for a pinion and a rack, let it be required to determine if the path of contact can begin at *a* and go as far as *b*; to be solved without using the tooth curves. The solution is similar to that in § 193. For the contact to begin at *a* the face of the rack must reach to *a*. Draw the line *ac* perpendicular to the line of obliquity, giving *cd* as the arc of approach; draw *ae* parallel to the line of centres, and if *ce* is less than one-half the thickness of the rack tooth, the approaching action is possible without pointed teeth. Similarly for the recess, draw the line *bf* perpendicular to the line of obliquity, giving *df* equal to the arc of recess; make the arc *dg* on the pinion's pitch circle equal to *df*, then the face of the pinion's tooth will pass through *b* and *g*; draw the line *bh* to the centre of the pinion, and note the point *h* where it crosses the pinion's pitch circle. If *gh* is less than one-half the thickness of the tooth, the recess is possible without pointed teeth.

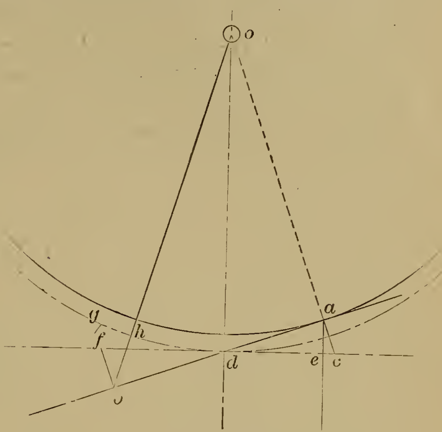


FIG. 305.

195. Involute Pinion and Annular Wheel.—Fig. 306 shows an involute pinion driving an annular wheel. This case is very similar to a pinion and rack. The addendum of the annular is limited by the tangent point *a* of the pinion's base circle and the line of obliquity, while the addendum of the pinion is unlimited except by the teeth becoming pointed. The base circle of the annular lies inside the annular, so that its point of tangency with the line of obliquity is at *b*. If we take some point on the line of obliquity, as *c*, and roll the tooth curves as they would appear in contact at that point, the teeth of the annular will be found to be *concave*, and the addendum of the annular will seem to be limited by the base circle of the annular where the curves end. But if these two teeth are moved back until they are in contact at *a*, it will be evident that the annular's tooth curve cannot be extended beyond *a* without interfering with the radial extension of the flank of the pinion. Therefore the addendum of the annular is limited by the point of tangency of the base circle of the pinion and the line of obliquity.

196. Possibility of Separating two Involute Wheels.—One of the most important features of involute gearing is the fact that two such wheels may be *separated*, within limits, without destroying the accuracy of the a.v. ratio. In this way the backlash may be adjusted, since the original pitch circles need not be in contact. To show that this is so, Fig. 301

may be redrawn using the same pitch circles and base circles, but separating them slightly, keeping the teeth in contact, as has been done in Fig. 307. Connect the base circles by the tangent *bc*. If now the line *bc* carries a marking-point, it will evidently trace the involutes of the two base circles, as *de* and *he*, and these curves must be the same as the tooth

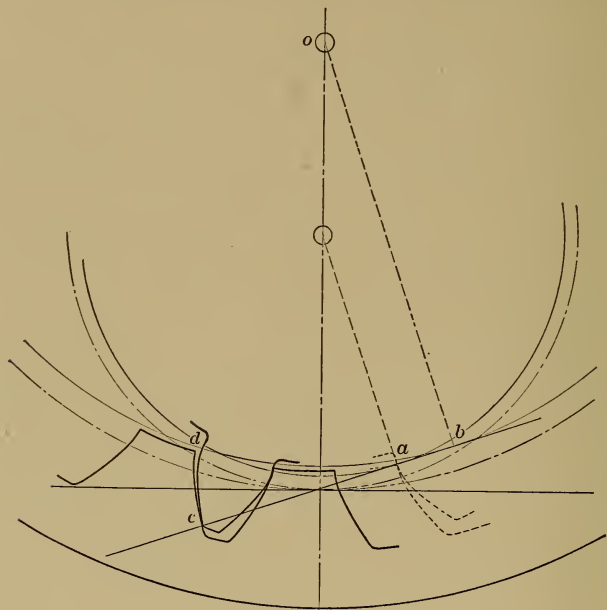


Fig. 306.

curves in Fig. 301. In Fig. 307 these curves *de* and *he* will give an a.v. ratio to the base circles inversely as their radii, but the radii of these base circles are directly as the radii of the original pitch circles (Fig. 301); hence in Fig. 307 the tooth curves *de* and *he* would give an a.v. ratio to the two wheels inversely as the radii of the original pitch circles, although these circles do not touch. The path of contact is now from *k* to *e*, which is considerably shorter than in Fig. 301; it is, however, greater than the normal pitch, so that the action is still sufficient. The limit of the separation will be when the path of contact is just equal to the normal pitch. The angle of obliquity is *bam*, which is greater than in Fig. 301. The backlash has also increased.

Theoretically the wheels have new pitch circles in contact at *a*, and a new angle of obliquity, also a greater circular pitch with a certain amount of backlash; and if we had started with these latter data, we should have obtained exactly the same wheels as in Fig. 301, only slightly separated. It will be seen that the radii of the new pitch circles are to each other as the radii of the respective base circles, and consequently as the respective original pitch circles. It will also be seen that the line of obliquity, which is the common normal to the tooth curves, passes through the new pitch point a so that the fundamental law of gearing is still fulfilled.

By the application of the preceding principles two or more wheels of different numbers of teeth, turning about one axis, can be made to gear correctly with one wheel or one rack; or two or more parallel racks

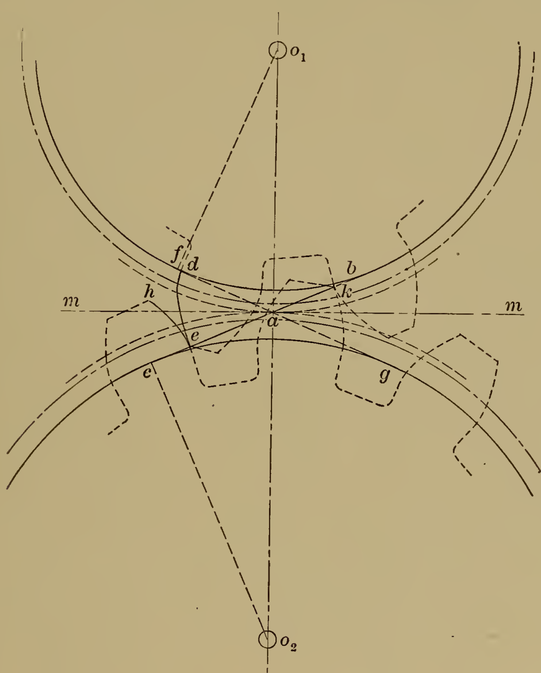


FIG. 307.

with different obliquities of action may be made to gear correctly with one wheel, the normal pitches in each case being the same. Thus differential movements may be obtained which are not possible with teeth of any other form.

The principal objection to the use of involute teeth for large gears is the great obliquity of action and the large number of teeth in the smallest wheel.

On the other hand, for smooth action, especially for light work, it may be an advantage to have this constant obliquity of action, as the side pressure will tend always to keep the axes at the greatest possible distance from each other, thus preventing jarring in case there be any looseness in the bearings.

197. Interchangeable Involute Gears.—Since the tooth curve in an involute wheel depends only on the base circle, or in other words on the diameter of the pitch circle and the angle of obliquity, wheels may be designed separately, keeping the *normal* pitch the same in each wheel of a set, the addendum and clearance being so taken that no interference will occur with the smallest wheel desired in the set.

Let it be required to design an involute wheel having 36 teeth and 6-P., with an obliquity of 15°, the smallest wheel of the set to allow arcs of approach and recess each equal to the pitch. In Fig. 308, A is the pitch circle, bac the line of obliquity, and B the base circle of the 36-tooth wheel. The pitch is approximately $0''.52$, and is laid off on the tangent on each side of a so that $ad = ae = 0''.52$. The desired path of contact will then be found to be bac, where $ba = ac =$ the normal pitch. The addendum for the 36-tooth wheel must pass through b, and should not be greater, for the smallest wheel of the set would theoretically have its base circle tangent to the line of obliquity at b, just allowing the path of contact to go to b. This would give the centre of the smallest pinion of the set at o_2, provided, however, that the distance ao_2 can be used with the given pitch. In this example the wheels are 6-P., so the radius of the pitch circle must be divisible into twelfths of inches. This will generally throw the actual centre a little beyond the centre found in the diagram. The addendum circle of the smallest wheel will pass through c; this will give the root circle C for

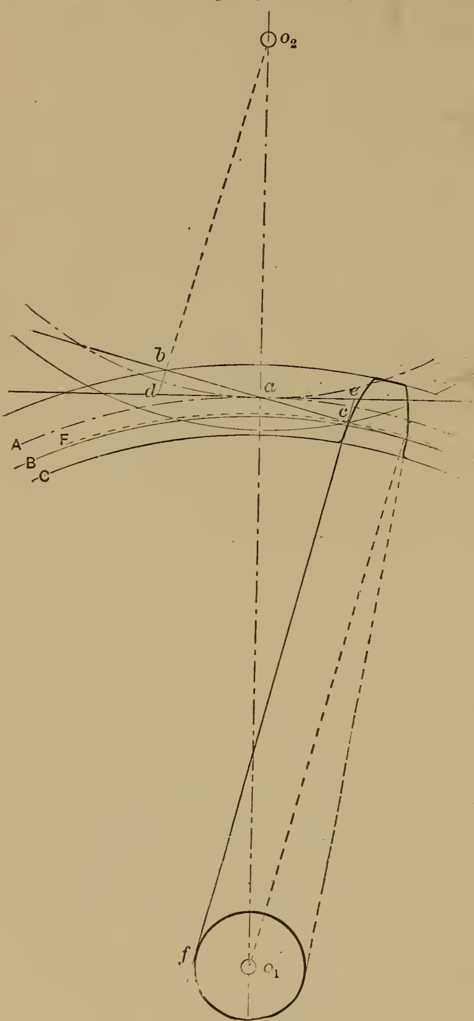


FIG. 308.

the 36-tooth pinion, the clearance being known. We have thus the complete diagram for the 36-tooth wheel and may proceed to roll the teeth.

Since the path of contact is not intended to go beyond c, the involutes need not be extended to the base circle, and instead of using radial extensions from the involutes to the root circle, straight lines tangent to the involutes at the points where a circle F, drawn through the point c, cuts them, may be used. If a tooth were just ready to begin action at c, as shown in Fig. 308, the line of obliquity would, at that position of the tooth, be normal to the acting involute; therefore if we draw the line cf from c perpendicular to the line of obliquity, it will give the direction of the desired flank extension at that position. If a circle is now drawn with o_1 as a centre and tangent to the line cf, the flank extensions for all the teeth will be tangent to this circle, as shown in the figure for one tooth. This will result in a stronger form of tooth than that obtained with a radial extension.

When interchangeable gears are constructed with involute teeth, the addendum is made the same for all wheels, and is usually taken, as in the epicycloidal system, equal to the diametral pitch; but when the addendum is thus arbitrarily chosen, the teeth of the larger wheels, particularly of the rack, would be liable to conflict with the flank extensions of the smaller wheels; to avoid such interference the ends of the teeth must be rounded off.

198. Arbitrary Proportions for Involute Gears.—Rankine's rule to make the entire length of the tooth $0.75P$, or that of Willis to make it $0.7P$, will give good results. Experience has shown that for ordinary purposes the obliquity should not exceed from 15° to 17°. When it is no greater than this it is safe to say that these teeth are equal to the cycloidal or any others, even for heavy work; in addition to other advantages that have been mentioned, it is to be observed that the form of the tooth is a strong one.

199. Pin Gearing.—In this form of gearing the teeth of one wheel consist of cylindrical pins, and those of the other of surfaces parallel to cycloidal surfaces, from which they are derived.

In Fig. 309 let o_1 and o_2 be the centres of the pitch circles whose circumferences are divided into equal parts, as ce and cg. Now if we suppose the wheels to turn on their axes, and to be in rolling contact at c, the point e of the wheel o_1 will trace the epicycloid gp on the plane of the wheel o_2, and merely a point e upon the plane of o_1. Let cf be a curve similar to ge and imagine a pin of no sensible diameter—a rigid material line—to be fixed at c in the upper wheel. Then if the lower one turn to the right, it will drive the pin before it with a constant velocity ratio, the action ending at e if the driving curve be terminated at f as shown.

If the pins be made of a sensible diameter, the outlines of the teeth upon the other wheel are curves parallel to the original epicycloids, as shown in Fig. 310. The diameter of the pins is usually made about equal to the thickness of the tooth, the radius being, therefore, about $\frac{1}{4}$ the pitch arc. This rule is, however, not imperative, as the pins are often made considerably smaller.

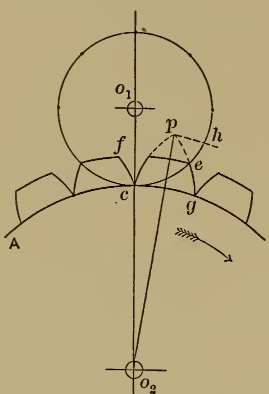


FIG. 309.

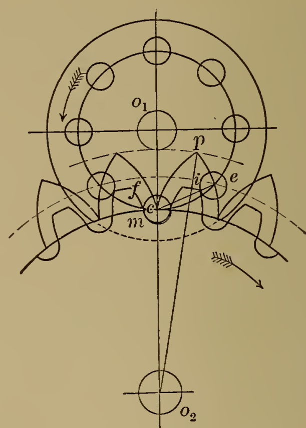


FIG. 310.

Clearance for the pin is provided by forming the root of the tooth with a semicircle of a radius equal to that of the pin, the centre being inside of the pitch circle an amount equal to the clearance required.

The pins are ordinarily supported at each end, two discs being fixed upon the shaft for the purpose, as shown in Fig. 274, thus making what is called a *lantern* wheel or pinion.

Mode of Action.—In wheel work of this kind the action is almost wholly confined to one side of the line of centres. In the elementary form (Fig. 309) the action is wholly on one side, and receding, since it cannot begin until the pin reaches c (if o_2 drives), and ceases at e; if o_1 is considered the driver, action begins at e, ends at c, and is wholly approaching. As approaching action is injurious, pin gearing is not adapted for use where the same wheel has both to drive and to follow; the pins are therefore always given to the follower, and the teeth to the driver.

When the pin has a sensible diameter, the tooth is shortened and its thickness is decreased; the line of action is also shortened at e, Fig. 310, and, instead of beginning at c, will begin at a point where the normal to the tooth curve, through the centre of the pin, first comes in contact with the derived curve mf. This normal's end will not fall at c, but at a point on the arc ce beyond, on account of a property of the curve parallel to an epicycloid. The parallel to the epicycloid is shown in Fig. 311, *cp* being the given epicycloid. The curve may be found by drawing a series of arcs *ss* with a radius equal to the normal distance between the curves, and with the centres on *cp*. The parallel curve first passes below the pitch curve *cm* and then rises, after forming a cusp, and cuts away the first part drawn: this is more clearly shown somewhat exaggerated at *mno*. Hence the part which would act on the tooth when its centre is at *c* is cut away, and, for the same epicycloid, the greater the diameter of the pin the more this cutting away. In Fig. 310 the pin *e* is just quitting contact with the tooth at *i* while *c* is at the pitch point, and, according to the above property of the parallel to the epicycloid, is not yet in contact with the tooth *m*. Strictly speaking, then, the case shown is not a possible one, as the tooth should not cease contact at *i* until *m* begins its action. The above error is practically so small that it has been disregarded, especially for rough work.

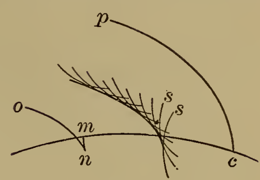


FIG. 311.

The following method may be used in determining a limiting case in pin gearing:

If we assume the pitch arc $=cg$ (Fig. 312), the greatest possible height of tooth is determined by the intersection of the front and back of the tooth at *p*; and if this height is taken, action will begin at *c* and end at *h*, the point in the upper pitch circle through which *p* passes. Now if *p* falls upon the pitch circle *ceh*, we should have a limiting case for a pin of no sensible diameter. If the pin has a sensible diameter and the pitch arc $cg=ce$ is assigned, bisect *cg* with the line o_2p and draw *ce* intersecting o_2p in *k*; assume a radius for the pin less than *ek* and draw the derived curve to cut o_2p in *j*, which will be the point of the tooth. Through *j* draw a normal to the epicycloid, cutting it at *s*; through *s* describe an arc about o_2 cutting the upper pitch circle at *t*, the position of the centre of the pin at the end of its action. Draw the outline *mf* of the next working tooth, find the point *m* at the cusp of the curve parallel to the epicycloid, and draw the normal *mn*; *m* is the lowest possible working point of the tooth. Through *n* describe an arc about o_2 cutting the original path of contact in *r*, which is the point that *n* must

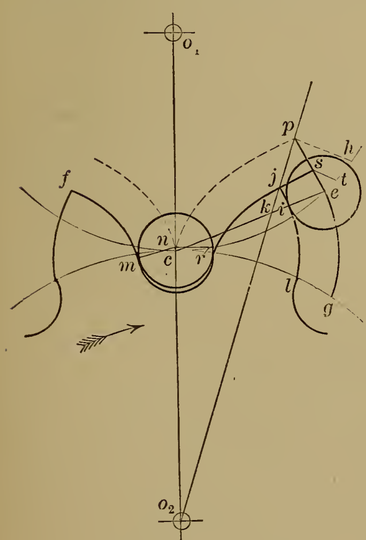


FIG. 312.

reach before the tooth will be in contact with the pin, or is the point that n must reach before the common normal to the pin and tooth curve passes through the pitch point.

Now action begins when the axis of the pin is at r and ends at t; if $rt = ce$, we have an exact limiting case and the assumed radius of the pin is a maximum; if $rt < ce$, the radius is too great; but if $rt > ce$, the case is practical. To get the exact limit a process of trial and error should be resorted to. When the pin is a point the methods used in cycloidal gearing may be applied; the correction for a pin of sensible diameter can then be made by applying the method of Fig. 312.

Wheel and Rack.—As the pins are always given to the follower, we have two cases.

1° *Rack drives*, giving the pin-wheel and rack, Fig. 313. Here the

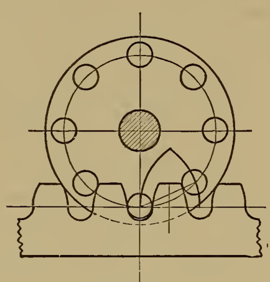


FIG. 313.

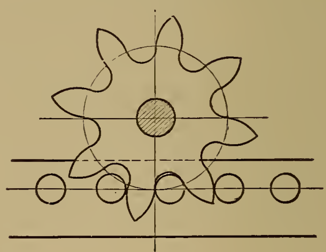


FIG. 314.

original tooth is bounded by cycloids generated by the pitch circle of the wheel.

2° *Wheel drives*, giving the pin-rack and wheel. Here (Fig. 314) the original tooth outline is the involute of the wheel's pitch circle.

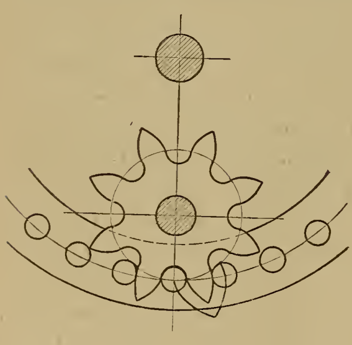


FIG. 315.

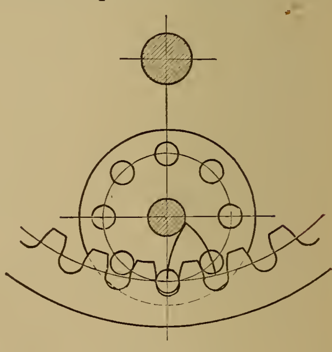


FIG. 316.

Inside Pin Gearing.—Here also there are two cases.

1° *Pinion drives* (Fig. 315). The original tooth outlines will be internal epicycloids generated by rolling the pitch circle of the annular wheel on the pinion's pitch circle.

2° *Annular wheel drives* (Fig. 316). Here the original tooth outline is the hypocycloid traced by rolling the pinion's pitch circle in the wheel's circle.

Path of Contact.—In the elementary form of tooth (Fig. 309) the path of contact is on the circumference of the pitch circle of the follower o_1, as ce. When a pin is used its centre always lies in this circumference, and its point of contact may be found by laying off a distance ei equal to the radius of the pin (Fig. 317) on the common normal. Drawing

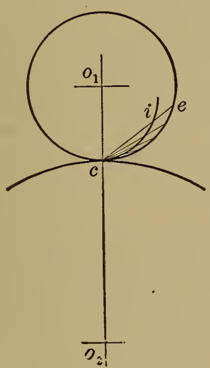


FIG. 317.

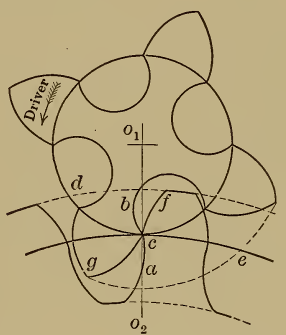


FIG. 318.

a number of these common normals, all of which must pass through the pitch point c, and laying off the radius of the pin ei on each, we have the path of contact ci known as the *limaçon.*

200. Double-point Gearing.—This form of gearing, shown in Fig. 318, gives very smooth action where not much force is to be transmitted. The pitch circles are here taken as the describing circles; the face cg of the pinion o_1 is generated by rolling the pitch circle o_2 on that of o_1, and the face cf is generated by rolling the pitch circle o_1 on o_2. If o_1 is considered the driver, action begins at d, the point c of o_1 sliding down the face cf while c travels from d to c. In the receding action the point c of the tooth of o_2 is acted on by the face cg while c moves from c to e. The spaces must be so made as to clear the teeth. This combination reduces friction to a minimum and gives the obliquity of action less than in any case except pin gearing, but the teeth are much undercut and weakened by the clearing curves, and if much force is to be transmitted the line of contact will soon be worn away.

Shrouded Wheels.—When the teeth of a wheel, as o_1, Fig. 318, are undercut and weak they are sometimes united at their ends by annular rings cast with the wheel, and the wheel is then said to be *shrouded.*

This shrouding strengthens the teeth and is usually applied to the pinion, where the wear is greater; it may extend the whole depth of the teeth of the pinion, or both pinion and wheel may be shrouded to half the length of the teeth. The latter arrangement is seldom adopted on account of the difficulty of casting the wheels.

201. Sang's Theory.—A conjugate curve has already been defined, § 177. If in Fig. 278 we assume any tooth outline, as aa_1, we may construct a series of wheels, assuming different pitch circles, that will work with the wheel having the assumed tooth outline. Then any one of the series may be taken and a second series of wheels made from its tooth outline. From the method of generating the tooth outlines it follows that any wheel of the first series will gear with any one of the second. Now if the conjugate tooth outlines on any two equal wheels be made the same, the two series will be identical and the wheels will all be interchangeable. If the tooth and space are taken equal to each other, two racks formed from the above equal wheels will exactly fit into each other, the tooth of one filling the space of the other and being at the same time equal to that of the other.

Hence we may assume the outline of a rack tooth such that it is made up of four equal lines in alternate reversion (thus giving a symmetrical tooth about its centre line) and from it derive an interchangeable set of wheels. If the rack be made up of equal cycloidal arcs, the cycloidal system with a constant describing circle will be obtained; if it be bounded by oblique straight lines, the involute system will be reproduced.

The rack may be arbitrarily assumed, provided it fulfils the above conditions, and different series of wheels may be derived. This method of constructing the tooth outline, which is practically the reversal of the ordinary way, is due to Professor Sang.

Should we now shape a cutter to the exact outline of a rack tooth and then give to it a reciprocating motion, parallel always to the axis of a gear-wheel blank, which is made to move in reference to the rack tooth just the same as it would if the pitch surface of the rack and that of the wheel were in rolling contact with each other, the rack tooth would shape the space of the wheel; it being understood that the wheel is not moved while the rack tooth is cutting, and that it is only rolled fast enough to give a light chip to cut each time the tooth moves in the cutting direction. One space having been cut, the blank is turned through the pitch angle and the operation is repeated until the wheel is formed. This method has been applied in a bevel-gear-cutting engine.

202. Unsymmetrical Teeth.—In all the figures hitherto given the teeth are symmetrical, so that they will act equally well whether the wheels are turned one way or the other. In cases where the action is always one way it may be advantageous to make the teeth otherwise, as shown in Fig. 319. Here the lower wheel, o_2, is the driver, and the acting outlines of both wheels are of the cycloidal form; the describing circles p_1 and p_2 have been taken large to reduce the obliquity to a minimum. If the other sides of the teeth were made the same, we should have a weak tooth at the root. To avoid this the backs of the teeth may be made involutes of considerable obliquity, the radii of the base circles being o_1a and o_2b. It can be seen that this gives a very strong form of tooth.

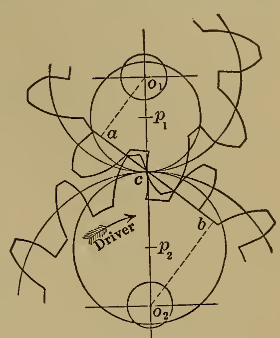


FIG. 319.

203. **Twisted Gearing.—Hooke's Stepped Wheels.**—If a pair of spur-wheels are cut transversely into a number of plates, and each plate is rotated through an angle, equal to the pitch angle divided by the number of plates, ahead of the adjacent plate, as shown in Fig. 320, we shall have a pair of *stepped wheels*, first introduced by Dr. Hooke. By this device we obtain the effect of increasing the number of teeth without diminishing their strength; the number of contact-points is also increased, and the interval between their times of crossing the line of centres, where the action is best, is correspondingly diminished. The upper figure shows a section on the pitch line AA. The action for each pair of plates is the same as that for spur-wheels having the same outlines. In practice there is a limit to the reduction in the thickness of the plates, depending on the material of the teeth and the pressure to be transmitted, since too thin plates would abrade. The number of divisions is not often taken more than two or three, and the teeth are thus quite broad. These wheels give a very smooth and quiet action.

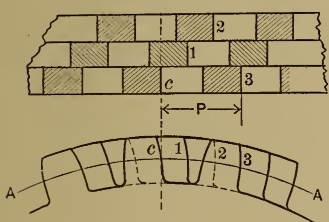


FIG. 320.

Hooke's Twisted Gearing.—If the number of plates be taken infinite, the effect is the same as that explained in § 174, 4°. The twisting being uniform, the elements of the teeth become helices, all having the same pitch. The line of contact between two teeth will have a helical form, but will not be a true helix; the projection of this helix on a plane perpendicular to the axis will be the ordinary path of contact. It can easily be seen that the common normal at any point of contact can in no case lie in the plane of rotation, but will make an angle with it. The line of action then can in general have three components: 1° A component producing rotation, perpendicular to the plane of the axes; 2° A component of side pressure, parallel to the line of centres; 3° A component of end pressure parallel to the axes. When the point of contact is in the plane of the axes the second component is zero; advantage may be taken of this, as will be shown, so that there may be no sliding action between the teeth. The end pressure is neutralized as explained in § 174, 4°.

Sliding Friction Eliminated.—In this case the angle of twist is at least equal to the pitch angle and should be a little more. In Fig. 321, which represents a transverse section of a pair of twisted wheels, suppose the original tooth outlines to have been those shown dotted. Then cut away the faces as shown by full lines having the new faces tangent to the old ones at the pitch point c; we now have lost proper contact except that at c for the section shown, but by twisting the wheels this contact can be made to travel along the common element of the pitch cylinders through c from one side of the wheel to the other. A simple construction to use in this case is to make the flanks of the wheels radial and the faces semicircles tangent to the flanks. The action here is purely rolling and is very smooth and noiseless; but for heavy work it is best to use the common forms of teeth with sliding action, so that the pressure may be distributed over a line instead of acting at a point.

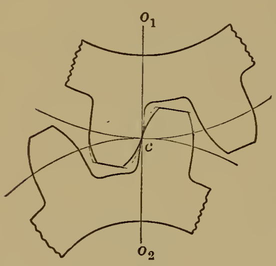


FIG. 321.

204. Approximate Forms of Teeth.—To secure perfect smoothness of action in toothed wheels, it is necessary that the tooth outlines should be accurately laid out, as explained in the preceding pages, and that the teeth when constructed should conform exactly with the outlines found. If the teeth are to be cut, the exact curves should be used, as when the cutter is once made it will cut the accurate shape as well as any other. When, however, the teeth are to be cast, or for some other reason perfect accuracy is not required, the exact curves may be replaced by others which approximate to them more or less closely, but which are simpler to construct. This is possible as, the teeth being short, only a small part of the theoretical curve is used. In these approximations the proportions of the teeth are usually governed by one of the sets of arbitrary proportions given in § 188. The two principal methods of approximation are: 1° by circular arcs, and 2° by curved templates.

Approximation by Circular Arcs.—*Willis Method.*—Let o and o_1 (Fig. 322) be the centres of two pitch circles in contact at c. Draw a line qcq_1 making an angle θ

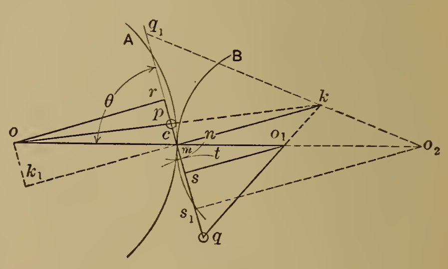


FIG. 322.

with the line of centres, and through c draw the line ck perpendicular to qq_1. On ck assume any point, as k, and through this point draw the lines ko and ko_1q intersecting qq_1 at p and q respectively. These points may now be taken as limiting the length of the connecting-rod of a four-bar linkage $opqo_1$, the links op and o_1q turning about o and o_1 respectively, k being the instantaneous axis of pq_1. For the a.v. ratio we have

$$\frac{\text{a.v. } op}{\text{a.v. } o_1q} = \frac{o_1c}{oc},$$

which is the same as that for the rolling pitch circles. This angular velocity ratio is also momentarily constant, as ck is perpendicular to pq (§ 98, page 75); and for a slight angular movement of the links either way from their present position pq would still pass through c. If now through any point, as m, on pq we draw two circular arcs, as mn and mt, with p and q as centres respectively, they will do for tooth curves, since they will retain p and q at a distance $= pm + mq = pq$ apart, thus replacing the link, and will also have, for a limited motion, their common normal at the point of contact passing through the pitch point c. In the figure mn might be considered the face of o, and mt the flank of o_1. Had the point m been taken outside of pq, both arcs would have been convex the same way. If o_1 be placed so that the angle ko_1c is acute, as, for example, at o_2, then q will fall at q_1 on the same side of c as p, and this will make the flank mt concave instead of convex. But if ko_1 becomes parallel to ps, then q will fall at an infinite distance from p, and mt will become perpendicular to ps.

Application to Involute Teeth, where the outline of the tooth consists of a single arc.—In Fig. 322 let ck become infinite; then op and o_1q will become perpendicular to pcq, and the points p and q will be found at r and s respectively. Let the arcs of the teeth be struck through c, and let $ocr = \theta$, the radius of the wheel $oc = R$, and the required radius of tooth outline $cr = D$; then $D = R \cos \theta$, which is independent of the wheel o_1, as well as of the pitch and number of teeth of o. If, then, the angle θ and the pitch be made constant in a set of wheels, any two wheels of the set will work together.

Assume $\theta = 75° \ 30'$, a very convenient value; then

$$D = R \cos 75° \ 30' = R \times 0.25038 = \frac{R}{4}, \text{ very nearly.}$$

To apply this approximation, let oc (Fig. 323) be the radius of the pitch circle of the proposed wheel. Draw cp, making an angle $ocp = 75° \ 30'$ with oc, and drop the perpendicular op upon cp, or, better, describe a semicircle on oc, and make the chord $cp = \frac{1}{4}oc$; then p will be the centre for the tooth outline acb drawn through c. The tooth may now be completed as shown. The centres of all the curves are found on the circumference of a circle of a radius op, and the lengths of the teeth should be kept within the limits mentioned in § 191.

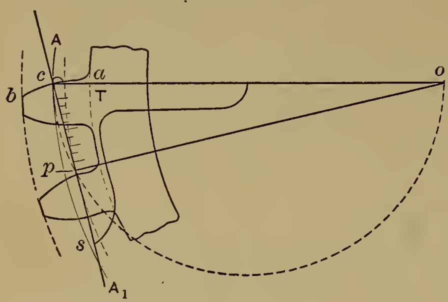


FIG. 323.

For convenience a bevel template, as shown at T, may be made, the angle ocs being 75° 30′. The edge cs can then be graduated $\frac{1}{4}$ size; now, knowing the radius of the wheel, the position of p may be found directly by adjusting the template as shown, and noting the point p at the radius reading on the scale.

Application to Cycloidal Teeth where the side of the tooth consists of two circular arcs. Let o (Fig. 324) be the centre of the given wheel,

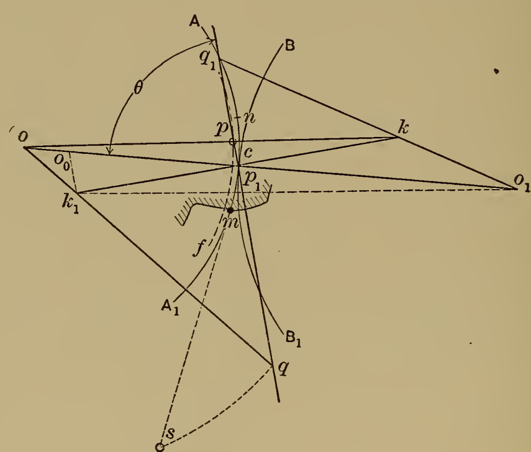


FIG. 324.

o_1 that of a wheel with which it is to gear, and c the pitch point. Draw qcq_1, making an angle θ with oo_1, and through c draw the perpendicular kck_1, making $kc = k_1c$ and less than either oc or o_1c. Draw ok and o_1k, cutting qc at p and q_1 respectively. Lay off $cm = \frac{1}{2}$ pitch on the pitch circle AA_1 on the side of c opposite p and q_1; then p is the centre and pm the radius of the face of o drawn through m, which face will work with a flank of o_1 with a centre q_1 and radius q_1m. Lines from o_1 and o through k_1 locate the tooth centres p_1 and q; then laying off $cn = \frac{1}{2}$ pitch on A, we have p_1n, the radius of the faces of o_1, and qn, the radius of the flanks of o. Circles pf and qs drawn through p and q about o will locate the face and flank centres respectively for the wheel o, and circles through p_1 and q_1 about o_1

will locate the face and flank centres for the wheel o_1. If now the points k and k_1 remain fixed, changing the radius o_1c will not affect p and q, the centres of the tooth curves for o; hence any number of wheels may be designed, using different values for o_1c, that will work with the wheel o. To find the limit of ck_1 for a given value of oc, we see from the figure that when o approaches c, cq increases, becoming infinite when ok_1 is parallel to cq, thus giving flanks perpendicular to cq through n. If o approach still nearer c, the flanks become convex (q then appearing above c), which would give an awkward tooth form. The greatest value given to ck_1 is then that which makes ok_1 parallel to cq, and the smallest wheel of the set will have straight flanks. If the radius of this smallest wheel is represented by R_0, and if D represents the distance pc, and d the distance qc, then, by assuming values for R_0 and θ in a set of wheels, the corresponding values of D and d for different pitches and numbers of teeth may be calculated and arranged in tabular form. Professor Willis assumed $\theta = 75°$, and took twelve as the least number of teeth to be given to any wheel, the flanks of this wheel being radial.

The Willis Odontograph consists of two thin strips T (Fig. 325) making an angle of 75° with each other; the edge nr corresponds to oc, and nq to cq (Fig. 324). The edge nq is graduated with equal divisions beginning at n and going both ways. The graduations to the right of n are for face centres, and to the left of n for flank centres; these graduations are made to suit the tables calculated by the method suggested above.

Fig. 325 illustrates the method of applying the instrument. The pitch and number of teeth being known, the radius of the pitch circle oc can be found; make mn equal to the pitch arc and bisect it in c; find from the tables the values $D = mp$ and $d = nq$, which locate the centres p and q respectively for the face ca and the flanks cb. The method of using the instrument can easily be seen from the figure.

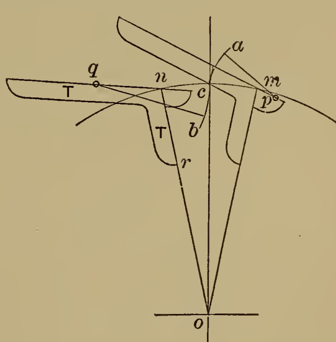


Fig. 325.

Wheels laid out with the odontograph resemble the cycloidal wheels with a constant describing circle, of a diameter one-half that for a twelve-toothed pinion. The outlines of the teeth show an angle at the pitch points of the teeth.

The approximate radii of the face and flank curves for teeth, with the radii of their centre circles pf and qs, Fig. 324, may be calculated from the values of D and d; tabulating these, we may get along without the odontograph. This has been done by Mr. George B. Grant, who has arranged a table which gives the radii of tooth curves and the radial distances between the pitch circle and the face, and the flank centre-circles.

Mr. Grant has also arranged a table, known as "Grant's odontograph table," in which the approximate circular arcs are made to conform more nearly to the theoretical shape than by the Willis method. The Willis arc lies wholly within the true curve, while the Grant arc intersects the tooth face in three points; viz., at the pitch line, at the addendum line, and at a point midway between. The above tables may be found in "A Handbook on the Teeth of Gears," by George B. Grant.

Robinson's Template Odontograph.—This ingenious instrument, the invention of Prof. S. W. Robinson, gives the outline of the tooth direct, and may be used in the pattern-shop for laying out gear patterns. It was found that the curve, to satisfy the mathematical conditions in what precedes, 1° must be one of rapidly changing curvature, approximating very closely to the epicycloid; 2° it must be very nearly perpendicular to the pitch circle at the middle point of the tooth outline; and 3° it must intersect the addendum circle at the same point as the epicycloid; in short, it must coincide with the epicycloidal face. The curve most completely satisfying these conditions was found to be a logarithmic spiral.

The odontograph consists of a thin brass plate *fgh* (Fig. 326), graduated on the edge *gh*, the figure showing the instrument about one-sixth size. Accompanying the instrument are tables varying according to the kind of tooth desired. Fig. 326 shows the method of using the odontograph to lay out a wheel belonging to an interchangeable series. The table is here arranged in four columns, giving: 1° Diameter in inches; 2° Number of teeth; 3° Face settings; 4° Flank settings.

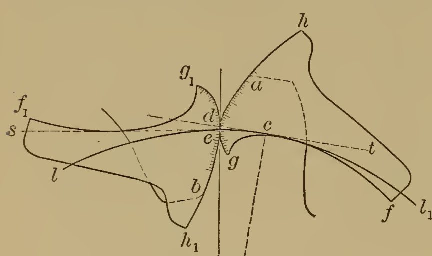


FIG. 326.

Let lcl_1 be the pitch circle, which is known when the pitch and number of teeth are given; assume c the middle point of a tooth, and lay off the arc ce=its half-thickness. Draw tangents ct and es to the pitch circle at c and e. Set the instrument in the position fgh, the proper division on the scale, found from the column of face settings, being brought to d while at the same time the curved edge fg is tangent to ct, and c is on the edge gh; now draw the face ea. To draw the flank the instrument is placed in the position $f_1g_1h_1$, the proper flank reading being at e, and the curve f_1g_1 being tangent to es.

Professor Robinson's paper may be found in Van Nostrand's Magazine, Vol. 15.

Prof. J. F. Klein has recently introduced a method of finding correct tooth outlines by means of tables specially prepared for the different systems of gearing. The method consists in giving, by table, the distances of points of the tooth outline from each of two perpendicular reference lines XX and YY (Fig. 327) drawn through some easily fixed point in the tooth outline. Each of the two sets of distances is expressed in simple decimal fractions of the pitch or diameter, and, for ease in tabulation, computation, and laying out, one of these sets is arranged in groups of equidifferent values. The computations from the tables are very simple, only short division or multiplication being necessary for determining ordinary outlines. After making the computations, the drawing of the outline only involves the use of the square and the ability to lay off distances accurately. Tables are also given by means of which two reference lines, one on each side of a tooth, may be located in proper relation to each other, thus making it unnecessary to use compasses. This method is especially useful in laying out the outlines of the teeth of large wheels where it would be inconvenient to roll up the curves in the ordinary way.

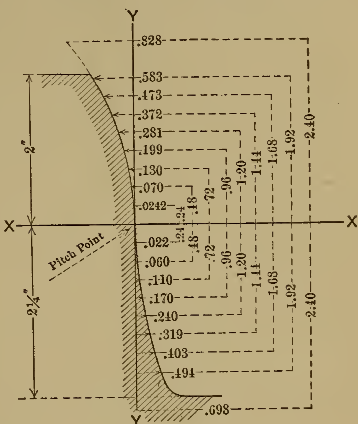


FIG. 327.

205. **Bevel Gearing.**—In the discussion on the teeth of spur-wheels, the motions were considered as taking place in the plane of the paper, and we have thus dealt with lines instead of surfaces. But the pitch and describing curves, and also the tooth outlines, are but traces of surfaces acting in straight-line contact, and having their elements perpendicular to the plane of the paper. In bevel gearing the pitch surfaces are cones, and the teeth are in contact along straight lines, but these lines are perpendicular to a spherical surface, and all of them pass through the centre of the sphere, which is at the point of intersection of the two pitch cones.

As in spur gearing an element of the same rolling cylinder generates the faces of the teeth of the driver, and the flanks of the teeth of the follower, so in bevel gearing an element of the same rolling cone, having its apex at the point of intersection of the axes of the two pitch cones, generates the faces of the driver's and the flanks of the follower's teeth and *vice versa*.

Let *o–uc* and *sct–o* (Fig. 328) be the pitch cones of a pair of bevel gears in contact on the line *oc*. Let *o–ec* be a third cone in contact externally with *sct–o*, and internally with *o–uc* on the line *oc*. The above three cones, being in contact on *oc*, will have their axes in a plane passing through *oc*. Suppose the bases of the cones to be circular portions of a spherical surface whose centre is at *o* and whose radius is *oc*, and let the three cones turn in rolling contact on *oc*, their axes being fixed; then the point *f* on the small cone will describe a spherical epicycloid *fh* on the spherical base of the cone *sct–o* produced, and a spherical hypocycloid *fg* on the base of the cone *o–cu*; or the element *co* of the cone *o–ce* would generate the proper surfaces for the faces of the teeth on *sct–o* and for flanks on *o–uc*, these surfaces being the same as those formed by allowing a right line to pass through *o* and move along *fh* and *fg* respectively as directrices. As the common normal plane to the two tooth surfaces generated by the above method always passes through the common element of contact of the two pitch cones, viz., *oc*, therefore they are suitable tooth surfaces, and will maintain by their sliding contact the same a.v. ratio as the pitch cones would maintain by rolling contact. Since the above method of drawing the shapes of bevel gears on a true spherical surface involves much labor, the following approximate method, given by Tredgold, is extensively used where absolute accuracy is not required.

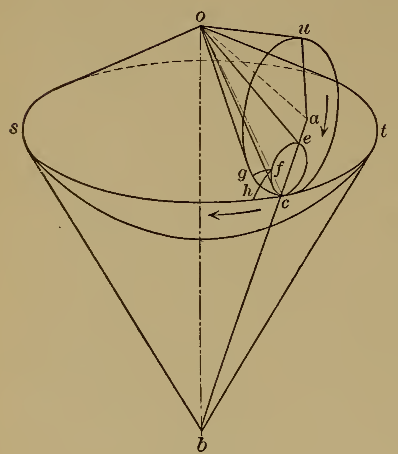


Fig. 328.

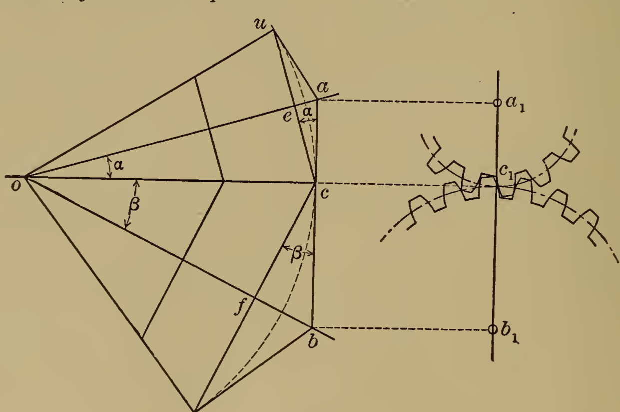


Fig. 329.

Tredgold's Approximation.—In the plane of the axes of the two rolling cones (Fig. 328) draw *acb* perpendicular to *oc*, intersecting the axes in a and b; then cb is an element of the cone b–sct, tangent to the sphere at the circle sct, and ac is an element of the cone a–cu, tangent to the sphere at the circle uc. Since narrow zones of the sphere near the circles sct and uc will so nearly coincide with cones tangent at these circles, the conical surfaces may be substituted for the spherical ones without serious error, and, as the tooth outlines are always comparatively short, they may be supposed to lie in the conical surfaces b–sct and a–cu, which are called the *normal cones*, they being normal to the pitch cones. These normal cones may now be developed, and the process of drawing the tooth outlines will be the same as for a pair of spur gears of the required pitch with ac and bc as radii. The method of drawing the normal cones and obtaining the tooth outlines is shown in Fig. 329, which is lettered the same as Fig. 328.

The demonstrations and methods belonging to the teeth of spur-

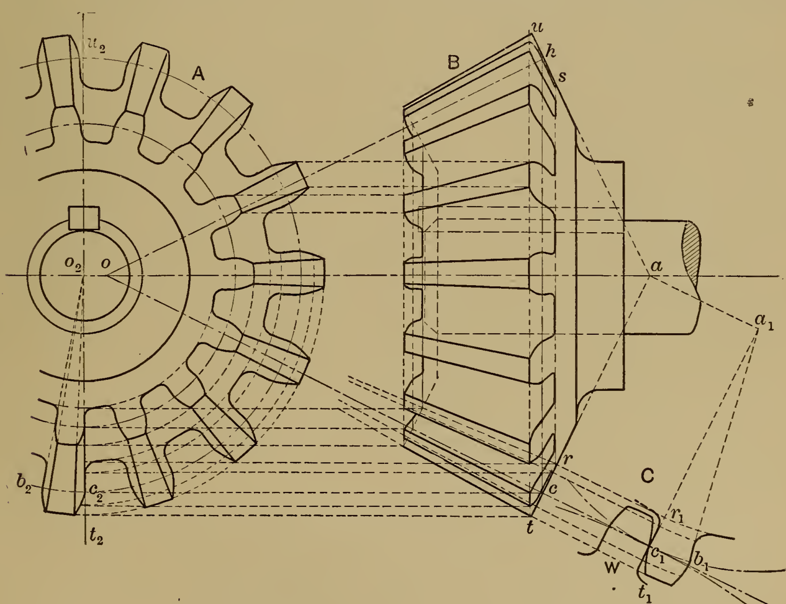


FIG. 330.

wheels may be applied to the development of bevel-gear teeth with the exception of pin gearing, which requires another system now obsolete.

Of the various forms of teeth, that is, radial-flank, parallel-flank, curved-flank, or involute teeth, the first form—radial-flank—is commonly used. Bevel gears being generally made in pairs, radial flanks are preferred as they have the simplest form and give the least obliquity of action. The involute tooth is now coming into use for cut bevel gears, and very good results have been obtained by its adoption.

The method of finding the tooth outline upon the normal cone graphically is shown in Fig. 330, where A is an end view, B a side view, of the gear-wheel, and C the development of the tooth outline. Since the tooth projects beyond the pitch circle to t_1, the normal cone will extend to t, which fixes the extreme diameter of the "blank" tu; projecting r_1 to at, we find the diameter of the projected root circle rs. The points t, c, r, etc., in revolving about the axis ao, describe circles which appear as straight lines in B, and as circles in A, projected in their true size. It is obvious that the length of the arc which measures the thickness of the tooth at the addendum, root, pitch, or any other projected circle will be the same in the end view A as in the development, which enables us to draw the outlines of the teeth in the end view as shown, all outlines being the same. From these outlines those on the side view B may be found by the principles of projections.

The teeth are limited at their smaller ends by another normal cone on which the outline will have the same form as on the large end. Since all elements of the teeth run to the vertex o, this second outline may be found from the first, the method of obtaining it being sufficiently indicated in the figure. W is the development of the tooth outline of a wheel to work with the one shown. Both wheels have radial flanks, and the development is conveniently located for drawing both wheels at the same time; one may be shown in gear with the other.

Construction of the Correct Tooth Outline.—Let doc (Fig. 331) be the pitch cone, fae the normal cone indefinitely extended; and let oh be the axis of a describing cone cob, tangent to the pitch cone on the line co, and intersecting the normal cone on the curve $crstb$. The right-hand figure is a projection on a plane at right angles to ao, and the circle cgd is shown in its true size, $c_1g_1d_1$; the curve of intersection appears as $c_1r_1s_1t_1b_1$, the method of obtaining two of its points being clearly shown.

Suppose the axes of the pitch and describing cones to be fixed, and suppose the cones to turn in rolling contact in the directions shown by the arrows; then the element co of the describing cone will sweep up the outline for the tooth face, and will always pierce the normal cone in some point of the curve ctb, the normal cone being fixed as far as its relation to the curve is concerned. Knowing the ratio of the bases of the pitch and describing cones, the angular motion of one can be found from that of the other, either graphically as shown in the figure, or better by calculation. If the pitch cone turns through an angle $c_1a_11 = mp1$, the describing cone will turn through an angle $mn1$, the element will be found at or, giving us the point r_1 and the small portion of the projected tooth outline $1r_1$. To find another point, suppose the turning to go on to 2, giving the point s_1, r_1 having now gone to u, a point found on the arc r_1u about a_1 by making the angle $r_1a_1u = 1a_12$. In the same way other points may be found, giving the projected outline $3vwt_1$ which is to be developed, as shown at D, before it can be applied to the normal cone to fix the tooth outlines. In order to accurately fix the outline, great care must be used and several points should be found intermediate to those shown in the figure.

In laying out the teeth of internal bevel wheels by Tredgold's process, it is evident that the size of the describing circles must be fixed with

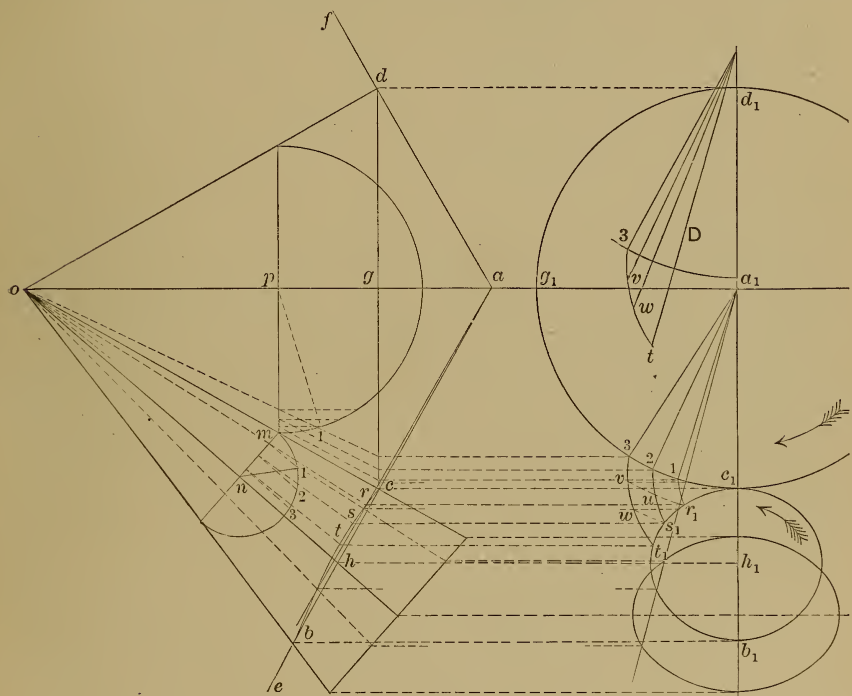


FIG. 331.

due regard to the limits obtained for annular spur-wheels, to prevent interference upon the development of the normal cone. From the above we deduce, as a safe practical rule for the size of the describing cones used in the exact process, that the diameters of their bases should not be greater than those of the describing circles used in the approximate method.

Cone and Flat Disc.—When a cone rolls on a flat disc the normal cone becomes a cylinder, and if Tredgold's process be applied, a case like that of a rack and spur-wheel presents itself. It is to be observed that if we start with such a flat bevel-wheel, making its developed rack teeth of equal curves in alternate reversion, any two bevel-wheels gearing with it will gear with each other (§ 201).

Involute Wheels.—Tredgold's process is here applied to the developed base circles of the normal cones. The exact outline of the teeth on the normal cone would be found by noting on it the intersecting path of a line carried by a plane, in rolling contact with two base cones, and turning on an axis passing through the apexes of the pitch cones perpendicular to the plane.

Method of Cutting the Teeth.—The tooth surfaces being conical, their outlines are constantly changing; it is then impossible to cut them accurately with an ordinary milling-cutter. This method, however, is often used for small bevel gears. To distribute the unavoidable errors as uniformly as possible, it is the practice to make the cutter conform to the middle section of the tooth, and to make it travel on the element of the tooth on the pitch cone, where the face and flank join, and at the same time along the root cone. The cutter is made narrower than the smallest space, and only one side of the tooth is cut at a time. With this method the flank of the tooth at the large end is too full and the face not full enough; at the small end the errors are reversed; the surfaces also are cylindrical and not conical as they should be. Messrs. Brown and Sharpe make the clearance the same at both ends of the teeth; the cutting angle is thus the complement of the face angle of the gear with which the one being cut is to work; they also shape the cutter to be correct for a point on the tooth one-third of its breadth from the large end.

The system of diametral pitch is also applied to small bevel gears, the same rules holding, it being always understood that by the pitch circle is meant the largest, or that of the base of the rolling pitch cone. There are, however, machines which will cut true bevel gears.

Twisted Bevel-wheels.—It is to be noticed that bevel gears may be stepped in the same way as spur gears, and the advantages arising would be the same; but there are practical reasons why this arrangement is not employed. The wheels may have the process of twisting applied to them as in twisted gearing; in such case the only objection is the difficulty of forming the teeth: as far as outline goes, any outline that is suitable before twisting will also be after twisting.

206. Screw Gearing.—Worm and Wheel.—The most familiar example is that of the Worm and Wheel, where the axes are situated in planes at right angles to each other, as shown in Fig. 332. Here let *o* be the centre of a pitch circle through *c*, and *tt* the pitch line of a rack. In the plane of the paper construct teeth on these pitch lines of any proper form for spur gearing. If now the rack outline be taken as the meridian section of a screw whose pitch is equal to that of the rack, one turn of the screw or worm will advance the wheel one tooth, just as though we considered the screw to act as a rack, and to be moved along its axis a distance equal to the pitch; the wheel being made very thin, the screw action of the successive equal meridian sections as they come into the plane of the paper is the same as that of a uniformly moving rack tooth driving the wheel. Hence the screw may be considered as a rack which advances by rotation along the axis *ss*. The line *tt*, revolving about the axis *ss*,

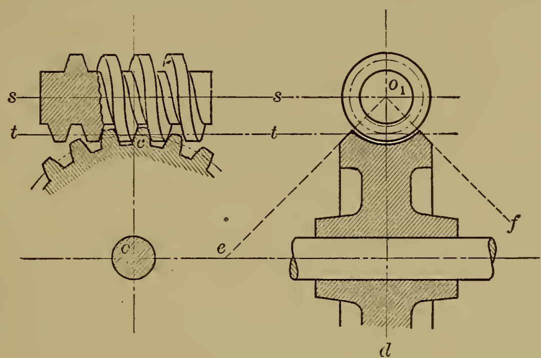


FIG. 332.

generates the pitch cylinder of the screw, and this is tangent to the pitch cylinder of the wheel at *c*.

The action in screw gearing may be distinguished from that of twisted gearing by three characteristics. 1° The velocity ratio depends wholly upon the screw pitch and not on the relative diameters of the pitch cylinders. 2° The directional relation depends upon the twist, one motion being given by a right-handed, and the opposite by a left-handed screw. 3° The end thrust of the screw causes the motion of the wheel.

To give the wheel of Fig. 332 sensible thickness, determine the common tangent plane of the two pitch cylinders at *c*, as shown at *MN* (Fig. 333), on which the helices of the screw will develop into straight parallel lines, as *ab*. The development of the screw helix passing through *c* will, when the plane *MN* is wrapped upon the pitch cylinder of the wheel, become another helix lying on that surface; these two helices tangent at *c* will be either both right-handed or both left-handed. Now consider the helix on the wheel's pitch cylinder as a directrix for the wheel section, shown in Fig. 332, the pitch point *c* moving along the helix;

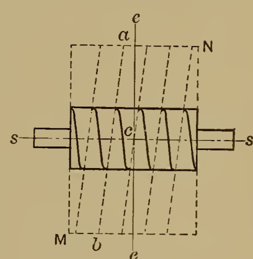


FIG. 333.

then the section will, in its motion, form a twisted wheel which will work with the worm. Here the teeth only touch on points in the central transverse plane and thus the wear is excessive. The thickness of the wheel depends upon its material and the pressure to be transmitted. The teeth of these wheels are sometimes cut straight across the wheel with an ordinary milling-cutter, at an angle with the elements of the pitch cylinder equal to that between *ab* and *ee*.

207. Close-fitting Worm and Wheel.—To make such a wheel, an exact copy of the screw is made of steel, and then it is fluted and hardened, similar to a tap, so as to become a cutting-tool, which may be used to finish the teeth, usually roughed out by the method of Fig. 333. Placing this cutting-tool in proper position in reference to the axis of the wheel, and in the notches previously made, it can be made to cut out the wheel by its rotation, the axes being pressed nearer together as the cutting goes on. Worm-wheel cutting-machines are now made where the wheel can be given the proper rotation in relation to the worm by independent mechanism. When the worm is allowed to cut all of the material away, no guiding notches being made, the wheel will have more teeth than wanted, as the cutting begins on a cylinder larger than the pitch cylinder; the tooth form is also unsatisfactory.

The involute form of tooth is usually applied in worm gearing, as it gives a straight-sided screw, and a change in the distance between the axes does not affect the velocity ratio.

For a close-fitting worm-wheel the blank is usually of the form shown at the right in Fig. 332, where the lines o_1f and o_1e through the axis of the worm describe cones on the axis *oe*, which limit the teeth.

Since all sections of a screw on planes parallel to and equidistant from its axis are alike, they will act the same as the meridian section of Fig. 332. This enables us to draw the outline for the teeth of a close-fitting worm, as shown in Fig. 334, where the view at the left corresponds to the section at the right of Fig. 332. The teeth of the wheel follow the circle of the worm through an angle 2α, which ought not to exceed 60°. The pitch point *o*, to secure the strongest tooth on the wheel, should be located half-way between *f* and *h*, in which case the teeth of the wheel will be cut away much less at their points than those shown. Now pass a plane through *cd* parallel to *ab*; it will cut from the screw the outline of a rack as shown at *B*, Fig. 334; the conjugate of this rack tooth will give the shape of the wheel's tooth on the plane *cd*. In the same way, other planes may be passed parallel to *cd*. The contour of the teeth on the conical sides of the wheel may be found by developing the cones and applying a method similar to Tredgold's, used in drawing bevel gears. The several sections found must be properly located relatively to each other, and a sufficient number of outlines will enable the wheel pattern to be made.

It has been found in practice that the worm-wheel, to give good results, should not have less than 25 teeth; the obliquity of action for an involute wheel tooth may be taken about 15°.

Hindley Worm.—Fig. 335 shows the close-fitting Hindley worm and wheel. Here the contour of the worm corresponds with that of the root circle of the wheel at its central plane. The worm is cut with a tool shaped to the contour of its thread (in this case straight-sided), but,

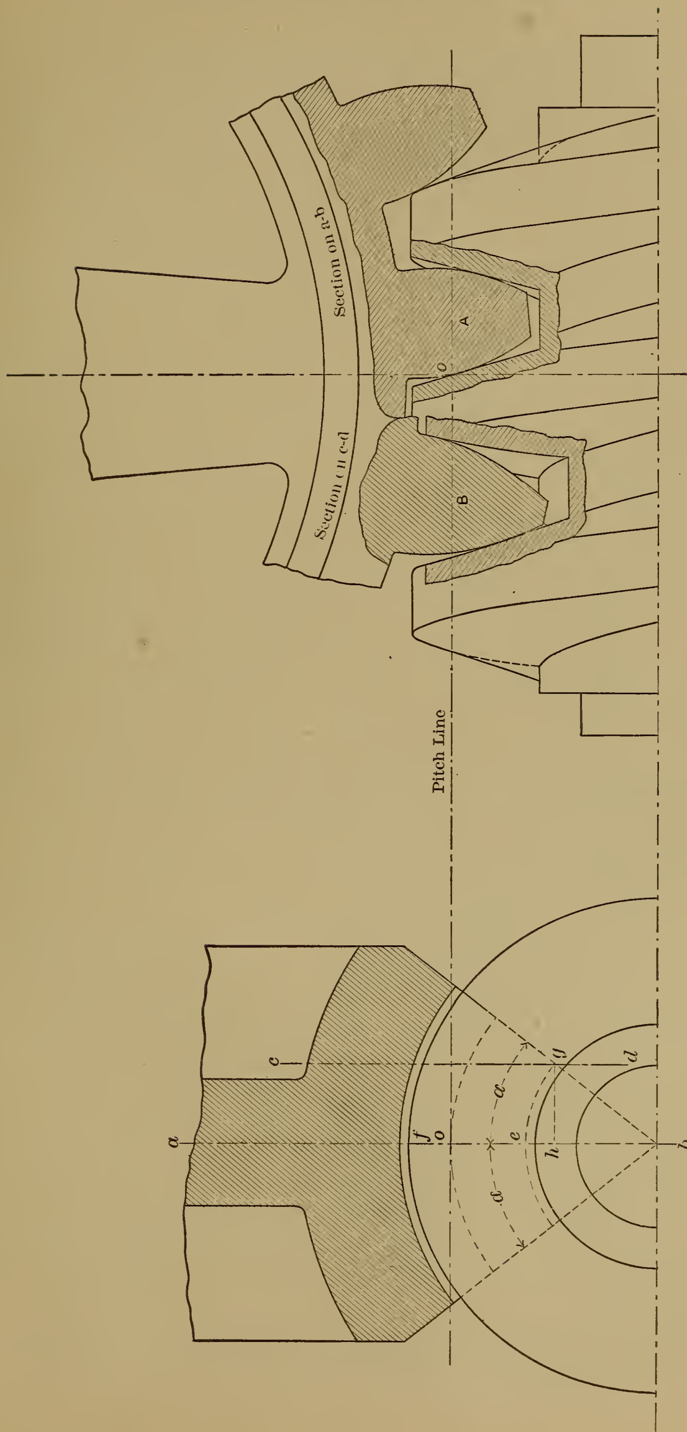


Fig. 334.

instead of being advanced uniformly parallel to the axis of the worm, the tool is here made to turn uniformly about an axis having the same position relative to the axis of the worm as the wheel to be driven. This angular motion for one rotation of the worm is the same as that desired in the wheel. After turning the worm it may be made to cut a close-fitting wheel in the manner previously described for the ordinary close-fitting worm and wheel. This worm when properly made has a greater bearing surface than the ordinary form, and hence the pressure and wear on the teeth of the wheel are both distributed and thereby reduced. It is extensively used in driving elevator drums.

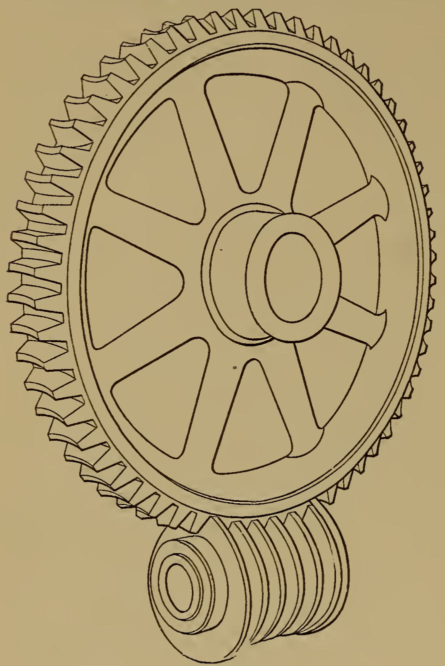


FIG. 335.

Close-fitting worms should always be well lubricated, and are for that reason usually placed under their wheels, so that they may run in a bath of oil, the worm and wheel being enclosed in a suitable tight casing.

Multiple-threaded Screw-wheels.—So far the screw has been supposed to be single-threaded, its pitch being that of the fundamental rack tooth. If now we double the helical pitch, the angular velocity of the thin wheel (Fig. 332) will be doubled, and only alternate teeth will come into action. To bring the remaining teeth into action, the screw can be made double-threaded, and this will at the same time reduce the pressure upon each tooth. In the same manner the helical pitch may be made any number of times as great as the tooth pitch, the number of threads being increased accordingly; the diameter of the screw in such case should be made great enough to avoid excessive obliquity of action. The screw may then have as many threads as there are teeth upon the wheel, or more; the combination will then appear as shown in Fig. 273. When the number of teeth on the wheel becomes infinite, the wheel becomes a rack, and its teeth will have an outline like a portion of an ordinary nut.

208. **Oblique Screw Gearing.**—The axis of the screw may cross the plane of the wheel obliquely, and give motion to the wheel by its end

thrust; the fundamental principle is here the same, the screw being still a rack which advances by rotation.

Oblique Rack and Wheel.—Suppose MN (Fig. 336) to be a broad plate with teeth cut across it parallel to ca and in gear with the wheel o_1. On moving the plate in the direction cb, the wheel o_1 will be turned through an angle depending on the component ce of the plane's motion, perpendicular to the axis of the wheel, and the sliding of the teeth, in a direction parallel to the axis of the wheel, will be represented by the component ca. The sections of the teeth on a plane perpendicular to the axis will be the same as in spur gearing, and their action will be the same, excepting the additional sliding ca. It is easily seen that the inclined rack, running between suitably shaped guides, will act the same as the wide plate. If ce is taken as the pitch arc of the wheel o_1, then cb will represent the movement of the rack while the wheel turns through the pitch angle.

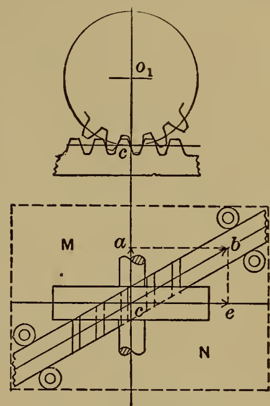


FIG. 336.

Oblique Worm and Wheel.—Assume the distance cb (Fig. 336) as the helical pitch of an oblique single-threaded worm to replace the rack. A worm thus constructed is shown in Fig. 337, where A is its pitch cylinder, B that of the wheel, and p the point of tangency. Let p be the present point of contact between a thread and a tooth, as shown at p' below, which gives an intersection of the wheel and worm on the plane lm normal to the axis of the wheel. Also let the section of the thread have the form of the rack tooth of Fig. 336, and make the wheel tooth conjugate to it. As the screw turns, there will always be a section of its thread, like p, similarly situated with respect to its axis, travelling along with uniform speed, as shown by the straight arrow, and advancing for one turn of the screw to the position o. To keep the velocity ratio constant, this moving section of the thread must always act on a tooth outline of the same form. Hence in every normal section of the wheel the teeth will have the same outline, and will be of the same length when the outside of the wheel is cylindrical.

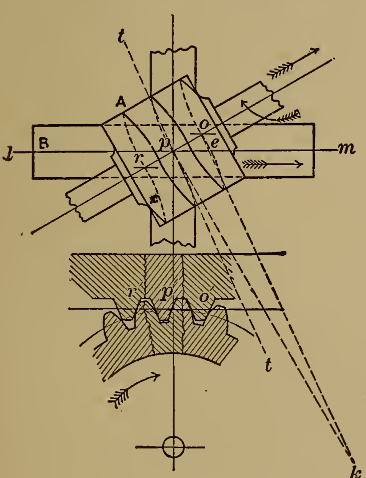


FIG. 337.

The twist of the wheel is here found by the same method as that shown in Fig. 333. Developing the helical line through p_1 (the pitch point), as shown at *pko* (*pk* being equal to the circumference of *A* and perpendicular to *po*), we have the angle *pok* that the common tangent *tt* of the helices in contact at *p* makes with *po*. The normal sections *r′* and *o′* of the wheel, on the planes through *r* and *o*, would be the same as *p′* and the wheel would thus be a simple twisted one if made from a cylindrical blank. The length of the pitch arc on the wheel is *pe*, corresponding to *ce* (Fig. 336), and this must be an aliquot part of the pitch circumference.

Action on Wheel.—It may be seen in Fig. 337 that one turn of the worm will drive the wheel through more than the original pitch angle, although the pitch of the screw is equal to the diagonal pitch of the rack in Fig. 336. In that case the rack tooth always acted against a surface with rectilinear elements perpendicular to the plane of rotation; but here the worm acts against helices of the same pitch, crossing the plane of rotation obliquely. The velocity ratio being constant, we may confine our attention to the helices upon the pitch cylinders, and study their action as represented in their developments on the common tangent plane to the pitch surfaces. In Fig. 338 let *cc* and *dd* be elements of the pitch cylinders of the wheel and worm respectively, intersecting at *p* and fixing the tangent plane; also let *po* be the pitch of the worm, *pk* its pitch circumference, and *ok* the developed helix, as in Fig. 337. *tt*, parallel to *ok*, is the common tangent to the two helices in contact at *p*. On one turn of the worm *p* goes to *o*, while the point *p* of the wheel must move in the direction *pf*. If we consider *pm* as the helix of the wheel, and suppose the screw to be pushed along *pd*, acting as a rack, *po* may be resolved into two components *pm* and *pe*. The first of these, *pm*, is simply a sliding component and is ineffective; but *pe* represents the linear motion of the wheel, due to the motion *po* of the screw. Now let *pk* represent the linear motion of the driving point *p* acting against the helix *pm*. Its normal and tangential components are *pg* and *pj* respectively. The motion of *p* in the wheel is along *pf*, and its normal component must also be *pg*, which would again give us *pe* as the linear motion of the wheel.

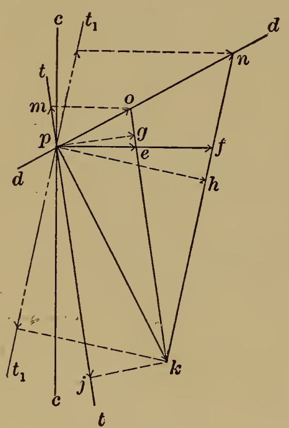


FIG. 338.

Suppose now that a double thread is desired upon the worm, without changing the subdivision of the wheel: in such case we must double the pitch arc *pe*, and then *pn* will be the pitch of the worm, found by making $pf = 2pe$, and drawing kfn. t_1t_1 is then the common tangent to the two helices, and ph the common normal component. Now in Fig. 338 both ko and kn will form right-handed helices upon the pitch cylinder of the screw; but on wrapping the tangent plane down upon the pitch cylinder of the wheel, kn will become a right-handed and ko a left-handed helix. Hence there must be an intermediate position in which the developed worm helix will be parallel to cc and will therefore become a rectilinear element of the wheel's pitch cylinder. Such a case is obtained by the proportions shown in Fig. 339 lettered the same as Fig. 338. If pe and the obliquity cpo are given, the pitch and circumference of the screw are found by drawing through e a perpendicular to pe, cutting po at o, and pk (a perpendicular to po) at k. If pe and pk are given, draw an arc about p with a radius pe and also draw po perpendicular to pk; then ke tangent to the arc fixes the pitch po and the obliquity cpo. The wheel in this case becomes a common spur-wheel, as shown in Fig. 340.

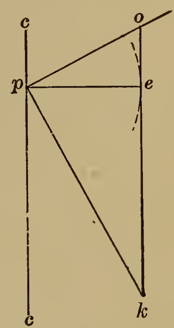


FIG. 339.

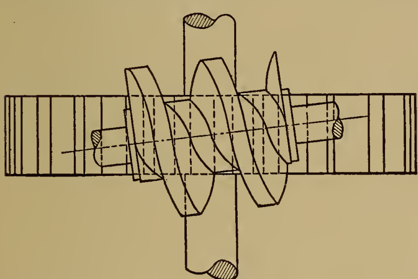


FIG. 340.

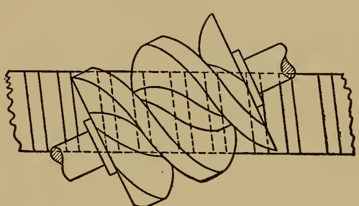


FIG. 341.

Oblique Screw and Rack.—If the diameter of the oblique worm-wheel be increased indefinitely, it will become a rack whose tooth surfaces are made up of rectilinear elements. Such a case is shown in Fig. 341.

An oblique worm could be made to cut its own wheel just as in the common case; the cylindrical blank may then be made to conform to the curvature of the screw, and the teeth be limited by conical frusta instead of transverse planes.

INDEX.

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